Final Exam - Micro I

(20 points) Decisions under Uncertainty:

Consider a decision maker who is an expected utility maximizer with a von Neumann-Morgenstern utility function $u(\cdot)$.

(a) Show that the following two statements are equivalent:

(i) The decision maker is risk averse.

(ii) The certainty equivalent of a random variable x which is distributed according to some cumulative distribution function $F(\cdot)$ is smaller than its expected value, i.e. $c(F) < \int x dF(x)$ for all $F(\cdot)$.

(b) Suppose that there are two assets, a safe asset yielding a return of 1 Euro per Euro invested and a risky asset with a random return of z Euros per Euro invested. The random return z has a cumulative distribution function $F(z)$ which satisfies $\int z dF(z) > 1$. The decision maker has initial wealth w to invest which can be freely devided between the two assets. Let α and β denote the amounts invested in the risky and safe asset, respectively:

(i) Show that a risk-averse decision maker will always invest a positive amount in the risky asset, i.e. $\alpha^* > 0$.

(ii) Consider a second expected utility maximizing decision maker with a von Neumann-Morgenstern utility function $v(\cdot)$. Assume that this second decision maker is strictly more risk averse than the first decision maker in the sense that $v(\cdot)$ is a concave transformation of $u(\cdot)$. Show that this second decision maker invests less in the risky asset than the first decision maker, i.e. $\alpha_v^* < \alpha_u^*$.

Final Exam - Microeconomics II, January 2015

Prof. Piero Gottardi

Problem 1

(13) Consider a production economy with two commodities: a consumption good and labor. The consumption good is produced by a (representative) firm, with the technology $y = \sqrt[2]{l}$, where y denotes the units of good produced and l the units of labor used as input. There is then a (representative) consumer with preferences over consumption and labor (hours worked) $U(c, L) = c(\bar{L} - L)$ and an initial endowment given by $\bar{L} = 24$ units of time, to be allocated to work or leisure. The consumer also owns the firm.

a. (5) Find the Pareto optimal allocations of this economy and represent them graphically.

b. (8) Find the competitive equilibrium allocation and prices of this economy.

Problem 2

(7) Consider a pure exchange economy under uncertainty with a single commodity and two states of nature (S=2). Suppose that each consumer h has VNM preferences $\mathbb{E}_\pi u^h(x)$ with identical beliefs π and $u^h(.)$ strictly concave, and that a complete set of Arrow securities is available for trade, at the prices $q(s)$, s=1,2. Explain why, if there is no aggregate risk, equilibrium prices are equal to the probabilities: $q(s) = \pi(s), \ \ s = 1, 2.$

John Moore’s Microeconomics Final Exam Questions

January 2015

Answer 2 out of the following 5 questions

(70 points)

Please note that each question is worth the same number of points

1. Consider an Akerlof model in which workers' individual productivities θ are uniformly distributed on [1, 10]. Each worker privately knows her own θ. A worker with productivity θ has an opportunity cost $r(\theta) = 1 + (4\theta/5)$. Firms are risk neutral and have additive technologies.

(a) What is the first-best allocation of labour, the allocation that maximises aggregate surplus?

(b) What is the laissez-faire equilibrium allocation of labour?

(c) To try to implement first-best, a benevolent government levies a tax on (or pays a subsidy to) every worker who works, and pays a subsidy to (or levies a tax on) every worker who does not work. Assume that the government must balance its budget. Show it cannot succeed in implementing first-best.

(d) Can the government nevertheless use some tax/subsidy scheme to increase aggregate surplus relative to the laissez-faire equilibrium allocation?

(e) How could a malevolent government use a balanced-budget tax/subsidy scheme to implement first-worst, the allocation that *minimises* aggregate surplus (the exact opposite allocation to first-best)?

2. Consider the following Spence signalling model. There are three equal-sized populations of workers, distinguished by which musical instrument they play as a hobby: accordionists (who play the accordion), bassoonists (who play the bassoon), and cellists (who play the cello). A worker privately knows which of these three types of musician he is – i.e. which type of instrument he plays. To employers, accordionists each have productivity 88, bassoonists each have productivity 100, and cellists each have productivity 160. All three types of musician can acquire a general-purpose music qualification by taking a single examination that is open to everyone. The private cost of taking this examination and acquiring the qualification is 50 to an accordionist, 45 to a bassoonist, and 40 to a cellist. The qualification has no intrinsic benefit. Employers compete in wages to hire a worker, after seeing whether or not he has the qualification.

(a) Find an equilibrium in which some workers acquire the qualification and others do not.

(b) Find a pure pooling equilibrium. Does it satisfy the Cho-Kreps Intuitive Criterion?

3. Consider a competitive screening model with many firms and workers. A fraction $\pi > ½$ of the workers are productive, each with productivity 1. The remaining workers are unproductive. Each of the productive workers is capable of doing a fixed task, at a cost c that is independently and uniformly distributed on (0, 1) across these workers. The task cannot be done by any of the unproductive workers, and has no intrinsic benefit. The timing of the model is as follows. First, each worker privately learns whether she is productive or unproductive; if productive, she also privately learns her c. Then, the firms offer contracts comprising a choice of wage w_1 or w_0, depending on whether or not the task is done. Finally, each worker chooses which contract, if any, to accept.

(a) Why is $w_1 = 1$, $w_0 = 0$ not an equilibrium?

(b) Find the equilibrium.

4. A monopolist, with 600 units of an indivisible good available, serves three types of customer, $i = 1$, 2 and 3, and there are 100 customers of each type. The monopolist cannot observe a customer's type. A type i customer derives monetary benefit of v_i per unit purchased, up to a capacity of 2 units, where $v_1 > v_2 > v_3 = 1$. Find the monopolist's revenue maximizing selling policy, as a function of v_1 and v_2, if she is unrestricted – i.e. she offers a menu of contracts, $\{(m_i, x_i)\}$, where a customer of type i pays a total amount m_i for x_i units of the good, and $x_i = 0, 1 \text{ or } 2$.

5. A risk-neutral Principal contracts to hire an Agent on a project. The project either succeeds and yields $\pi > 0$ dollars revenue, or fails and yields nothing. The Agent is risk-neutral with respect to non-negative income, but has no money of his own with which to pay anything out (i.e. negative wages aren't feasible). The Agent has a reservation utility (an outside opportunity) equivalent to v dollars. If the Agent accepts the Principal's contract, he then chooses an effort level which determines the probability f that the project succeeds. When the Agent exerts 1 dollar's worth of effort (a privately-observed non-cash expenditure, netted from his wage to calculate his utility), $f = 2/3$; whereas when the Agent exerts no effort, $f = 1/3$. (Only these two effort levels are feasible.) Find the optimal contract – that is, the wage w that the Principal pays the Agent if the project succeeds – as a function of π and v.

MICROECONOMICS IV: GAME THEORY

Exam January 2015

Jörgen Weibull

(No documents allowed.)

There are 3 main tasks, the first two carrying 33 points each, and the last carrying 34 points. The maximum total score is thus 100. Please divide your answers in 3 separate bundles, one for each main task, and please write clearly and concisely. You have to define formally the terms written in *italics* (no need to define more than once in the exam), and you have to clearly and correctly derive and motivate your answers. Please state your answers clearly! You may refer to established theorems and known results without proving them, but you then have to state these theorems/results precisely and correctly. Good luck!

1. Consider the following two-player game $G(a)$ with parameter $a \in \mathbb{R}$:

	A'	B'	C'
A	$2+a, 2+a$	$2,2$	$2,2$
B	$2,2$	$3,3$	$0,4$
C	$2,2$	$4,0$	$1,1$

For any given $a > 0$:

(a) Find all pure strategies that are *strictly dominated* (by a pure or mixed strategy).

(b) Find all *rationalizable* pure strategies.

For any given $a \geq 0$:

(c) Find all pure-strategy *Nash equilibria.*

(d) Find all pure-strategy *perfect equilibria.*

(e) Find all *evolutionarily stable* pure strategies.

2. Suppose that the game $G(a)$ in task 1, for some a such that $0 < a < 1$, is played twice with perfect monitoring between the two rounds, that is, the players play simultaneously within each round, and observe each others' actions in the first round before taking their actions in the second round. Suppose that each player's goal is to maximize the (expected) sum of his or her payoffs in the two periods. This defines an extensive-form game $\Gamma(a)$. Does there exist a *subgame-perfect equilibrium* (SPE) of $\Gamma(a)$ in which each player earns payoff 3 in both periods? Does the answer depend on a? Either prove that there is no such SPE or else specify precisely such an SPE, for any given $0 < a < 1$.

(a) Find the *pure-strategy minmax point* $v^o \in \mathbb{R}^2$ for the game $G(a)$ in task 1, for any given a such that $0 < a < 1$. Identify the set $V^*(a) \subset \mathbb{R}^2$ of *feasible* and *strictly individually rational* payoff pairs in $G(a)$ (still for $0 < a < 1$), and illustrate this set in a diagram.

(b) Now consider the infinitely repeated play of the stage game $G(a)$ (still for $0 < a < 1$), with perfect monitoring and with a common discount factor $\delta \in (0,1)$, for the case when players never randomize. State the one-shot deviation principle and apply it to answer the following question: For what range of δ-values can the payoff 3 be obtained by each player in each period on the path of a *subgame-perfect* strategy profile? Specify precisely such a strategy profile, based on temporary mutual minmaxing, and derive your upper bound on δ.

3. Consider a sender-receiver game in which there are two equally likely states of nature, $\omega = A$ and $\omega = B$. Player 1, the sender, observes (without error) the state of nature and sends one of two messages, a or b, to player 2 (who cannot observe the state). Having received 1's message, 2 takes one of two actions, α or β. Both players receive payoff 2 if action α is taken in state A, or if action β is taken in state B. Otherwise both players receive payoff zero.

   (a) Write up an extensive-form game, Γ, for this interaction, with "nature" (player 0) as first mover, and specify each player's set of behavior strategies.

   (b) Write up the pure-strategy (two-player) normal form G of the game Γ, in the form of a bimatrix with player 1 choosing row and player 2 choosing column. (Recall that nature is not a player in normal-form games.)

   (c) How many *sequential equilibria* of Γ does there exist in which both players obtain payoff 2 for sure? Either prove that there are none or specify precisely all such equilibria.

   (d) How many *sequential equilibria* of Γ does there exist in which each player obtains expected payoff 1? Either prove that there are none or specify one such equilibrium.

   (e) Find all *perfect equilibria*, in pure strategies, in the normal-form game G.