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The Zero Lower Bound on Deposit Rates, Monetary Policy and Bank Insolvency Risk*

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Abstract

I develop a banking model with monopolistic competition to analyze the effect of reserve rate policy on bank insolvency risk, when banks are constrained by a zero lower bound on deposit rates. When binding, the lower bound compresses the solvency relevant deposit spread and causes it to be a function of the policy rate, which results in the policy rate affecting bank default. The policy rate has an impact through two separate effects: The direct effect increases default probability unambiguously as a lower policy rate decreases the deposit spread for every realization of credit risk. By contrast, the (indirect) risk effect may increase or decrease default probability depending on the hazard function of credit risk. The risk effect arises because banks endogenously adjust their solvency threshold in response to a policy rate change. This novel result suggests that in a low interest rate environment, the relationship between competition in the banking sector, monetary policy and financial stability cannot be isolated from the underlying credit risk distribution.

JEL classification: D40, E43, E52, G21.

Keywords: Zero lower bound, Reserve rate policy, Bank insolvency, Credit risk, Monti-Klein, Deposit spread

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1 Introduction

In the aftermath of the 2008 Financial Crisis monetary policy entered into uncharted territory: Several central banks implemented a negative nominal interest rate policy for the first time in modern economic history.[1] This unprecedented policy experiment exposed that the relevant *zero* lower bound does not relate to the policy rate, but rather to deposit rates as many commercial banks did not fully pass-through negative interest rates to their depositors. Evidence of this zero lower bound on deposit rates is well documented by Eggertsson et al. (2024) for Switzerland, Sweden and Japan (among other countries).

How does the zero lower bound on deposit rates and the policy rate impact bank insolvency risk when banks have market power? This paper develops a partial equilibrium model of banking to answer this question and provides new insights by identifying a novel interaction between credit risk and the policy rate that arises due to the lower bound constraint.

The theoretical framework models monopolistically competitive banks subject to a zero lower bound on deposit rates. In order to asses the effect on bank solvency, I introduce credit risk arising from borrower default. The lower bound constraint introduces a kink into the deposit supply function which causes the relative deposit spread to become a function of the policy rate once the constraint binds. I find that this generates an effect of the policy rate on the probability of bank insolvency and I identify two separate effects that account for the total effect of the policy rate: There is a *direct* effect, which is related to the deposit spread, and this effect is always negative: The lower the policy rate, the lower the deposit spread, which reduces a bank's net value because it earns less on every unit of deposits. Hence lowering the policy rate increases the probability of bank insolvency for every realization of credit risk. But there is also an endogenous *risk* effect, which changes the set of credit risk realizations that trigger bank default, because banks respond optimally to a change in the policy rate by adjusting their solvency threshold. This risk effect is related to the hazard function and can be

[1] A brief overview about the implementation of negative nominal interest rates can be found in Section 2 of Berentsen et al. (2020).

positive or negative, depending on the particular credit risk distribution. Moreover, a negative risk effect can give rise to a reversal effect of the policy rate through the bank lending channel of monetary policy, such that policy rate cuts become contractionary.

Due to the interaction of both effects, the total effect of the policy rate on the probability of bank insolvency is theoretically ambiguous. This precise decomposition of the policy rate effect is a novel result and indicates that the relationship between competition in the banking sector, monetary policy and financial stability is complex and cannot be analyzed in isolation without taking into account the underlying credit risk distribution.

Related Literature. Following the seminal work of Klein (1971) and Monti (1972), the theoretical framework in this paper is a modified version of the canonical Monti-Klein model (an in-depth overview of the standard Monti-Klein model can be found in Section 3.2 of Freixas and Rochet (2008)). The research question however is closely related to two branches of research: The theory on banks' risk-taking implications of conventional interest rate policy and the literature studying the relationship between financial stability and competition in the banking sector.

Regarding the first branch of research, Dell'Ariccia et al. (2017) provide an extensive overview (important contributions include Adrian and Shin (2010), Borio and Zhu (2012) and Bruno and Shin (2015), among others): The relationship between safe rates (which entail policy rates) and banks' risk-taking behavior is theoretically ambiguous. Traditional portfolio theory suggests that when safe rates are lowered, then risky investments become more attractive and banks increase the overall risk exposure. This portfolio effect postulates a negative relationship between safe rates and risk-taking. On the other hand models of asymmetric information, as in the seminal paper on credit rationing by Stiglitz and Weiss (1981), predict a positive relationship. Credit risk may depend on loan rates, such that borrowers are more likely to default if loan rates are high. Because loan rates tend to be positively correlated with policy rates, the higher the policy rate, the more likely is borrower default. In line with the traditional portfolio theory, my model abstracts from asymmetric information. I contribute to this literature by showing that even absent any incentive problems like asymmetric information, the relationship between safe policy rates and the probability of bank failure, which captures banks' risk-taking, can be ambiguous and fundamentally depends on the underlying distribution of credit risk.

The second branch of research that is relevant is the literature analyzing the relationship between financial stability and banking competition. Early theoretical work by Keeley (1990) argues that in less competitive markets banks incentives for risk-taking are lower, because of comparatively higher monopoly rents and therefore more competition among banks increases bank failure. In contrast, Boyd and De Nicolo (2005) develop a theoretical model that incorporates the ideas from Stiglitz and Weiss (1981) such that the riskiness of loans is a positive function of loan rates. In their model, more bank competition suppresses loan rates and leads to a reduction in the probability of loan default. Therefore they find that more competition reduces the risk of bank failure. But Martinez-Miera and Repullo (2010) argue that this result does not necessarily translate to the (according to them) more realistic case of imperfect loan-default-correlation among borrowers and they show that in this case, the relationship between competition and the risk of bank failure is U-shaped. For the specific case of monopolistic competition Damjanovic (2013) also finds this U-shaped relationship. This is consistent with Allen and Gale (2004), where the authors consider a variety of theoretical models and emphasize that the relationship between competition and financial stability is complex and non-trivial. My work complements these findings by highlighting that this relationship is not obvious and conditional on the distribution of credit risk, but in contrast to the majority of the literature my research incorporates market power in both the lending and deposit market. These theoretical findings are in line with the empirical literature reviewed by Boyd and De Nicolo (2005), where they describe the empirical evidence as mixed.

Unsurprisingly, the effect of safe rates, financial stability and banking competition are closely interconnected, which ties both branches of research together. Martinez-Miera and Repullo (2020) find that in a Cournot environment, the effect of safe rates on the probability of bank failure depends on competition in the lending market via the pass-through of financing cost to loan rates. If financial intermediaries have high market power, then this pass-through is low and there is a positive relationship between the probability of bank failure and safe rates, whereas in very competitive markets with low market power this relationship is negative. In contrast to this result I do not find that the degree of competition alone is able to account for a change in this relationship.

A broader perspective on market power is provided by Wang et al. (2022), where they use structural estimation to find that the degree of bank market power explains much of the imperfect transmission of policy rates. Closely related to the transmission of policy rates is the more recent research on the limited pass-through of (negative) interest rates and the role of a zero lower bound. The theoretical analysis in my paper is motivated by the empirical evidence in Eggertsson et al. (2024) which documents a zero lower bound on deposit rates for households in Sweden.[2] This is in line with Heider et al. (2019), where they find that in the Euro area banks were reluctant to pass on negative rates to depositors. Consistently, Basten and Mariathasan (2018) and Schelling and Towbin (2020) find similar empirical evidence for Switzerland and Hong and Kandrac (2022) report this phenomena also for Japan. However Altavilla et al. (2022) suggest that there is heterogeneity among banks regarding the zero lower bound and the limited pass-through, because the transmission mechanism can be more impaired for less healthy banks. Such a dimension is not accounted for in my model, because due to symmetry, all banks will have the same balance sheet structure in equilibrium and thus there is no heterogeneity among them.

On the theoretical side Ulate (2021) provides both a static banking model as well as a dynamic DSGE model with New Keynesian features to study the transmissions effects of negative interest rates onto the economy. In the former model, the author introduces market power in lending and deposit markets as the key friction affecting the

[2]The work of Eggertsson et al. (2024) studies negative nominal interest rate policies in a DSGE model, but their benchmark specification assumes perfectly competitive banks. By including money storage costs and central bank reserves they can capture a disconnect between the policy and the deposit rate at the lower bound, which is different from a deposit spread that arises due to market power. Interestingly, in their Appendix D.9.2, when they introduce monopolistic competition in the deposit market, the deposit rate does no longer enter the first-order condition for lending, which is different to my model. The authors conclude that depending on the parametrization, under monopolistic competition negative interest rates can have positive or negative effects on the bank lending channel of monetary policy.

banking sector and a zero lower bound on deposit rates as a constraint. Market power is essential in order to have the bank lending channel of monetary policy at work, because only then are banks able to lower their lending rate when deposit rates are stuck at zero. In this static setup, interest rate cuts are never contractionary, which is no longer the case in the DSGE section of the paper, where the author introduces additional frictions to allow for a net-worth channel of monetary policy, which can give rise to contractionary effects. Such an reversal effect is studied in Abadi et al. (2023), where the authors theoretically derive the reversal rate at which accommodative monetary policy becomes contractionary for lending. In their framework, the key frictions are deposit market power and bank lending that is constrained by net worth. In my model there can also be a reversal effect in a low or negative interest rate environment, but the source of this reversal effect is fundamentally different because it is related to credit risk.

Lastly, since the existence of a deposit spread is crucial for the causal mechanism of the zero lower bound, my research is in line with the recent literature that acknowledges and emphasizes the deposit market structure and its role for banking. In Drechsler et al. (2017) the authors describe a deposit channel as a novel mechanism for monetary policy transmission and this channel arises due to market power. Furthermore Whited et al. (2021) find a quantitatively important mechanism, based on deposit market power, that can cause banks to take on more risks when interest rates are low.

The rest of this paper is organized as follows: Section 2 lays out the theoretical framework. Section 3 characterizes the equilibrium. Section 4 evaluates the impact of the zero lower bound, analyzes comparative statics of the policy rate and discusses the reversal effect and the role of market power. Section 5 concludes.

2 Model

I modify the canonical Monti-Klein setup – which is a nominal, static one period partial equilibrium banking model with non-competitive banks – along several dimensions:[3]

[3]Hence the model cannot capture certain aspects of banking such as maturity transformation. Furthermore, the one period setup emphasizes the short-term character of the model.

First, there are no costs of managing loan or deposit volume. Thus the deposit spread only arises due to market power, which helps to clearly identify the causal mechanism of the zero lower bound. *Second*, banks' market power is modeled using the Dixit and Stiglitz (1977) framework of monopolistic competition, which eases the formal derivations considerably. *Third*, I introduce a zero lower bound on the deposit rate following Ulate (2021). *Fourth*, I extend the model by incorporating exogenous loan default risk similar to Taggart and Greenbaum (1978), which endogenizes the probability of bank insolvency.[4] *Fifth*, in the balance sheet of a bank, the difference between its liabilities and its loans is not divided into cash and an interbank market position. In my model, this difference is solely given by a bank's reserve holdings at the central bank.

2.1 Environment

There is a continuum of monopolistically competitive financial intermediaries called banks, indexed by $j \in [0, 1]$. At the beginning of the period, each bank is endowed with an identical level of equity, denoted by E. In the lending market, the bank lends to an aggregate borrower and receives the gross lending rate $(1 + l_j)$. In the deposit market, it attracts deposits by paying the gross deposit rate $(1 + d_j)$ to an aggregate depositor. The role of the two aggregators is to collect bank individual quantities, aggregate them and engage in lending with a representative firm and attract deposits from a representative household. Both representative agents are not modeled explicitly.

Due to market power a bank faces a price-quantity trade-off in each market: Setting a higher loan rate reduces the quantity of loans while offering a higher deposit rate increases the quantity of deposits. Because of this price-quantity trade-off, a bank wants to hold a bounded loan and deposit position in its balance sheet. So at the beginning of the period, bank j observes the loan demand function $L_j(l_j)$, with $L'_j < 0$, and the deposit supply function $D_j(d_j)$, with $D'_j \geq 0$, and decides upon the loan and

[4] In Gordy (2003), the author discusses policy relevant frameworks to model loan portfolio risk. This is a complex topic in finance and there might not exist analytical closed form solutions for these frameworks. Therefore I opt for the more general approach as in Taggart and Greenbaum (1978), which has the advantage of being a multiplicative formulation of credit risk. This is complementary to the setup in Dixit and Stiglitz (1977), where the optimal loan (deposit) rate can be written as the product of a markup (markdown) over the gross policy rate, which is also a multiplicative formulation. This multiplicativity generates the tractability of the model.

deposit rate which maps into outstanding loan and deposit volume, $(1 + l_j)L_j$ and $(1 + d_j)D_j$ respectively.[5] Deposits D_j and equity E can be invested in either loans L_j or central bank reserve holdings H_j.[6] Central bank reserves pay a safe interest rate i that acts as the *policy rate* in this model. Similar to Eggertsson et al. (2024) and Ulate (2021), I assume that these reserves are non-negative, $H_j \geq 0$, and thus ignore borrowing from the central bank.[7] Therefore, at the beginning of the period, the balance sheet identity of bank j is given by

$$H_j + L_j = D_j + E. \tag{1}$$

At the end of the period, the bank receives the loan repayment and pays back its outstanding deposits. Here I introduce the notion of risk following Taggart and Greenbaum (1978): The loan repayment is stochastic and a bank might not receive full loan repayment, which potentially causes the bank to be unable to reimburse its outstanding deposit. At the end of the period, bank j receives the ex-ante uncertain loan repayment

$$\theta(1 + l_j)L_j, \tag{2}$$

where the random variable θ has support $[0, 1]$, a probability density function (PDF) $f(\theta)$, a cumulative probability function (CDF) $F(\theta)$ and an unconditional expectation $\mathbb{E}[\theta] = \int_0^1 \theta f(\theta)\, d\theta \in (0, 1)$, where the distribution is known to the bank. Hence θ is the stochastic share of performing loans at the end of the period.[8] Intuitively, we can think of $f(\theta)$ as the distribution of the project quality of a bank's risky investments. Note that the stochastic share θ is not bank specific, i.e. each bank will be repaid the same share. There is a threshold value of θ, denoted by $\hat{\theta}_j$, which determines the minimum loan revenue bank j needs in order to be solvent. This threshold is implicitly defined

[5] The weak inequality for the deposit supply function arises because of the zero lower bound on deposit rates; see the deposit supply function in (9).

[6] A bank will execute all of its market power to maximize its loan and deposit spread and any left over liabilities will be placed as central bank reserves.

[7] See e.g. footnote 7 in Ulate (2021).

[8] The multiplicative nature of θ in expression (2) makes the model tractable. Together with the multiplicative solutions for the loan and deposit rate, it implies that the key equilibrium equation of the model given by equation (20) is also *multiplicative*. This will allow to clearly separate the total effect of the policy rate into a risk and a direct effect.

by the *solvency constraint*:

$$\hat{\theta}_j(1+l_j)L_j + (1+i)H_j = (1+d_j)D_j. \tag{3}$$

The threshold $\hat{\theta}_j$ is the *solvency threshold*: Hence in this model, bank solvency refers to a bank's ability to fully reimburse depositors at the end of the period. Since a realization $\theta \leq \hat{\theta}_j$ triggers bank insolvency and causes a bank to default, $Pr(\theta \leq \hat{\theta}_j) = F(\hat{\theta}_j)$ is the *probability of bank insolvency*.[9] The solvency threshold $\hat{\theta}_j$ is endogenous, because it depends on the loan rate l_j and the deposit rate d_j, both of which are choice variables. Thus also the probability $F(\hat{\theta}_j)$ is endogenous and this probability is the measure of bank insolvency risk (or risk-taking) in the model.

Realize that in this setup the willingness to repay loans is never an issue, only the ability to do so. Without loss of generality, I assume that non-performing loans are lost resources and cannot be recovered by the bank – so in the case of bank insolvency, the bank seizes whatever is left and pays out its deposits as best as possible. I further assume that the remaining outstanding deposits are covered by a deposit insurance: If a bank has negative net value and is therefore not able to fully repay its deposits, then there is a transfer from a deposit insurance institution that covers the difference. Thus I implicitly assume that there are always enough resources in the economy to cover insurance transfers. So in every possible state of the world, at the end of the period the aggregate depositor always receives $(1+d_j)D_j$ from each bank and hence the deposit insurance causes deposits to be risk free. I abstract from modeling such an insurance institution explicitly as it is not needed to get the main insights.[10] Finally, note that credit risk θ does not depend on the loan rate. This implies that I also abstract from adverse selection that may arise in credit markets. Hence this setup does not account

[9] In this model a bank's solvency risk corresponds to its liquidity risk. This follows from the the one-period setup and because there is only deposit and equity funding.

[10] The main causal channel of the zero lower bound is not affected by this assumption and would not change, if e.g. we assumed a risk-independent deposit insurance premium. This would only change the level of the optimal deposit rate, but not the interaction of the ZLB, the policy rate and risk. See Appendix C for more formal details. Furthermore, Dermine (1986) considers different deposit insurance schemes and finds that a deposit insurance alters the independence result of the canonical Monti-Klein model only in case of an over- or underpriced risk-dependent premium to the extend that the loan rate becomes dependent on the deposit rate. But this dependency already occurs in my setup, because of the endogenous solvency threshold. Therefore also a risk-dependent deposit insurance scheme would not affect the main results. For further discussion on the independence result see Appendix E.

for any incentive problems as described in Stiglitz and Weiss (1981).

A bank behaves in the best interest of its equity holders: The objective of the bank is to maximize its end of period equity or net value NV_j, which is given by

$$NV_j = \theta(1 + l_j)L_j + (1 + i)H_j - (1 + d_j)D_j,$$

for a particular realization of θ. If we combine this equation with the solvency constraint in (3) we get the expression $NV_j = (\theta - \hat{\theta}_j)(1 + l_j)L_j$. This formulation of net value reveals that a bank's ex-post net value is affected by the realization of credit risk as long as $L_j \neq 0$ and that its net value is non-negative as long as $\theta \geq \hat{\theta}_j$.

Due to the stochastic nature of loans, the best a bank can do is to maximizes its expected end of period net value. Combining the equation for expected net value with the balance sheet identity in (1) yields the objective function:

$$\mathbb{E}[NV_j] = \int_{\hat{\theta}_j}^{1} \left[(\theta(1 + l_j) - (1 + i))\, L_j + (i - d_j)D_j + (1 + i)E\right] f(\theta)\, d\theta. \tag{4}$$

The fact that the integral is only evaluated from the solvency threshold $\hat{\theta}_j$ to 1 (rather than over the whole support of θ) is due to limited liability: For its optimal decision, a bank only considers the states of the world in which it is solvent.

Loan Demand and Deposit Supply. Following Dixit and Stiglitz (1977), monopolistic competition is modeled using a CES-framework: There is a final loan producer, which acts as the aggregate borrower, and a final deposit producer, which acts as the aggregate depositor. Each producer uses a CES-technology to aggregate bank individual quantities into an aggregate volume. The final loan producer borrows from each bank and lends the aggregate loan volume to a representative firm whereas the final deposit producer attracts funds in form of a deposit from a representative household and allocates it as individual deposits among all banks. Both the representative firm as well as the household are not modeled explicitly. Monopolistic competition in each market implies that loans and deposits are imperfect substitutes: The individual contribution from each bank to the final loan or final deposit is essential and cannot be fully substituted, but at the same time each bank is too small to affect aggregate interest rates and quantities.[11]

I assume that credit risk exogenously affects the representative firm and that the final loan aggregator fully passes through this risk to each bank. The CES-aggregation technology for loans is given by

$$L^A = \left(\int_0^1 L_j^{\frac{\varepsilon^l - 1}{\varepsilon^l}} \, dj \right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}},$$

where L^A denotes the aggregate loan volume. In equilibrium $\sigma_L \equiv \frac{\varepsilon^l}{\varepsilon^l - 1}$ corresponds to the *expected* relative markup, where ε^l is the elasticity of substitution with $\varepsilon^l > 1$.[12] The parameter σ_L also governs the curvature of the aggregation technology: In the limiting case $\lim_{\varepsilon^l \to \infty} \sigma_L = 1$, the aggregation technology is linear and loans from different banks are perfect substitutes – this is the case of perfect competition with marginal cost pricing. The other limiting case of full monopoly is achieved if $\lim_{\varepsilon^l \to 1} \sigma_L = \infty$ and hence monopolistic power implies $\varepsilon^l > 1$. I show in Appendix A.1 that in this setup the solution to the optimization problem of the final loan producer yields a standard CES-loan demand function:

$$L_j = \left(\frac{(1 + l_j)}{(1 + l^A)} \right)^{-\varepsilon^l} L^A, \tag{5}$$

where l^A is the economy wide loan rate. By imposing a zero-profit condition for the

[11] Even though there is empirical evidence for market power of banks, it is not obvious why deposits are not perfect substitutes. One rationale could be that there are bank specific benefits for customers (for instance better lending conditions). Another reason could be a certain idleness on the customer side due to switching cost. Regarding loans, one argument for imperfect substitution are syndicated loans: There, a consortium of banks grants a loan to a single borrower and hence each bank contributes to the final loan. The important take-away is that the CES-framework is a mathematically convenient, reduced form way to capture monopolistic competition, but it is no micro foundation for it.

[12] The parameter ε^l corresponds to the elasticity of substitution between a loan from bank j and bank k, which is given by the absolute value of $\frac{\partial\left(\frac{L_j}{L_k}\right)}{\partial\left(\frac{(1+l_j)}{(1+l_k)}\right)} \frac{\frac{(1+l_j)}{(1+l_k)}}{\frac{L_j}{L_k}}$ for $j \neq k$. The CES-framework also implies that this elasticity of substitution coincides with the gross price elasticity of loan demand, where the latter is given by the absolute value of $\frac{\partial L_j}{\partial(1+l_j)} \frac{(1+l_j)}{L_j}$.

final loan producer, we can derive the aggregate lending rate as

$$(1 + l^A) = \left(\int_0^1 (1 + l_j)^{1-\varepsilon^l} \, dj \right)^{\frac{1}{1-\varepsilon^l}}. \tag{6}$$

On the other hand, the aggregation technology for deposits is given by

$$D^A = \left(\int_0^1 D_j^{\frac{\varepsilon^d - 1}{\varepsilon^d}} \, dj \right)^{\frac{\varepsilon^d}{\varepsilon^d - 1}},$$

where D^A denotes the aggregate deposit volume, the parameter $\varepsilon^d < -1$ is the elasticity of substitution for deposits and $\sigma_D \equiv \frac{\varepsilon^d}{\varepsilon^d - 1}$ is the markdown.[13] The limiting case of perfect competition is $\lim_{\varepsilon^d \to -\infty} \sigma_D = 1$ and full monopoly power is achieved if $\lim_{\varepsilon^d \to -1} \sigma_D = \frac{1}{2}$. The deposit insurance ensures that the deposit side of the model is unaffected by risk and thus apart from the restriction on the elasticity of substitution parameter, the setup is formally identical to the one for loans. Therefore the optimal supply function for deposits is given by

$$D_j = \left(\frac{(1 + d_j)}{(1 + d^A)} \right)^{-\varepsilon^d} D^A, \tag{7}$$

where d^A denotes the aggregate deposit rate in the economy. Again, imposing a zero-profit condition for the final deposit producer implies the following aggregate deposit rate:

$$(1 + d^A) = \left(\int_0^1 (1 + d_j)^{1-\varepsilon^d} \, dj \right)^{\frac{1}{1-\varepsilon^d}}. \tag{8}$$

Banks take the aggregate quantities $L^A > 0$ and $D^A > 0$ as given.

Zero Lower Bound on Deposit Rate. In order to account for the zero lower bound (ZLB) on deposit rates, following Ulate (2021), I impose an exogenous bound on a bank's deposit rate. This ZLB introduces a kink into the deposit supply function and is crucial for the results. The ZLB modifies the deposit supply function in (7) such

[13] In contrast to the markup, the markdown exhibits no uncertainty because of the deposit insurance.

that D_j is a composite function:

$$D_j = \begin{cases} \left(\frac{(1+d_j)}{(1+d^A)}\right)^{-\varepsilon^d} D^A & \text{if } d_j \geq 0 \\ 0 & \text{if } d_j < 0. \end{cases} \tag{9}$$

Intuitively, if bank j sets a negative deposit rate it attracts no deposits from households (because they may hold cash instead of deposits). Notice that at $d_j = 0$, the bank still attracts positive deposits. The deposit supply function in (9) captures the ZLB without a micro-foundation for it and thus is a reduced form way to think about what is observed in the data: Commercial banks behaved *as if* they faced such a deposit supply function.

2.2 Bank Problem

A bank chooses the deposit rate d_j and the loan rate l_j in order to maximize its expected net value (4) subject to the solvency constraint (3), the deposit supply function (9), the loan demand function (5) and the non-negativity constraint on central bank reserves $H_j \geq 0$. The maximization problem that an individual bank j solves is therefore as follows:[14]

$$\begin{aligned} \max_{\{d_j, l_j\}} \mathbb{E}[NV_j] &= \int_{\hat{\theta}_j}^{1} \left[\left(\theta(1+l_j) - (1+i)\right) L_j + (i-d_j)D_j + (1+i)E \right] f(\theta)\, d\theta \\ \text{s.t.} \quad \hat{\theta}_j &= \frac{1}{(1+l_j)L_j}\Big((1+i)L_j - (i-d_j)D_j - (1+i)E\Big), \\ D_j &= \begin{cases} \left(\frac{(1+d_j)}{(1+d^A)}\right)^{-\varepsilon^d} D^A & \text{if } d_j \geq 0 \\ 0 & \text{if } d_j < 0, \end{cases} \\ L_j &= \left(\frac{(1+l_j)}{(1+l^A)}\right)^{-\varepsilon^l} L^A, \\ H_j &\geq 0. \end{aligned} \tag{10}$$

Optimal Deposit Rate. Because of the deposit insurance, uncertainty about borrower's loan repayments does not affect the deposit side of the model. If the zero lower

[14] I have substituted the balance sheet identity into the solvency constraint and into the objective function and hence H_j is no longer a choice variable.

bound on the deposit rate is not binding, the first-order condition corresponds to the standard market power optimality condition and yields the following solution for the optimal gross deposit rate:

$$(1 + d_j) = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i), \tag{11}$$

where $\sigma_D \equiv \frac{\varepsilon^d}{\varepsilon^d - 1} \in (0, 1)$ is the markdown relative to the gross reserve rate $(1 + i)$.[15] Realize that the expression in (11) constitutes the solution for the deposit rate as it characterizes the deposit rate only in terms of the exogenous variable i and the parameter ε^d. Note further that the deposit rate does *not* depend on the loan rate.

On the other hand, if the zero lower bound constraint is binding, then the best a bank can do is to set $d_j = 0$.

Optimal Loan Rate. The first-order condition for the loan rate implies that the solution for the optimal gross loan rate is characterized by

$$(1 + l_j) = \frac{\varepsilon^l}{\varepsilon^l - 1} \frac{1}{\mathbb{E}[\theta | \theta \geq \hat{\theta}_j]}(1 + i), \tag{12}$$

where the expectation of θ conditional on $\theta \geq \hat{\theta}_j$ is defined as

$$\mathbb{E}[\theta | \theta \geq \hat{\theta}_j] \equiv \frac{\int_{\hat{\theta}_j}^1 \theta f(\theta) \, d\theta}{1 - F(\hat{\theta}_j)}. \tag{13}$$

Realize that this expression is not yet the solution for the loan rate as it still depends on the endogenous solvency threshold $\hat{\theta}_j$. There are some interesting observations regarding optimality condition (12). *First*, the expected marginal revenue relative to marginal cost is given by $\frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}_j](1+l_j)}{(1+i)} = \frac{\varepsilon^l}{\varepsilon^l - 1}$. Thus $\sigma_L \equiv \frac{\varepsilon^l}{\varepsilon^l - 1} > 1$ is the expected markup, which is solely governed by a banks' market power on the lending side. *Second*, the conditional expectation arises due to the stochastic loan repayments – that is, because there is a non-zero probability that borrowers will default on their loan

[15] Recall that a bank *pays* the deposit rate to its depositors. Therefore market power on the deposit side manifests by the fact that the bank pays less to depositors relative to what it would pay under perfect competition.

contracts. Since $\mathbb{E}[\theta] \leq \mathbb{E}[\theta|\theta \geq \hat{\theta}_j] \leq 1$, the loan rate is higher when there is credit risk relative to a world without risk. This is in line with basic economic intuition: A bank prices in the possibility of borrower default. *Third*, the loan rate depends on the deposit rate through the solvency threshold $\hat{\theta}_j$ since this threshold depends on d_j. Therefore the classic *independence* result of the canonical Monti-Klein model – namely that a bank's optimal loan and deposit rate decision are independent – does no longer hold.[16] *Fourth*, the uniqueness of the solution for the loan rate depends on the uniqueness of the solvency threshold $\hat{\theta}_j$, because the conditional expectation is a function of this threshold. This implies that the only reason why banks would choose different loan rates are different solvency thresholds, since all other objects on the right-hand side of equation (12) are independent of j. In general, the solution for the solvency threshold may not be unique.

3 Equilibrium

There are three possible regimes depending on whether bank j sets a strictly positive, zero or negative deposit rate. Regime 1 corresponds to the unconstrained benchmark model of monopolistic competition, where the zero lower bound on deposit rates is not binding. In regime 2, the zero lower bound binds. There will be a upper threshold for the policy rate, denoted by $\bar{i}$, such that if the policy rate falls below this threshold, bank j switches from regime 1 to regime 2. The third regime applies when some banks do no longer want to attract deposits, which is the case if the policy rate falls below a lower threshold $\underline{i}$. I will not analyze this third regime – where a bank sets a negative deposit rate – in detail, because I focus on environments in which intermediaries accept deposit funding and act as traditional depository banks.[17]

In regime 1, each bank chooses the same deposit rate $d_j = d$ because the right-hand side of equation (11) is independent from bank-specific variables. This is also the case

[16] The fact that risky loans break the independence result of the Monti-Klein model is discussed in Dermine (1986). Even though I model credit risk formally different than Dermine (1986), the notion of risk is similar and the collapse of the independence result also occurs. I provide an extensive commentary on the relationship between the Monti-Klein model and credit risk in Appendix E.

[17] Moreover, in reality banks still accepted deposits and did not turn away depositors during the zero lower bound period. Some details on this third regime are discussed in Appendix A.2.

in regime 2, since each bank sets $d_j = 0$. But this symmetry does not necessarily apply for the loan rate: Even though its solution is characterized by optimality condition (12) in both regimes, the optimal loan rate depends on the solvency threshold and this threshold may not be unique. If there are multiple solutions for $\hat{\theta}_j$, then there is no mechanism in the model that pins down a specific solution. But since there is symmetric behavior among banks concerning the optimal deposit rate, it appears reasonable to assume that they also choose the same optimal lending rate (as banks are identical among all other dimensions). Thus we restrict the analysis to *symmetric equilibria* by imposing symmetry with respect to banks lending behavior:

Assumption 1. *All banks choose the same loan rate.*

Assumption 1 implies that all bank choose the same solvency threshold $\hat{\theta}$. This is an important conceptual difference between the optimal deposit and the optimal lending rate: For the former, symmetry is an outcome of the model while for the latter, we impose it.

The endogenous variables of the model are $\{(l_j, l^A, L_j), (d_j, d^A, D_j), H_j, \hat{\theta}_j\}$, the exogenous variables are $\{i, L^A, D^A, E\}$ and there are two parameters $\{\varepsilon^l, \varepsilon^d\}$. We can now define the equilibrium concept of this model:

Definition 1 (Symmetric Partial Equilibrium)**.** *Given Assumption 1, a symmetric partial equilibrium, conditional on the exogenous variables and parameters, consists of*

(i) the solution for the deposit rate given by expression (11) in regime 1 and given by $d_j = 0$ in regime 2,

(ii) the solutions for the other endogenous variables that solve conditions (3), (5), (6), (8), (9), (12) as well as the non-negativity constraint $H_j = D_j + E - L_j \geq 0$ for each regime,

where the solution for any endogenous variable is identical for each bank $j \in [0, 1]$.

I proceed by presenting the approach to solve for an equilibrium. Then I analyze the key equilibrium equation which characterizes the differences across regimes and allows to identify the impact of the zero lower bound on the equilibrium. Lastly, I discuss existence and uniqueness of equilibria. In Section B.1 of the Appendix, I provide the model solution for $f(\theta) \sim U[0,1]$ and also review other distributions.

3.1 Model Solution

3.1.1 Regime 1: $d_j > 0$

Deposits. When bank j sets a positive deposit rate $d_j > 0$, the zero lower bound on deposit rates is not binding. The solution for the optimal gross deposit rate is given by expression (11) and all banks choose the same deposit rate $d_j = d$. Since all banks behave identically, the CES-setup implies that a bank's individual rate and quantity is identical to the aggregate rate and quantity, respectively. From equation (8) it immediately follows that $d^A = d$ and from the deposit supply function in (7) it then follows $D_j = D^A$. Moreover notice from the optimization problem in (10) that the deposit spread is a relevant object in the solvency constraint. It is given by

$$(i - d) = \frac{1}{1 - \varepsilon^d}(1 + i) > 0. \tag{14}$$

Note that this deposit spread is strictly positive. Banks want to stay in this regime and set a strictly positive deposit rate $d > 0$ as long as the policy rate i is higher than the exogenous threshold $\bar{i}$:

$$\bar{i} \equiv -\frac{1}{\varepsilon^d}. \tag{15}$$

Since $\varepsilon^d < -1$, this threshold is strictly positive and strictly smaller than 1, i.e. $0 < \bar{i} < 1$. The upper threshold for the policy rate is solely driven by a bank's market power for deposits, since it only depends on the elasticity ε^d. Because the bank charges a markdown, the zero lower bound becomes binding when the policy rate is strictly positive.[18]

[18]Market power is the reason why the threshold is non-zero: Under perfect competition $\varepsilon^d \to -\infty$ and thus $\bar{i} \to 0$. Hence the ZLB would only bind if $i < 0$.

Loans. Given assumption 1, banks do not only choose the same deposit rate, but also the same loan rate. Hence equation (12) implies that they also choose the same solvency threshold $\hat{\theta}_j$. Therefore we omit the index j for both the loan rate and the solvency threshold, i.e. $l_j = l$ and $\hat{\theta}_j = \hat{\theta}$. The right-hand side of equation (12) is then independent of bank-specific choices. Using this symmetry argument, we can derive the aggregate gross loan rate from equation (6) as $l^A = l$ and the loan demand function in (5) yields the solution for loans as $L_j = L^A$. Due to its market power, it is optimal for the bank to hold a bounded amount of loans on which it earns a positive loan spread. The non-negativity constraint and the balance sheet identity then pins down the solution for central bank reserves, where the reserve holdings of bank j are given by $H_j = D^A + E - L^A \geq 0$. Each bank holds the same amount of central bank reserves and the non-negativity constraint implies $D^A + E \geq L^A$.

Solvency Threshold. To derive the solution for $\hat{\theta}$, we combine the solvency constraint as written in the maximization problem (10) and plug in the expression for the deposit spread in (14) to yield the equation that characterizes the solution for the solvency threshold:

$$\hat{\theta} = \frac{(1+i)}{(1+l)} \left(\frac{L^A - E}{L^A} - \frac{1}{1-\varepsilon^d} \frac{D^A}{L^A} \right).$$

This equation together with the optimality condition for the loan rate in (12) constitutes a system of two equations in the two unknowns $\{l, \hat{\theta}\}$. Combining both equations yields the key equation to determine the optimal solvency threshold:

$$\hat{\theta} = \mathbb{E}[\theta | \theta \geq \hat{\theta}] \cdot \kappa_1, \tag{16}$$

where $\kappa_1 \equiv \frac{\varepsilon^l - 1}{\varepsilon^l} \left(\frac{L^A - E}{L^A} - \frac{1}{1-\varepsilon^d} \frac{D^A}{L^A} \right)$. Note that κ_1 only contains exogenous variables and parameters. Hence equation (16) is one equation in one unknown $\{\hat{\theta}\}$. In regime 2, there will be a similar equation; see equation (18). Because these are the key equations to determine the existence and uniqueness of an equilibrium, we will study the characteristics of these two equations in more detail in Section 3.2.

3.1.2 Regime 2: $d_j = 0$

Deposits. The second regime prevails when $i \leq \bar{i}$ and therefore the zero lower bound on deposit rates is binding. The unconstrained optimization in regime 1 would prescribe that bank j sets a negative deposit rate, but the bank is constrained and thus the best it can do is to set a zero deposit rate, $d_j = 0$. Given the demand function in (9), the bank still attracts a positive amount of deposits. Note that there is still symmetry on the deposit side and hence $d_j = d$ and $d^A = d_j = 0 \; \forall \, j$. The deposit spread thus equals

$$(i - d) = i. \tag{17}$$

Due to symmetry and the CES-deposit supply function, the solutions for the economy wide deposit rate is $d^A = d$ and the solution for bank individual deposits is $D_j = D^A$.

Loans. The optimal loan rate is still characterized by equation (12). Only *the level* of the loan rate is affected as the solution for the loan rate depends on the deposit rate through the solvency threshold $\hat{\theta}$. Due to Assumption 1 and the CES-loan demand function, the solution for the endogenous variables are formally identical to regime 1 and thus $l^A = l, L_j = L^A$ and $H_j = D^A + E - L^A \geq 0$.

Solvency Threshold. To derive the solution for the solvency threshold, we again combine the solvency constraint as written in the maximization problem (10) and plug in the expression for the deposit spread in (17). The solution for $\hat{\theta}$ is therefore characterized by the following equation:

$$\hat{\theta} = \frac{1}{(1+l)} \left[(1+i) \left(\frac{L^A - E}{L^A} \right) - i \frac{D^A}{L^A} \right].$$

Again, combining this equation with the optimality condition for the gross loan rate yields the key equation to determine the optimal solvency threshold in regime 2:

$$\hat{\theta} = \mathbb{E}[\theta | \theta \geq \hat{\theta}] \cdot \kappa_2, \tag{18}$$

where $\kappa_2 \equiv \frac{\varepsilon^l - 1}{\varepsilon^l}\left(\frac{L^A - E}{L^A} - \frac{i}{(1+i)}\frac{D^A}{L^A}\right)$.

Regime Switch. An individual bank j could deviate from this regime 2-behavior, choose a negative deposit rate and attract no deposits.[19] There exist a cut-off value of the policy rate at which such a behavior becomes profitable and generates a higher expected net value, because accepting deposits is no longer optimal since the deposit spread is too low. I postpone the in-depth derivations to Appendix A.2, but analyzing such a deviating bank yields the threshold for the policy rate that induces a regime switch to regime 3. An individual bank j is better-off by not deviating as long as the policy rate is higher than the threshold $\underline{i}$:

$$\underline{i} \equiv \frac{-\left[1 + \frac{1}{\varepsilon^l - 1}\frac{L^A}{E} - \frac{\varepsilon^l}{\varepsilon^l - 1}\left(\frac{L^A}{E}\right)^{\frac{1}{\varepsilon^l}}\right]}{1 + \frac{1}{\varepsilon^l - 1}\frac{L^A}{E} - \frac{\varepsilon^l}{\varepsilon^l - 1}\left(\frac{L^A}{E}\right)^{\frac{1}{\varepsilon^l}} + \frac{D^A}{E}}. \tag{19}$$

This exogenous lower threshold for the policy rate is identical to the one found in Ulate (2021), where the author proves that $-1 < \underline{i} < 0$.[20] As mentioned by Ulate (2021), this lower threshold is only related to monopoly power on the lending market and not to monopoly power in the deposit market as only the parameter ε^l affects this threshold. The next figure summarizes at which policy rate the regime switches occur:

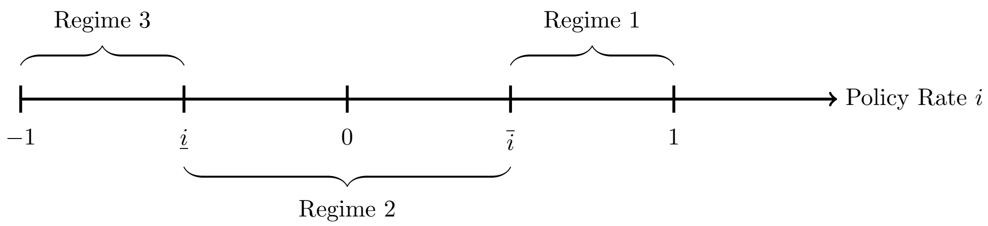

Figure 1: Regimes as Functions of the Policy Rate i

Notes: This figure depicts which regime prevails at given values of the policy rate i. The upper threshold $\bar{i}$ is given by expression (15) and the lower threshold $\underline{i}$ is given by expression (19).

[19] In regime 1, no bank has an incentive to deviate: Each bank choose the optimal deposit rate that maximizes its expected net value and this rate is strictly positive as long as $i > -1$.

[20] The author assumes $L^A > E$ in his model, which is also used in the proof.

3.2 Key Equilibrium Equation

The two key equations (16) and (18), which in essence represent the solvency constraints for each regime, can be generalized as follows: For each regime $R = \{1, 2\}$ the *key equation* that determines the solution for the solvency threshold $\hat{\theta}$ is given by

$$\hat{\theta} = \mathbb{E}[\theta|\theta \geq \hat{\theta}] \cdot \kappa_R, \tag{20}$$

where $R = 1$ if $i \geq \bar{i}$ with $0 < \bar{i} < 1$, and $R = 2$ if $\underline{i} \leq i < \bar{i}$ with $-1 < \underline{i} < 0$, and where

$$\kappa_R \equiv \frac{1}{\sigma_L}\left(\frac{L^A - E}{L^A} - \varphi_R \frac{D^A}{L^A}\right) \tag{21}$$

$$\varphi_R \equiv \frac{(i-d)}{(1+i)} = \begin{cases} \frac{1}{1-\varepsilon^d} > 0 & \text{if } R = 1 \\ \frac{i}{(1+i)} \gtreqless 0 & \text{if } R = 2 \end{cases} \tag{22}$$

$$\sigma_L \equiv \frac{\varepsilon^l}{\varepsilon^l - 1} > 1.$$

Across regimes, the only difference in the key equation (20) appears in the coefficient κ_R. This coefficient incorporates the exogenous variables and parameters of the solvency constraint in (3) that are *not* related to the distribution of θ. In particular, only the deposit spread *relative* to the gross policy rate, denoted by φ_R and defined in (22), differs across regimes: In regime 1, this relative spread is not a function of the policy rate, whereas in regime 2, it depends positively on the policy rate, i.e. $\varphi_2(i)$ with $\frac{\partial \varphi_2(i)}{\partial i} = \frac{1}{(1+i)^2} > 0$. Hence the relative deposit spread is the only object that depends on the policy rate, but only in regime 2. Note further that in regime 1 the relative deposit spread is strictly positive, i.e $\varphi_1 > 0$, since $\varepsilon^d < -1$. In regime 2 the sign of $\varphi_2(i)$ depends on the sign of the policy rate and thus if $i \neq 0$, then there is always a relative spread in this regime.

Result 1 (Relative Deposit Spread and Policy Rate). *The zero lower bound on deposit rates only affects the relative deposit spread in equilibrium. This solvency relevant deposit spread is given by* $\varphi_R \equiv \frac{(i-d)}{(1+i)}$ *for* $R = \{1, 2\}$.

(1.1) If the zero lower bound on deposit rates is not binding, then $R = 1$ *and* $\varphi_1 = \frac{1}{1-\varepsilon^d}$*.*

Therefore the relative deposit spread is independent of the policy rate.

(1.2) If the zero lower bound on deposit rates is binding, then $R = 2$ *and* $\varphi_2(i) = \frac{i}{(1+i)}$*.*

Therefore the relative deposit spread depends positively on the policy rate

This result determines how balance sheet positions interact with the policy rate: From the definition of κ_R in (21) we see that the ratio involving equity, $\frac{L^A-E}{L^A}$, is never proportional to the policy rate, whereas the deposit-to-loan ratio $\frac{D^A}{L^A}$ acts as a proportionality factor to φ_R.

Lastly, to gain economic intuition for the role of the relative deposit spread, notice that in (21) the coefficient in front of $\frac{D^A}{L^A}$ can be written as $\frac{\varphi_R}{\sigma_L} = \frac{1}{\sigma_L}\frac{(i-d)}{(1+i)}$. From the expression for the optimal loan rate given by (12), we realize that the denominator of this ratio is equal to the expected loan rate, conditional on being solvent – that is $\sigma_L(1 + i) = \mathbb{E}[\theta|\theta \geq \hat{\theta}](1 + l)$. Therefore the ratio $\frac{\varphi_R}{\sigma_L}$ is the deposit spread relative to the conditional, expected loan rate. This ratio is the solvency relevant interest rate object in the model and in this ratio, the relative deposit spread φ_R fully governs how the policy rate i affects the equilibrium.

3.3 Existence and Uniqueness

Whether equation (20) has a solution depends on both the properties of the conditional expectation $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$ as well as on the coefficient κ_R, where the coefficient must satisfy the following equilibrium condition:

Proposition 1. $0 \leq \kappa_R \leq 1$ *is a necessary condition such that a well-defined solution to equation* (20) *exists.*

Proof. See Appendix A.2.1. □

Both sides of the key equation can be expressed as a function of the solvency threshold: Let me denote the left-hand side by $v(\hat{\theta}) = \hat{\theta}$ and since $v'(\hat{\theta}) = 1 > 0$ it is strictly increasing in $\hat{\theta}$. The right-hand side also depends on the solvency threshold, because the conditional expectation is a function of $\hat{\theta}$, and I denote it by $g(\hat{\theta}) = \mathbb{E}[\theta|\theta \geq \hat{\theta}]\kappa_R$. A solution for the solvency threshold $\hat{\theta}$, call it $\hat{\theta}^*$, is a fixed point that satisfies $v(\hat{\theta}^*) = g(\hat{\theta}^*)$.

Since the function $g(\hat{\theta})$ is scaled by the coefficient κ_R, any solution $\hat{\theta}^*$ is regime specific. To understand the impact of the conditional expectation on the existence and uniqueness of a solution $\hat{\theta}^*$, emphasizing some of its properties is insightful:

Lemma 1. *The conditional expectation* $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$ *has the following properties:*

(i) Its lower bound is given by the unconditional expectation $\mathbb{E}[\theta]$ *and its upper bound by 1, that is* $0 < \mathbb{E}[\theta] \leq \mathbb{E}[\theta|\theta \geq \hat{\theta}] \leq 1$.

(ii) It is not smaller than the solvency threshold $\hat{\theta}$, *that is* $\hat{\theta} \leq \mathbb{E}[\theta|\theta \geq \hat{\theta}]$.

(iii) It is a non-decreasing function of $\hat{\theta}$, *that is* $\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}} \geq 0$.

Proof. See Appendix A.2.2. □

Statement (*iii*) of Lemma 1 implies that also the function $g(\hat{\theta})$ is non-decreasing. Since both functions $v(\hat{\theta})$ and $g(\hat{\theta})$ are non-decreasing in $\hat{\theta}$, it is not guaranteed that there exists a single value $\hat{\theta}^*$ such that $g(\hat{\theta}^*) = v(\hat{\theta}^*)$. Intuitively, since both functions are non-decreasing, the graphs of these functions do not necessarily intersect. Therefore there can be no, one or multiple solutions for the solvency threshold.[21] I address the two corner solutions in Appendix A.2; hence let us focus on interior solutions $0 < \hat{\theta}^* < 1$.[22] The case with multiple interior solutions to equation (20) is illustrated in the next figure:

[21] This maps into no, one or multiple solutions for the probability of bank insolvency $F(\hat{\theta})$ as well as for the gross loan rate; from equation (12) we know that the gross loan rate depends on the solvency threshold through the conditional expectation.

[22] $\hat{\theta}^* = 0$ is a knife-edge case (or the limiting case of loan market monopoly). Moreover $\hat{\theta}^* = 1$ cannot be optimal, because then the bank has negative net value in every state of the world and hence the bank would be better off by stopping to operate.

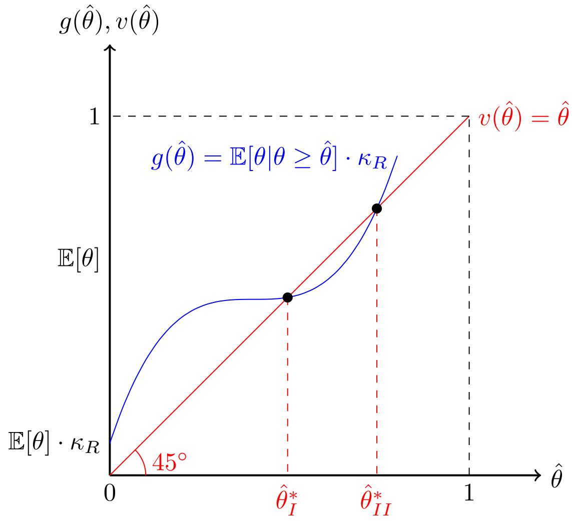

Figure 2: Non-Unique Equilibrium for Solvency Threshold $\hat{\theta}$

Notes: This figure qualitatively represents equation (20) for regime R. The function $v(\hat{\theta})$ is depicted in red and the function $g(\hat{\theta})$ is depicted in blue.

In Figure 2 there are two interior solutions $\hat{\theta}_I^*$ and $\hat{\theta}_{II}^*$ and hence the equilibrium is non-unique. The first solution $\hat{\theta}_I^*$ corresponds to an equilibrium with a low solvency threshold and a low probability of insolvency $F(\hat{\theta}_I^*)$ relative to the second equilibrium $\hat{\theta}_{II}^*$. Note that $\mathbb{E}[\theta|\theta \geq 0] = \mathbb{E}[\theta]$ whereas $\mathbb{E}[\theta|\theta \geq 1]$ is not well-defined, because then $F(1) = 1$ which implies that the denominator in the definition of the conditional expectation in (13) is zero.

The functional form of the conditional expectation $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$, which determines the specific shape of $g(\hat{\theta})$, is solely determined by the probability density $f(\theta)$. This emphasizes that different distributional assumptions about the share of loan repayments can give rise to different solutions for the solvency threshold and therefore affect the existence and uniqueness of the equilibrium. Realize however that in each regime, there is a unique solution for the deposit rate, because it is independent of the solvency threshold. In contrast, we see from expression (12) that the solution for the loan rate depends on $\hat{\theta}$ through $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$. Therefore in both regimes, the existence and uniqueness of the solution for the loan rate depends on the distribution of credit risk $f(\theta)$.

Lastly, although there can be multiple solutions for $\hat{\theta}$, Assumption 1 guarantees that each bank will choose the same solvency threshold. However in such a case, there is no mechanism to pin down which of these the multiple solutions is chosen and hence this is a situation with multiple symmetrical equilibria.

4 Analysis

4.1 Impact of the Zero Lower Bound

Notice from equations (14) and (17) that in both regimes, the absolute deposit spread $(i-d)$ depends on the policy rate i. However, as indicated by Result 1, for the solution of the solvency threshold $\hat{\theta}$ the *relative* deposit spread $\varphi_R \equiv \frac{(i-d)}{(1+i)}$ is relevant. The only impact of the zero lower bound is that it causes this relative deposit spread to be a function of the policy rate once the zero lower bound on deposit rates becomes binding in regime 2. Figure 6 in the appendix depicts the *relative* deposit spread as a function of the policy rate, but to gain intuition it is insightful to present the absolute deposit spread as illustrated in the next figure:

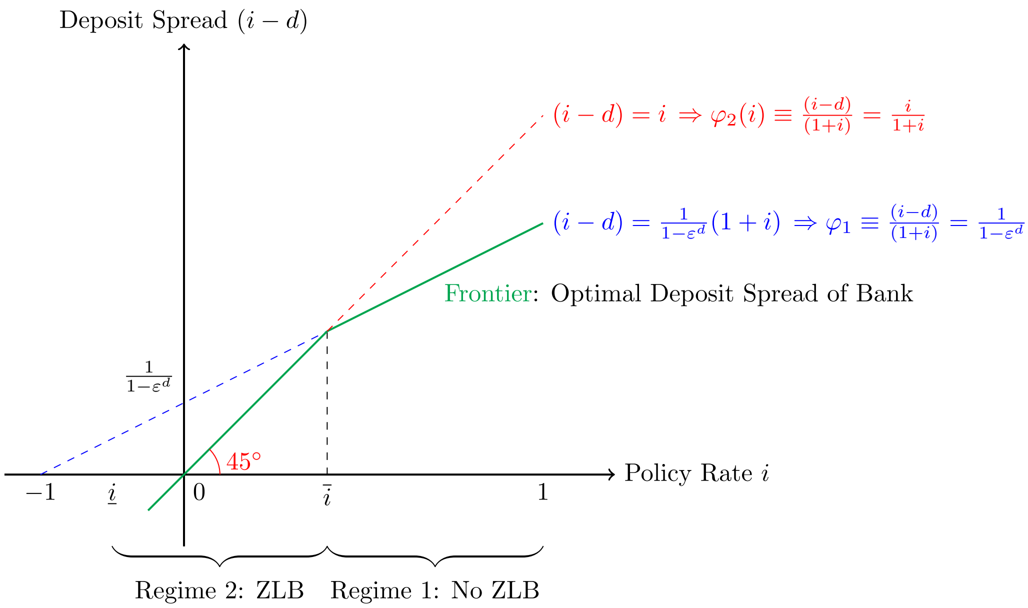

Figure 3: Deposit Spread as Function of Policy Rate

Notes: This figure depicts the solution for the deposit spread as a function of the policy rate for the interval $i \in [-1, 1]$.

In Figure 3, the deposit spread in regime 1 is plotted as the blue dashed line and given by $(i - d) = \frac{1}{1-\varepsilon^d}(1 + i)$, but the *relative* deposit spread is constant and given by $\varphi_1 \equiv \frac{(i-d)}{(1+i)} = \frac{1}{1-\varepsilon^d} > 0$.[23] Notice that in an unconstrained world, the deposit spread is strictly positive for any value of the policy rate $i > -1$. On the other hand, in regime 2, the deposit spread is given by $(i - d) = i$ and plotted as the red dashed line, which coincides with the 45-degree line. In this regime, the *relative* deposit spread is a function of the policy rate and given by $\varphi_2(i) \equiv \frac{(i-d)}{(1+i)} = \frac{i}{1+i}$. In this regime, if the policy rate is lowered below zero, then the (relative) deposit spread becomes negative. If the policy rate is negative, then for every unit of deposit a bank accepts, it loses money since the deposit spread is negative, i.e. $(i - d) = i < 0$. Why is it still optimal to obtain deposit funding in this case? Because attracting deposit allows the bank to extend its asset side of the balance sheet – especially their loan position as they still want to exploit their market power for loans. However, at some point this becomes too costly, precisely in regime 3 when $i < \underline{i}$, such that the bank stops accepting any deposits. In this case, banks would prefer to have a negative reserve position at the central bank, which is ruled out by assumption in my model as I impose a non-negativity constraint on central bank reserves $H \geq 0$. This is different to the basic Monti-Klein model, where banks can hold a positive or negative net interbank position.[24] To sum up, the green frontier in Figure 3 is the optimal deposit spread as a function of the policy rate. The zero lower bound on deposit rates introduces a kink into the optimal deposit spread function at $i = \bar{i}$ and compresses the deposit spread (relative to the unconstrained regime 1).

4.2 Impact of the Policy Rate

To study the main predictions of the model, using comparative statics we will evaluate for each regime how the policy rate affects the solvency threshold $\hat{\theta}$ in equilibrium. This allows us to implicitly study how the probability of bank failure $F(\hat{\theta})$ is affected:

[23] The bank can achieve a constant relative spread by adjusting its deposit rate in response to reserve rate changes. But when the zero lower bound is binding, the bank looses this margin as the deposit rate is set to zero for any value of the policy rate.

[24] A negative position means that a bank can be indebted at other banks or at the central bank; the latter would correspond to $H < 0$. As long as the interest rate on the net interbank positions is equal to the policy rate, the results are nor affected. However if banks could get funds from other institutions that cost them less than deposits, banks would not accept these costly deposits. In Figure 3, the green frontier would be identical to the x-axis for $i \leq 0$.

Recall that the derivative of the CDF is the PDF, $F'(\cdot) = f(\cdot) \geq 0$ and hence the probability of bank insolvency $F(\hat{\theta})$ is a non-decreasing function of $\hat{\theta}$. Therefore, if we know how an exogenous variable affects the solvency threshold, then we know if this variable increases or decreases the probability of bank failure. Given a solution for the solvency threshold $\hat{\theta}^*$, the *total effect* of the reserve rate i on this threshold, denoted by $\frac{d\hat{\theta}^*}{di}$, is given by

$$\underbrace{\frac{d\hat{\theta}^*}{di}}_{\text{Total Effect}} = \underbrace{\frac{d\hat{\theta}^*}{d\kappa_R}}_{\text{Risk Effect}} \times \underbrace{\frac{d\kappa_R}{di}}_{\text{Direct Effect}}.$$

The total effect is the result of an interaction between two different effects. The two differentials on the right-hand side allow to distinguish them: A indirect risk effect, characterized by $\frac{d\hat{\theta}^*}{d\kappa_R}$, which relies on the assumption about the distribution $f(\theta)$, and a direct effect of the policy rate that works through the relative deposit spread, characterized by $\frac{d\kappa_R}{di}$.

The intuition for the two effects is as follows: In regime 2, the direct effect of the policy rate is always negative: Ceteris paribus, the lower the policy rate, the lower the (relative) deposit spread and thus the lower a bank's net value, because it earns less on its deposits. This increases the probability of bank insolvency for every realization of credit risk. However, there is also the endogenous risk effect, which changes the set of credit risk realizations that induce bank default, because banks respond optimally to a change in the policy rate by changing their solvency threshold. Depending on the distribution, this adjustment of the solvency threshold can go in either direction and thus the risk effect is analytically ambiguous. In contrast, in regime 1 the direct effect of the policy rate is zero – because the solvency relevant deposit spread is constant and independent of the policy rate – and therefore the total effect is also zero.

I proceed by discussing the two effects formally and then present the results for the total effect.

Risk Effect. By totally differentiating the solution to key equation (20) with respect to κ_R, we get the following expression for the risk effect:

$$\frac{d\hat{\theta}^*}{d\kappa_R} = \frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{1 - \left(\kappa_R \times \frac{d\mathbb{E}[\theta|\theta\geq\hat{\theta}]}{d\hat{\theta}}|_{\hat{\theta}=\hat{\theta}^*}\right)}, \tag{23}$$

where one can show that

$$\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}} = h(\hat{\theta})\left(\mathbb{E}[\theta|\theta \geq \hat{\theta}] - \hat{\theta}\right) \geq 0 \tag{24}$$

and where $h(\hat{\theta})$ denotes the hazard function which is given by

$$h(\hat{\theta}) = \frac{f(\hat{\theta})}{1 - F(\hat{\theta})} \geq 0. \tag{25}$$

Because the numerator of the right-hand side in expression (23) is the conditional expectation, which is non-negative, the risk effect can be positive or negative depending on whether the product $\kappa_R \times \frac{d\mathbb{E}[\theta|\theta\geq\hat{\theta}]}{d\hat{\theta}}|_{\hat{\theta}=\hat{\theta}^*}$ in the denominator is bigger or smaller than one. Thus the risk effect is closely linked to the derivative $\frac{d\mathbb{E}[\theta|\theta\geq\hat{\theta}]}{d\hat{\theta}}$ and notice from the expression in (24) that this derivative is governed by the hazard function.[25] For a given κ_R, if the hazard function is sufficiently large, such that $\kappa_R \times \frac{d\mathbb{E}[\theta|\theta\geq\hat{\theta}]}{d\hat{\theta}}|_{\hat{\theta}=\hat{\theta}^*} > 1$, then the denominator of the right-hand side in expression (23) is negative which causes the risk effect to be negative. Crucially however, the hazard function is *not* restricted to be smaller than one since it is not a probability. Hence the size of the hazard function determines the sign of the risk effect.[26]

Proposition 2. *Given a solution* $\hat{\theta}^*$ *and the hazard function* $h(\hat{\theta}) = \frac{f(\hat{\theta})}{1-F(\hat{\theta})}$,

(i) $h(\hat{\theta}^*) < 1$ *is a sufficient, but not a necessary condition for a positive risk effect.*

(ii) $h(\hat{\theta}^*) > 1$ *is a necessary condition for a negative risk effect.*

Proof. See Appendix A.3.1. □

[25]Statement (i) and (ii) of Lemma 1 imply $0 \leq \mathbb{E}[\theta|\theta \geq \hat{\theta}] - \hat{\theta} \leq 1$. Therefore it is solely the hazard function $h(\hat{\theta})$ that can cause the derivative in expression (24) to be larger than one.

[26]Intuitively, the hazard function tells us – in case of solvency – how likely it is that the realization of θ was "close" to the solvency threshold $\hat{\theta}$, which is a notion of insolvency-hazard in this model.

Notice the subtle difference in statement (i) and (ii) of Proposition 2; for instance, if credit risk follows a uniform distribution, then $h(\hat{\theta}^*) > 1$ but the risk effect is positive (see Appendix B.1). The risk effect reflects the distributional assumption about credit risk, because the functional form of the hazard function is driven by this assumption.[27] The model endogenously determines $\hat{\theta}$, but for the size of the hazard function the PDF and CDF around this point, $f(\hat{\theta})$ and $F(\hat{\theta})$ respectively, are relevant. However, they are solely determined by the assumption about the probability density $f(\theta)$. Without specifying a particular distribution, the sign of the risk effect is therefore analytically ambiguous.[28]

Direct Effect. The direct effect of the policy rate on the coefficient κ_R is given by

$$\underbrace{\frac{d\kappa_R}{di}}_{\text{Direct Effect}} = \underbrace{\frac{d\kappa_R}{d\varphi_R}}_{\text{Spread Effect}} \times \underbrace{\frac{d\varphi_R}{di}}_{\text{ZLB Effect}}. \tag{26}$$

This direct effect of the policy rate arises due to the interaction of the relative deposit spread with the zero lower bound constraint, where $\frac{d\kappa_R}{d\varphi_R}$ is the effect of the relative deposit spread on κ_R and where $\frac{d\varphi_R}{di}$ captures the effect of the zero lower bound. The former is computed by differentiating the definition of κ_R in (21):

$$\frac{d\kappa_R}{d\varphi_R} = -\frac{1}{\sigma_L}\frac{D^A}{L^A} < 0.$$

Since $0 < \frac{1}{\sigma_L} < 1$ and $\frac{D^A}{L^A} > 0$, the effect of the relative deposit spread is strictly negative and not regime specific. Regarding the ZLB effect, we know from Result 1 that in regime 1 the relative deposit spread φ_1 is independent of the policy rate and in regime 2, $\varphi_2(i)$ is an increasing function of the policy rate on the interval $i \in (-1, 1)$. Therefore $\frac{d\varphi_1}{di} = 0$ and $\frac{d\varphi_2}{di} > 0$. Hence we can rewrite the direct effect in equation (26)

[27] This is another advantage of the multiplicative nature of the model: Since the right-hand side of equation (20) can be written as a product of the conditional expectation $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$ and the coefficient κ_R, we know that the *form* of the solution $\hat{\theta}^*$ only depends on the functional form of the conditional expectation, which in turn is solely determined by the distribution of θ.

[28] Since the risk effect is linked to the solvency threshold, the risk effect is also connected to the limited liability assumption.

as

$$\frac{d\kappa_R}{di} = \underbrace{-\frac{1}{\sigma_L}\frac{D^A}{L^A}}_{\text{Spread Effect}<0} \times \underbrace{\frac{d\varphi_R}{di}}_{\text{ZLB Effect}\geq 0} \leq 0.$$

Conditional on the negative spread effect, the next table presents an overview across regimes:

Regime	Relative Spread φ_R	ZLB Effect $\frac{d\varphi_R}{di}$	Direct Effect $\frac{d\kappa_R}{di}$
1	$\frac{1}{1-\varepsilon^d} > 0$	0	0
2	$\frac{i}{(1+i)} \gtreqless 0$	$\frac{1}{(1+i)^2} > 0$	$-\frac{1}{\sigma_L}\frac{D^A}{L^A} \times \frac{1}{(1+i)^2} < 0$

Table 1: Impact of ZLB across Regimes

Notes: For each regime $R = \{1, 2\}$, this table shows how the relative deposit spread φ_R and thus the coefficient κ_R depend on the policy rate $i \in (-1, 1)$.

Total Effect. In regime 1, the direct effect of the policy rate on κ_1 is zero, because the relative deposit spread is independent of the policy rate:

$$\frac{d\kappa_1}{di} = 0.$$

Hence the *total* effect of the policy rate on the solvency threshold is always zero independently of the risk effect:

$$\frac{d\hat{\theta}^*}{di} = \underbrace{\frac{d\hat{\theta}^*}{d\kappa_1}}_{\text{Risk Effect}\gtrless 0} \times \underbrace{\frac{d\kappa_1}{di}}_{=0} = 0.$$

Since the solvency threshold is not affected by the policy rate, this implies that the probability of bank failure $F(\hat{\theta}^*)$ is also not affected.

Result 2 (Neutrality of Reserve Rate Policy in Regime 1). *If the zero lower bound on the deposit rate is not binding, then in equilibrium the policy rate has no effect* *on the solvency threshold, independently of the distribution of credit risk. Therefore reserve rate policy does not affect bank insolvency risk.*

In contrast, in regime 2 the relative deposit spread does depend on the policy rate. Therefore

$$\frac{d\kappa_2}{di} = -\frac{1}{\sigma_L}\frac{D^A}{L^A}\frac{1}{(1+i)^2} < 0. \tag{27}$$

Hence in regime 2, the *total* effect of the policy rate on the solvency threshold depends on the sign of the risk effect:

$$\frac{d\hat{\theta}^*}{di} = \underbrace{\frac{d\hat{\theta}^*}{d\kappa_2}}_{\text{Risk Effect} \gtrless 0} \times \underbrace{\frac{d\kappa_2}{di}}_{<0} \lesseqgtr 0.$$

Result 3 (Non-Neutrality of Reserve Rate Policy in Regime 2). *In regime 2, there are two cases in equilibrium:*

(3.1) If the risk effect is positive, then the total effect of the policy rate is negative. Hence the solvency threshold depends negatively on the policy rate. Thus decreasing the policy rate increases the probability of bank insolvency: The lower the reserve rate, the higher the risk of bank failure.

(3.2) If the risk effect is negative, then the total effect of the policy rate is positive. Hence the solvency threshold depends positively on the policy rate. Thus decreasing the policy rate decreases the probability of bank insolvency: The lower the reserve rate, the lower the risk of bank failure.

The intuition for this result is depicted in the Figure 4.

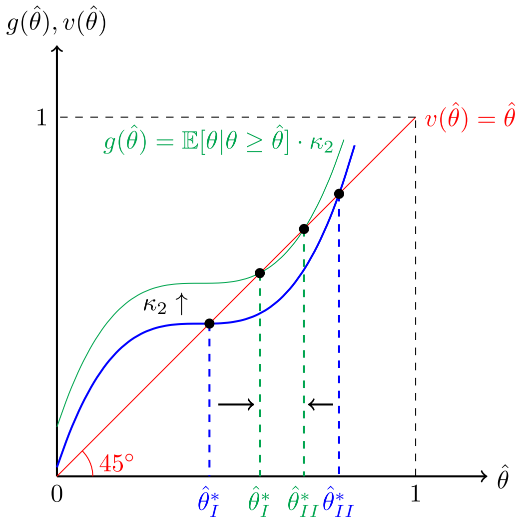

Figure 4: Illustration of Result 3

Figure 4 illustrates how both the risk effect, $\frac{d\hat{\theta}^*}{d\kappa_2}$, as well as the direct effect of the policy rate, $\frac{d\kappa_2}{di}$, impact the equilibrium value of the solvency threshold in regime 2. Both effects operate through the function $g(\hat{\theta}) = \mathbb{E}[\theta|\theta \geq \hat{\theta}]\kappa_R$. The risk effect is related to the slope of this function, which is not determined within the model, whereas the direct effect is related to the coefficient κ_R and shifts the graph upwards or downwards. Suppose that the initial situation is given by the blue schedule in Figure 4 with the corresponding two equilibria $\hat{\theta}^*_I$ and $\hat{\theta}^*_{II}$. Now consider a decrease in the policy rate $\Delta i < 0$. Since $\frac{d\kappa_2}{di} < 0$, this increases κ_2 and shifts the blue schedule up. The new schedule is depicted in green with the new corresponding equilibrium values.[29] Due to this exogenous shift in the policy rate, the first solution $\hat{\theta}^*_I$ increases and thus the probability of bank failure $F(\hat{\theta}^*_I)$ increases as well. This is the case when the risk effect is *positive* and corresponds to case (i) in Result 3. In contrast, the second solution decreases and so does the probability of bank failure, which is the case when the risk effect is *negative*. This demonstrates that the *risk effect is solution specific.*

4.3 Reversal Effect of the Policy Rate

Depending on the sign of the risk effect, the low or negative policy rate environment prevailing in regime 2 can cause a reversal effect of the policy rate such that interest

[29]Note that if the decrease in the policy rate – and thus the change in κ_2 – is large enough, the equilibrium can become unique or even non-existent.

rate cuts become contractionary.

Proposition 3. *If the zero lower bound on the deposit rate is binding and if*

1.) The risk effect is negative, i.e. $\frac{d\hat{\theta}^*}{d\kappa_2} < 0$,

2.) Central bank reserves holdings are strictly positive, i.e. $H > 0$,

3.) The bank lending channel is operative, i.e. $\frac{dl}{di} \neq 0$,

then the loan rate depends negatively on the reserve rate in equilibrium, i.e. $\frac{dl}{di} < 0$.

Proof. See Appendix A.3.2. □

Given Proposition 3 holds, a decrease in the policy rate (i.e. expansionary monetary policy) increases the loan rate. Suppose we impose more structure on the aggregate loan demand function:

Assumption 2. *The Aggregate loan demand* $L^A(l^A)$ *depends negatively on the aggregate loan rate, i.e.* $\frac{\partial L^A}{\partial l^A} < 0$.

In this simple model, only the bank lending channel of monetary policy is present and a reversal effect occurs if the loan rate is negatively correlated with the policy rate: If Proposition 3 holds, then lowering the policy rate increases the loan rate which reduces the loan demand a bank faces. Additionally, if Assumption 2 holds, then this translates into lower lending activity in the aggregate economy since in equilibrium $l^A = l$. Therefore there is a reversal effect of the policy rate: Decreasing the reserve rate is contractionary and reduces lending activity in the economy.

This reversal effect is related to the bank lending channel of monetary policy and this channel is closely related to market power in lending markets. See Appendix A.3 for more details on the bank lending channel.

4.4 Impact of Market Power

How does the market structure affect the equilibrium? After all, the deposit spread only arises due to banks' market power. Let us analyze what happens if we deviate from the monopolistic competition assumption by considering the limiting cases of perfect competition and full monopoly. In the model, market power manifests through the expected markup on the loan rate and the existence of a deposit spread. In the limiting case of perfect competition, there is marginal cost pricing, which implies $\sigma_L = 1$ for the expected loan rate markup and $\sigma_D = 1$ for the deposit rate markdown.

Loans. By using the definition of κ_R in (21), we can analyze the impact of loan market power as follows:

(i) Perfect Competition: $\lim_{\sigma_L \to 1} \kappa_R = \left(\frac{L^A - E}{L^A} - \varphi_R \cdot \frac{D^A}{L^A} \right)$.

(ii) Monopoly: $\lim_{\sigma_L \to \infty} \kappa_R = 0$.

Result 4 (Competition in Loan Markets). *If loan markets are perfectly competitive, then the zero lower bound affects the solvency threshold through the relative deposit spread* φ_R *in equilibrium. In contrast, if a bank has full market power, then the model collapses to the corner solution* $\kappa_R = \hat{\theta}^* = 0$.

Under monopoly, the model converges to the corner solution $\kappa_R = 0$, which in equilibrium is only consistent with $\hat{\theta}^* = 0$ meaning that the bank is always solvent. This implies that the conditional expectation attains its lowest value since $\mathbb{E}[\theta | \theta \geq \hat{\theta}^* = 0] = \mathbb{E}[\theta]$. Notice from the optimal loan rate in (12) that the lower the conditional expectation, the higher the gross loan rate: It makes sense that under monopoly the bank charges the highest possible loan rate and hence the gross loan rate is maximized if $\sigma_L \to \infty$.

Deposits. Assessing the market structure for deposits is more complex: Realize that the *degree* of deposit market power, which is related to the elasticity of substitution ε^d, only matters in regime 1 as it only affects φ_1 but not $\varphi_2(i)$. Thus in regime 2 only the *existence* of a deposit spread due to market power matters, but the *degree* of this market power is irrelevant (this is a result of the fact that in regime 2 the optimal deposit rate is always zero). This emphasizes the role of market power, because if banks operate in perfectly competitive deposit markets and have no market power at all, then they cannot set deposit rates which implies that there is no (relative) deposit spread, i.e. $\varphi_R = 0$ and $i = d$. In regime 1, the limiting cases are as follows:[30]

(i) Perfect Competition: $\lim_{\varepsilon^d \to -\infty} \kappa_1 = \frac{1}{\sigma_L}\left(\frac{L^A - E}{L^A}\right)$.

(ii) Monopoly: $\lim_{\varepsilon^d \to -1} \kappa_1 = \frac{1}{\sigma_L}\left(\frac{L^A - E}{L^A} - \frac{1}{2}\frac{D^A}{L^A}\right)$.

When there is no market power in regime 1, then the spread vanishes and all deposit related variables become irrelevant. We also see immediately that under perfect competition the coefficient κ_1 is strictly larger.[31]

Result 5 (Competition in Deposit Markets).

(5.1) If the zero lower bound on the deposit rate is not binding, then the degree of deposit market competition affects the equilibrium. The more competition, the higher the coefficient κ_1.

(5.2) If the zero lower bound on the deposit rate is binding, then only the existence of market power is relevant, but the degree of competitions does not affect the equilibrium.

[30] Recall from the optimal deposit rate given by expression (11) that the markdown σ_D is defined as

$$\sigma_D \equiv \frac{\varepsilon^d}{\varepsilon^d - 1} = \frac{1}{1 - \frac{1}{\varepsilon^d}} \in (0, 1).$$

The limiting values are $\lim_{\varepsilon^d \to -\infty} \sigma_D = 1$ and $\lim_{\varepsilon^d \to -1} \sigma_D = \frac{1}{2}$. The limit $\sigma_D = 1$ corresponds to the case of perfect competition, which implies marginal cost pricing and thus $d = i$.

[31] Therefore in regime 1, more competition in deposit markets is qualitatively isomorph to the analysis in Figure 4, because in both cases the coefficient κ_R increases, even though $R = 2$ in Figure 4.

5 Conclusion

This paper expands the canonical Monti-Klein model of banking by introducing credit risk from stochastic loan repayments and a zero lower bound on deposit rates. I perform a positive analysis to study the impact of this lower bound constraint and of the policy rate on bank insolvency risk. Monopolistic competition in deposits markets implies the existence of a deposit spread. When binding, the zero lower bound constraint makes the solvency relevant deposit spread an increasing function of the policy rate and causes an effect of the policy rate on bank solvency. This effect can be decomposed into two separate effects: For a given realization of risk, a lower policy rate increases the probability of bank default because of a lower deposit spread. This is the direct effect of the policy rate. However, there is also a risk effect as banks endogenously adjust their solvency threshold in response to a policy rate change – this alters the set of credit risk realizations that trigger bank default. Depending on the sign of this risk effect, which depends on the particular distribution of credit risk, the *total* effect of the policy rate on the probability of bank insolvency can be positive or negative. The novel result in this paper suggests that in a low interest rate environment, the link between the market structure of banking, monetary policy and financial stability cannot be isolated from the underlying credit risk distribution. The model retains generality because the results are independent of distributional assumptions about credit risk.

The analytical clarity of the model comes with a cost: It is limited to a static, partial equilibrium framework; however inter-temporal aspects and strategic interactions play an important role in banking. For instance, adverse information in loan markets may affect the risk effect because then loan repayments depend on loan rates and a richer model could account for other channels of monetary policy not present in my model. Further research could also embed the mechanism into a general equilibrium framework. Lastly, my model does not provide any micro foundations for the zero lower bound on deposit rates and thus formalizing such a foundation is a challenge for future research.

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A Appendix

A.1 Appendix to Section 2

Final Loan Producer. This paragraph clarifies that the final loan producer acts as the borrowing counter-party to banks. The precise notion of borrowing is as follows: With each bank j, at the beginning of the period the final loan producer engages in a loan contract $(1 + l_j)L_j$. That is, taking bank-individual loan rates l_j as given it chooses how much quantity L_j to borrow from each bank. Then using a CES-technology, it aggregates these loan quantities into an aggregated quantity L^A which it lends to a representative firm at the gross loan rate $(1 + l^A)$. The representative firm is not modeled explicitly and the final loan producer has no market power and therefore takes l^A as given. At the end of the period, uncertainty materializes and risk exogenously affects the representative firm as it only can repay a share $\theta(1 + l^A)L^A$ of its outstanding obligation to the aggregator. The final loan producer fully passes this risk through and only repays a share $\theta(1 + l_j)L_j$ to each bank. Formally, the final loan producer solves the following optimization problem:

$$
\begin{aligned}
&\max_{\{L_j\}} \int_0^1 \theta(1 + l^A)L^A f(\theta)\, d\theta - \int_0^1 \left(\int_0^1 \theta(1 + l_j)L_j f(\theta)\, d\theta \right) dj \\
&\text{s.t.} \quad L^A = \left(\int_0^1 L_j^{\frac{\varepsilon^l - 1}{\varepsilon^l}}\, dj \right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}} .
\end{aligned}
$$

Given that the stochastic loan repayments do not depend on the loan rate, we can rewrite the objective function of the final loan producer as

$$
\begin{aligned}
&(1 + l^A)L^A \underbrace{\int_0^1 \theta f(\theta)\, d\theta}_{=\mathbb{E}[\theta]} - \int_0^1 (1 + l_j)L_j \underbrace{\left(\int_0^1 \theta f(\theta)\, d\theta \right)}_{=\mathbb{E}[\theta]} dj \\
&= \mathbb{E}[\theta] \left((1 + l^A)L^A - \int_0^1 (1 + l_j)L_j\, dj \right),
\end{aligned}
$$

which reveals that the unconditional expectation $\mathbb{E}[\theta]$ multiplicatively scales the objective function and therefore will cancel out in the first-order condition.

Final Deposit Producer. The final deposit producer takes bank-individual deposit rates d_j and the aggregate deposit rate d^A as given and choose how much quantity of deposits D_j to place at each bank. It then aggregates D_j into the aggregate quantity D^A using a CES-technology. A representative household, which is not modeled explicitly, holds the deposit volume D^A at the final deposit producer and at the end of the period, this household is reimbursed the aggregate deposit contract $(1+d^A)D^A$. Formally, the final deposit producer solves the following optimization problem:

$$\begin{aligned} \max_{\{D_j\}} \quad & \int_0^1 (1+d_j)D_j \, dj - (1+d^A)D^A \\ \text{s.t.} \quad D^A = & \left(\int_0^1 D_j^{\frac{\varepsilon^d - 1}{\varepsilon^d}} \, dj \right)^{\frac{\varepsilon^d}{\varepsilon^d - 1}}. \end{aligned}$$

We can see how the implicit assumption about fully insured deposits simplifies the model considerably: Since the deposit insurance guarantees that the deposit contracts are safe and thus repaid in every state of the world (i.e. every draw of θ leads to full re-imbursement of deposits even though the representative firm defaults on its outstanding loan), the final deposit producers does not have to take into account any uncertainty and therefore just aggregates bank-individual deposits over all banks.

A.2 Appendix to Section 3

First-Order Conditions. It is helpful to rewrite the objective function:

$$\mathbb{E}[NV_j] = (1+l_j)L_j \int_{\hat{\theta}_j}^1 \theta f(\theta) \, d\theta + \big((i-d_j)D_j + (1+i)E - (1+i)L_j\big) \int_{\hat{\theta}_j}^1 f(\theta) \, d\theta.$$

Realize that combining the balance sheet identity (1) with the solvency constraint (3) yields $-\hat{\theta}(1+l_j)L_j = (i-d_j)D_j + (1+i)E - (1+i)L_j$. Furthermore,

$$\begin{aligned} \int_{\hat{\theta}_j}^1 \theta f(\theta) \, d\theta &\equiv \mathbb{E}[\theta, \theta \geq \hat{\theta}_j], \\ \int_{\hat{\theta}_j}^1 f(\theta) \, d\theta &= 1 - F(\hat{\theta}_j) \end{aligned}$$

and hence the objective function can be written as

$$\mathbb{E}[NV_j] = (1 + l_j)L_j \left(\mathbb{E}[\theta, \theta \geq \hat{\theta}_j] - \hat{\theta}\big(1 - F(\hat{\theta}_j)\big)\right). \tag{28}$$

Next we rewrite $\mathbb{E}[\theta, \theta \geq \hat{\theta}_j]$ using integration by parts:

$$\mathbb{E}[\theta, \theta \geq \hat{\theta}_j] \equiv \int_{\hat{\theta}_j}^{1} \theta f(\theta)\, d\theta = 1 - \hat{\theta}_j F(\hat{\theta}_j) - \int_{\hat{\theta}_j}^{1} F(\theta)\, d\theta.$$

Taking the derivative of this expression with respect to the solvency threshold yields

$$\frac{d\mathbb{E}[\theta, \theta \geq \hat{\theta}_j]}{d\hat{\theta}_j} = -\hat{\theta}_j F'(\hat{\theta}_j).$$

Since the solvency threshold depends on the loan and the deposit rate, the previous differential is helpful to get the optimality conditions. Using this information, from the objective function in (28), we can derive the first-order conditions, which for the loan rate is given by

$$\big((1 + l_j)L_j' + L_j\big)\mathbb{E}[\theta, \theta \geq \hat{\theta}_j] - \big(1 - F(\hat{\theta}_j)\big)(1 + i)L_j' = 0.$$

For the deposit rate, many terms cancel out and therefore the first-order condition is given by the standard market power optimality condition:

$$(i - d_j)D_j' - D_j = 0.$$

Upper Threshold $\bar{i}$. We see from equation (11) that the deposit rate is given by

$$d_j = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i) - 1.$$

Thus $d_j > 0$ is equivalent to $\frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i) - 1 > 0$ which can be solved for i to get

$$i = -\frac{1}{\varepsilon^d} \equiv \bar{i}.$$

Lower Threshold $\underline{i}$. Consider a bank j that deviates and sets $d_j < 0$ while all other banks still set $d = 0$. Taking the behavior of all other banks as given, a deviating bank attracts zero deposits $D_j = 0$ and thus its balance sheet identity is given by $H_j + L_j = E$. Therefore its maximal loan position is restricted by its equity and if $L_j = E$, then $H_j = 0$. Suppose $L_j = E$. Such a deviating bank still has to satisfy the loan demand it faces and lends out all of its available funds. Hence by solving the loan demand function (5) for the gross loan rate of a deviating bank, denoted by $(1 + l_j^{dev})$, we get

$$(1 + l_j^{dev}) = (1 + l^A) \left(\frac{E}{L^A} \right)^{-\frac{1}{\varepsilon^l}}. \tag{29}$$

Due to monopolistic competition, a deviating bank is too small to have an effect on aggregate variables and takes them as given. Therefore, given Assumption 1, we see from equation (6) that the aggregate loan rate is still identical to the loan rate of non-deviating banks, i.e. $(1 + l^A) = (1 + l)$. Using this result in equation (29) we get the expression for the gross loan rate of a deviating bank as follows:

$$(1 + l_j^{dev}) = (1 + l) \left(\frac{L^A}{E} \right)^{\frac{1}{\varepsilon^l}}. \tag{30}$$

Realize that $L^A > E$ is a sufficient condition such that the deviating bank charges a higher loan rate compared to non-deviating banks. Since $l > i$, this condition also implies $l^{dev} > i$. Therefore such a deviating bank would indeed not hold any central bank reserves and invests all its funds into loans.

The intuition for threshold $\underline{i}$ is to compare the expected return on equity (ROE) of a deviating bank to the one of a non-deviating bank in regime 2. For any bank, the expected end of period equity is equal to its expected net value. Therefore the expected gross return on equity is given by $\mathbb{E}[1 + ROE_j] \equiv \frac{\mathbb{E}[NV_j]}{E}$.

(a) ROE of Non-deviating Bank: In regime 2 we have in equilibrium $d_j = 0, D_j = D^A$ and $L_j = L^A$. The solution for the loan rate is characterized by equation (12) for a given $\hat{\theta}$. Therefore the objective function (4) of a non-deviating bank in a symmetric

equilibrium reads as

$$\mathbb{E}[NV_j] = (1+l)L^A \int_{\hat{\theta}}^{1} \theta f(\theta)\, d\theta + \left((1+i)E - (1+i)L^A + iD^A\right)(1 - F(\hat{\theta})).$$

We rewrite the optimality condition of the loan rate (12) by using the definition of the conditional expectation (13) as $(1+l)\int_{\hat{\theta}}^{1} \theta f(\theta)\, d\theta = (1-F(\hat{\theta}))\frac{\varepsilon^l}{\varepsilon^l - 1}(1+i)$. Using this expression, expected net value can be rewritten as

$$\mathbb{E}[NV_j] = (1 - F(\hat{\theta}))\left[(1+i)E + (1+i)\frac{1}{\varepsilon^l - 1}L^A + iD^A\right].$$

Hence the expected net return on equity is given by

$$\mathbb{E}[ROE] = (1 - F(\hat{\theta}))\left[(1+i)(1 + \frac{1}{\varepsilon^l - 1}\frac{L^A}{E}) + i\frac{D^A}{E}\right] - 1. \tag{31}$$

(b) ROE of Deviating Bank: Since $L_j = E$ and $D_j = 0$ the objective function (4) of a deviating bank is

$$\mathbb{E}[NV_j] = (1 + l^{dev})E \int_{\hat{\theta}}^{1} \theta f(\theta)\, d\theta$$

and by using expression (30) for $(1 + l^{dev})$ we find that the expected net return on equity is given by

$$\mathbb{E}[ROE^{dev}] = (1+l)\left(\frac{L^A}{E}\right)^{\frac{1}{\varepsilon^l}} \int_{\hat{\theta}}^{1} \theta f(\theta)\, d\theta - 1. \tag{32}$$

(c) Lower Threshold: Bank j is better-off by not deviating if a deviation is not profitable:

$$\mathbb{E}[ROE^{dev}] \leq \mathbb{E}[ROE].$$

By plugging in expressions (31) and (32) for the expected return on equity and by using the definition of the conditional expectation in (13), we get

$$(1+l)\mathbb{E}[\theta|\theta \geq \hat{\theta}]\left(\frac{L^A}{E}\right)^{\frac{1}{\varepsilon^l}} \leq (1+i)(1+\frac{1}{\varepsilon^l-1}\frac{L^A}{E})+i\frac{D^A}{E}.$$

We further use optimality condition (12) and rewrite the previous inequality as

$$\frac{\varepsilon^l}{\varepsilon^l-1}(1+i)\left(\frac{L^A}{E}\right)^{\frac{1}{\varepsilon^l}} \leq (1+i)(1+\frac{1}{\varepsilon^l-1}\frac{L^A}{E})+i\frac{D^A}{E},$$

which can be solved for i to yield the lower threshold for the policy rate:

$$i \geq \frac{-\left[1+\frac{1}{\varepsilon^l-1}\frac{L^A}{E}-\frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{L^A}{E}\right)^{\frac{1}{\varepsilon^l}}\right]}{1+\frac{1}{\varepsilon^l-1}\frac{L^A}{E}-\frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{L^A}{E}\right)^{\frac{1}{\varepsilon^l}}+\frac{D^A}{E}} \equiv \underline{i}.$$

See Appendix A.4 in Ulate (2021) for the proof that $\underline{i} < 0$. The author uses a slightly different notation, namely $L^A \equiv L, D^A \equiv D, E \equiv F$ and the proof relies on some assumptions; one of which is $L > F$.

Regime 3: $d_j < 0$ This is a regime where banks do not want to attract deposits anymore and it applies if $i < \underline{i}$. Ulate (2021) describes this regime well and argues that in this case fears of "disintermediation" become relevant as banks stop accepting deposits which are not profitable anymore. Furthermore, the author shows that this is no longer a symmetric equilibrium on the deposit side because there can be a fraction of banks that accept deposits while the remaining banks do not. If the policy rate becomes too low, it is too costly to hold reserves and bank behavior eventually becomes independent of central bank policy. In the baseline calibration of Ulate (2021), $\underline{i}$ is around -2.2 percent. This value is very low and to date such a low policy rate has not been implemented. Therefore the author argues that regime 3 is ignored and not discussed in his empirical analysis. Because the focus of my paper lies on regimes where banks still administer their traditional role of accepting deposits from customers, I also do not discuss this third regime in more detail.

A.2.1 Proof of Proposition 1.

Realize that the key equation in (20) rewrites as

$$\frac{\hat{\theta}}{\mathbb{E}[\theta|\theta \geq \hat{\theta}]} = \kappa_R. \tag{33}$$

Statement (ii) of Lemma 1 implies that the left-hand side of equation (33) is equal or smaller than one. For the equation to hold, this also has to be the case for the right-hand side of this equation and therefore $\kappa_R \leq 1$. Since $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$ and $\hat{\theta}$ are non-negative, it follows $\kappa_R \geq 0$. Therefore $0 \leq \kappa_R \leq 1$. This concludes the proof of Proposition 1.

Since κ_R differs across regimes, for each regime there is a set of restrictions such that Proposition 1 holds.

Proposition 4. *For regime $R = 1$, the conditions such that $\kappa_R \in [0, 1)$ are:*

(i) If $\frac{L^A - E}{D^A} \geq \varphi_1 \equiv \frac{1}{1-\varepsilon^d} > 0$, then $\kappa_1 \geq 0$.

(ii) It follows by assumption that $\kappa_1 < 1$.

Proof.

(i) Suppose $\kappa_1 \geq 0$. Since $\sigma_L > 0$ and $L^A > 0$, this is equivalent to $\frac{L^A - E}{D^A} \geq \frac{1}{1-\varepsilon^d} > 0$. Since $\varepsilon^d < -1$, this also implies $L^A \geq E$.

(ii) Suppose $\kappa_1 \leq 1$. This implies $(1 - \sigma_L)L^A \leq \frac{1}{1-\varepsilon^d} D^A + E$. Since L^A, D^A, E and $\frac{1}{1-\varepsilon^d}$ are strictly positive and because $\sigma_L > 1$ this inequality holds. In fact, since σ_L is strictly larger than 1, the inequality is strict.

□

Proposition 5. *For regime $R = 2$, the conditions such that $\kappa_R \in [0, 1]$ are:*

(i) If $\frac{L^A - E}{D^A} \geq \varphi_2 \equiv \frac{i}{1+i}$, then $\kappa_2 \geq 0$.

(ii) (a) If $i \geq 0$, then it follows by assumption that $\kappa_2 < 1$.

(*b*) *If* $i < 0$ *and* $\sigma_L + \frac{E}{L^A} + \frac{i}{(1+i)}\frac{D^A}{L} \geq 1$*, then* $\kappa_2 \leq 1$*.*

Proof.

(i) Suppose $\kappa_2 \geq 0$. Since $\sigma_L > 0$ and $L^A > 0$, this is equivalent to $\frac{L^A - E}{D^A} \geq \frac{i}{(1+i)}$.

(ii) Suppose $\kappa_2 \leq 1$. This implies $(1-\sigma_L)L^A \leq \frac{i}{(1+i)}D^A + E$.

(a) If $i \geq 0$, then this inequality holds and it is in fact strict.

(b) If $i < 0$, then it is a restriction on the parameters and exogenous variables. It can be rewritten as $\sigma_L + \frac{E}{L^A} + \frac{i}{(1+i)}\frac{D^A}{L^A} \geq 1$.

□

A.2.2 Proof of Lemma 1.

(i) For $\hat{\theta} \in [0, 1)$, the conditional expectation is defined by (13). Since $\hat{\theta} \in \theta$ and $\theta \in [0, 1]$, the solvency threshold $\hat{\theta}$ can attain a value between zero and one. To find the lower bound of the conditional expectation, we evaluate it at $\hat{\theta} = 0$, because statement (iii) of Lemma 1 implies that there the conditional expectation attains its lowest value. Since $F(0) = 0$ the lowest value is the unconditional expectation of θ:

$$\mathbb{E}[\theta|\theta \geq 0] = \int_0^1 \theta f(\theta)\, d\theta = \mathbb{E}[\theta],$$

where the second equality follows from the conventional definition of the unconditional expectation. However, to find the value $\mathbb{E}[\theta|\theta \geq 1]$ we cannot perform the same procedure, because in this case $F(1) = 1$ and thus the denominator in (13) is equal to zero, such that the conditional expectation is not well-defined. We choose a different approach and develop a proof by contradiction.

Suppose $\mathbb{E}[\theta|\theta \geq \hat{\theta}] > 1$. Using definition (13) this inequality is equivalent to

$$\int_{\hat{\theta}}^1 \theta f(\theta)\, d\theta > 1 - F(\hat{\theta})$$

and by using integration by parts, we write the integral on the left-hand side as

$$\int_{\hat{\theta}}^{1} \theta f(\theta)\, d\theta = 1 - \hat{\theta}F(\hat{\theta}) - \int_{\hat{\theta}}^{1} F(\theta)\, d\theta.$$

Therefore the following equivalence holds:

$$\begin{aligned} \mathbb{E}[\theta|\theta \geq \hat{\theta}] > 1 \quad &\Leftrightarrow \quad 1 - \hat{\theta}F(\hat{\theta}) - \int_{\hat{\theta}}^{1} F(\theta)\, d\theta > 1 - F(\hat{\theta}) \\ &\Leftrightarrow \quad F(\hat{\theta})(1-\hat{\theta}) > \int_{\hat{\theta}}^{1} F(\theta)\, d\theta. \end{aligned} \tag{34}$$

The last inequality in (34) is not a true statement, because $\int_{\hat{\theta}}^{1} F(\theta)\, d\theta$ is the area under the CDF from $\hat{\theta}$ to 1. The rectangular area $F(\hat{\theta})(1-\hat{\theta})$ is a subset of this integral, that is $F(\hat{\theta})(1-\hat{\theta}) \subset \int_{\hat{\theta}}^{1} F(\theta)\, d\theta$. Therefore $\mathbb{E}[\theta|\theta \geq \hat{\theta}] > 1$ cannot be true and thus $\mathbb{E}[\theta|\theta \geq \hat{\theta}] \leq 1$. This contradiction concludes the proof of (i) in Lemma 1.

(ii) Realize that if $f(x) \geq g(x)$ on an closed interval $x \in [a, b]$, then $\int_a^b f(x)\, dx \geq \int_a^b g(x)\, dx$. Since $\theta f(\theta) \geq \hat{\theta} f(\theta)$ on $\theta \in [\hat{\theta}, 1]$, it therefore follows

$$\int_{\hat{\theta}}^{1} \theta f(\theta)\, d\theta \geq \int_{\hat{\theta}}^{1} \hat{\theta} f(\theta)\, d\theta. \tag{35}$$

Because $\hat{\theta}$ is a constant and can be taken out of the integral and since $\int_{\hat{\theta}}^{1} f(\theta)\, d\theta = 1 - F(\hat{\theta})$, we can rewrite equation (35), using the definition of the conditional expectation in (13), as follows:

$$\int_{\hat{\theta}}^{1} \theta f(\theta)\, d\theta \geq \hat{\theta}(1 - F(\hat{\theta})) \quad \Leftrightarrow \quad \mathbb{E}[\theta|\theta \geq \hat{\theta}] \geq \hat{\theta}. \tag{36}$$

Strictly speaking, this argument does not hold at $\hat{\theta} = 1$, since then $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$ is not well-defined, but this last inequality concludes the proof of (ii) in Lemma 1.

(iii) Using integration by parts, we can rewrite the definition of the conditional expectation as

$$\mathbb{E}[\theta|\theta \geq \hat{\theta}] = \frac{1}{1 - F(\hat{\theta})} \left(1 - \hat{\theta} F(\hat{\theta}) - \int_{\hat{\theta}}^{1} F(\theta) \, d\theta \right).$$

Differentiating with respect to $\hat{\theta}$ yields

$$\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}} = \frac{f(\hat{\theta})}{1 - F(\hat{\theta})} \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}] - \hat{\theta} \right).$$

Realize that $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$ is non-decreasing in $\hat{\theta}$, if its first derivative is non-negative. The hazard function is non-negative, i.e. $h(\hat{\theta}) = \frac{f(\hat{\theta})}{1-F(\hat{\theta})} \geq 0$, because both its nominator and denominator are non-negative. Moreover in (36) we have shown that $\mathbb{E}[\theta|\theta \geq \hat{\theta}] \geq \hat{\theta}$ and thus $\mathbb{E}[\theta|\theta \geq \hat{\theta}] - \hat{\theta} \geq 0$. Therefore $\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}} \geq 0$, which implies that $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$ is a non-decreasing function of $\hat{\theta}$. This concludes the proof of (iii) in Lemma 1.

Corner Solutions. Let me address the two possibilities of the corner solutions $\hat{\theta}^* = 0$ and $\hat{\theta}^* = 1$ for the solvency threshold. First notice from (28) that $\frac{d\mathbb{E}[NV_j]}{d\hat{\theta}} = -\left(1 - F(\hat{\theta})\right) < 0$, i.e. expected net value is decreasing in the solvency threshold. This is intuitive: The higher the solvency threshold (and thus the higher the probability of bank failure), the lower expected net value. Realize that the bank always has the option to either choose $L_j = 0$, by setting a very high loan rate or by investing all its liabilities in central bank reserves, or to stop operating which implies a net value of zero.[32]

(i) $\hat{\theta}^* = 0$: In this case, the bank never declares insolvency. The key equilibrium equation (20) boils down to $0 = \mathbb{E}[\theta]\kappa_R$. Since the unconditional expectation $\mathbb{E}[\theta]$ is non-zero, this can only hold if $\kappa_R = 0$. From the definition of κ_R in (21) this implies

$$L^A = \varphi_R D^A + E. \tag{37}$$

[32]A bank that optimizes with $L_j = 0$ still chooses the same deposit rate as derived in Section 3, because the deposit side of the model is completely independent from the lending side. Hence the solutions for deposit side variables are not affected.

Recall that net value can be written as $NV_j = (\theta - \hat{\theta}_j)(1 + l_j)L_j$ and hence if $\hat{\theta}^* = 0$, then $NV = \theta(1+l)L^A$. Given the restriction on L^A in (37), the bank has non-negative net value and grants loans as long as

$$\varphi_R \geq -\frac{E}{D^A}. \tag{38}$$

This condition is always satisfied in regime 1 and it is satisfied in regime 2 as long as

$$i \geq -\frac{E}{D^A + E}. \tag{39}$$

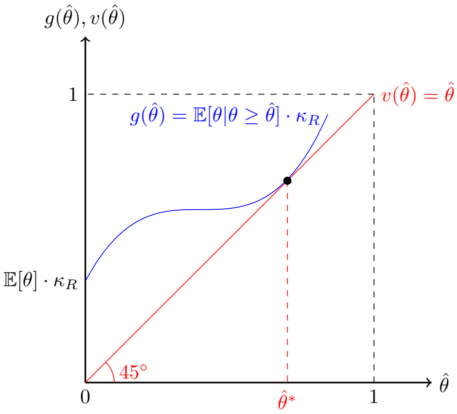
Hence choosing $\hat{\theta} = 0$ is only consistent with an equilibrium in the very particular knife-edge case that the exogenous variables L^A, D^A and E satisfy condition (37) and as long as the policy rate satisfies condition (39). Furthermore, from Result 4 it follows that $\hat{\theta}^* = 0$ is equivalent to the limiting case of loan market monopoly.

(ii) $\hat{\theta}^* = 1$: In this case, as long as the bank grants loans, it defaults for every realization of θ and thus always has negative net value. There are two potential deviation strategies: First, the bank can deviate from choosing $\hat{\theta}^* = 1$ by not granting loans and set $L_j = 0$, which implies $H_j = E + D^A$. By doing so, it has non-negative net value as long as

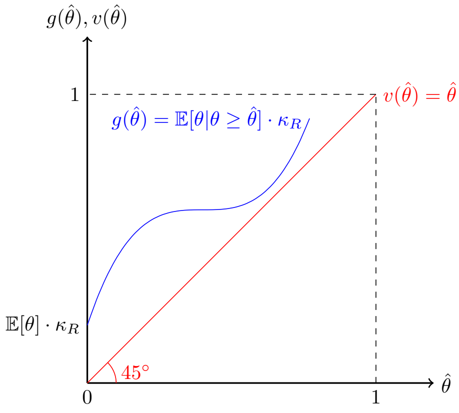
$$NV = (1 + i)E + (i - d)D^A \geq 0,$$

which also implies the inequality in (38). (Again, a sufficient condition for this inequality to hold is that the policy rate satisfies condition (39)). Second, the bank can also stop operating and always achieve a net value of zero. Therefore there always exist a profitable deviation and thus $\hat{\theta} = 1$ cannot be optimal and cannot constitute an equilibrium. Another way to see this is to realize that in this case the key equilibrium equation (20) is indeterminate, because for $\hat{\theta} = 1$ the conditional expectation is not well-defined.

Unique or No Solution. The two other cases, when there is one unique (interior) or no solution for $\hat{\theta}$, are depicted in the next figure. Since in theory there exists many probability distributions, a priori we cannot exclude the possibility depicted in Figure 5b, where no solution exists.



(a) Unique Solution



(b) No Solution

Figure 5: Unique and No Equilibrium for Solvency Threshold $\hat{\theta}$

Figure 5a qualitatively represents equation (20), when there exist a unique interior solution for $\hat{\theta}$. For instance, this is the case if credit risk is uniformly distributed. Figure 5b qualitatively depicts the case where no solution exists, in which case there is also no solution for the gross loan rate $(1 + l)$. This illustrates that $0 \leq \kappa_R \leq 1$ is a necessary, but not a sufficient condition for a solution to exist, because the functional form of $\mathbb{E}[\theta|\theta \geq \hat{\theta}]$, and thus the specific distribution of credit risk, is crucial as well.

A.3 Appendix to Section 4

The Deposit Spread Function $\varphi_R \equiv \frac{(i-d)}{(1+i)}$. Any effect from the policy rate on the solvency threshold works through the relative deposit spread. In regime 1 the relative deposit spread $\varphi_1 = \frac{1}{1-\varepsilon^d}$ is constant in the policy rate and therefore the policy rate does not affect the solvency threshold. Hence in an unconstrained world, optimal behavior prescribes that the bank keeps the relative deposit spread unchanged in response to a change in the policy rate. Since φ_R is the deposit spread relative to the expected, conditional loan rate, the bank can adjust the optimal deposit and loan rate to keep the relative spread constant. In regime 2 this is not the case as the relative deposit spread is no longer constant in the policy rate, i.e. $\varphi_2(i) = \frac{i}{(1+i)}$. This dependence on the policy rate occurs because if the zero lower bound is binding, then $d = 0$ and hence the bank looses one margin of adjustment: It can no longer vary the deposit rate in order to keep the relative deposit spread constant, whenever the policy rate changes. To contrast the relative deposit spread φ_R to the absolute deposit spread $(i - d)$ depicted in Figure 3, the next figure illustrates φ_R as a function of the policy rate.

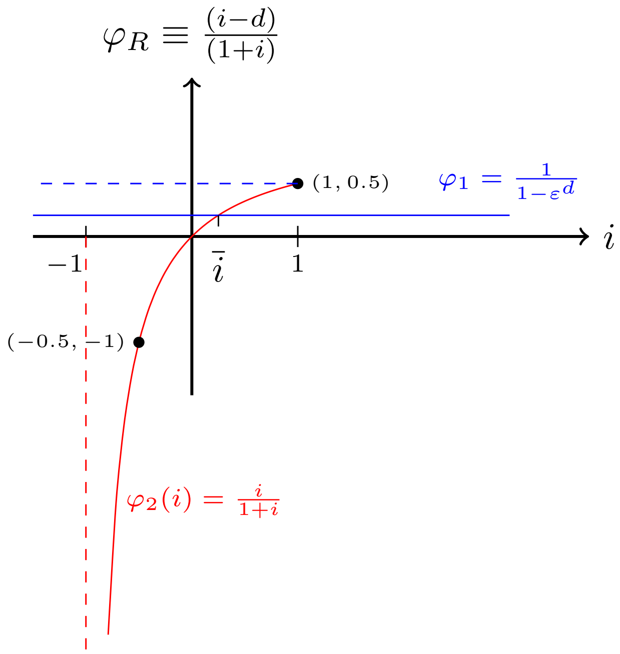


Figure 6: Relative Deposit Spread as Function of Policy Rate

Notes: This picture depicts the solution for the relative deposit spread $\varphi_R \equiv \frac{(i-d)}{(1+i)}$ as a function of the policy rate i. The origin corresponds to $(i = 0, \varphi_R = 0)$. In regime 1 the relative deposit spread $\varphi_1 = \frac{1}{1-\varepsilon^d}$ is depicted in blue. In regime 2 the relative deposit spread $\varphi_2(i) = \frac{i}{1+i}$ is depicted in red.

From Figure 6 we see that if the zero lower bound is not binding, then the relative deposit spread is a horizontal line because it is independent of the policy rate. The exact position of this line depends on the value of ε^d, but since $\varepsilon^d < -1$ it can be bound to the interval $\frac{1}{1-\varepsilon^d} \in (0, 0.5)$, which is indicated by the blue dashed line. Note that for the sake of comparison, φ_1 and $\varphi_2(i)$ are plotted over the entire interval $i \in [-1, 1]$. However the blue graph only applies if $i \geq \bar{i}$, whereas the red graph only applies if $\underline{i} \leq i < \bar{i}$. Also notice that at $i = \bar{i}$ both functions intersect, i.e. $\varphi_1 = \varphi_2(\bar{i})$.

Risk Effect and Hazard Function. The risk effect is characterized by expression (23). The slope function $S(\hat{\theta})$ of the conditional expectation is an important object that drives the risk effect. It is given by

$$S(\hat{\theta}) \equiv \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}} = h(\hat{\theta}) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}] - \hat{\theta} \right) \geq 0, \tag{40}$$

where $h(\hat{\theta}) = \frac{f(\hat{\theta})}{1-F(\hat{\theta})}$ is the hazard function. Using this definition of the slope function, the denominator of the risk effect can be written as

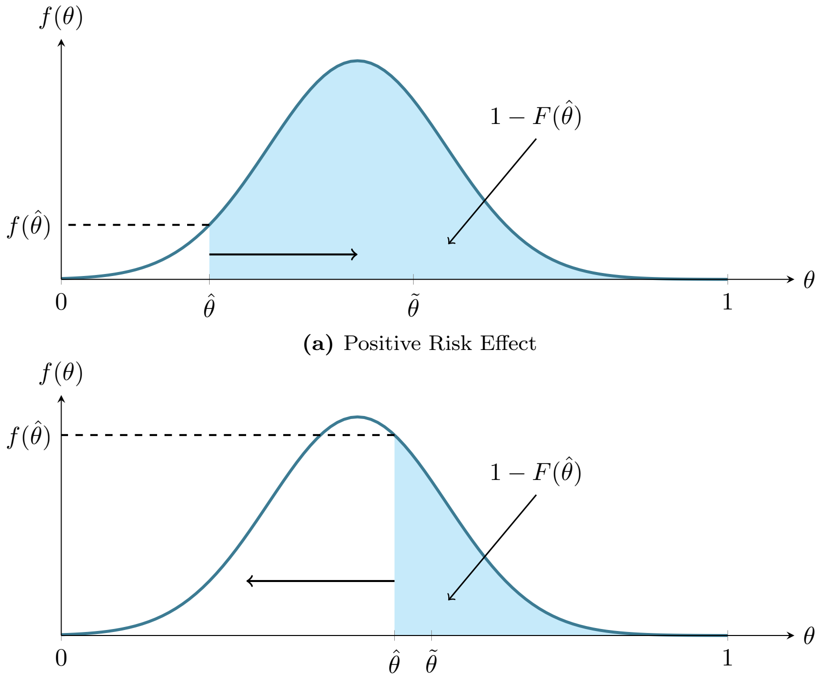
$$1 - \kappa_R \times \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}}|_{\hat{\theta}=\hat{\theta}^*} = 1 - \kappa_R \times h(\hat{\theta}^*) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^* \right) = 1 - \kappa_R S(\hat{\theta}^*) \tag{41}$$

and hence the the risk effect rewrites as

$$\frac{d\hat{\theta}^*}{d\kappa_R} = \frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{1 - \left(\kappa_R \times \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}}|_{\hat{\theta}=\hat{\theta}^*} \right)} = \frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{1 - \kappa_R S(\hat{\theta}^*)}.$$

This emphasizes that $S(\hat{\theta})$ is essential in determining the risk effect and that the slope function of the conditional expectation is closely related to the hazard function $h(\hat{\theta})$. The hazard function is a well known object in the statistical survival analysis literature. However, there the standard argument of $h(\cdot)$ is *time* t (and not a variable like $\hat{\theta}$ in our case). Thus the interpretation slightly differs here. To understand the relationship between the slope of the conditional expectation and the hazard function, it is important to understand the meaning of $h(\hat{\theta})$.

First consider the numerator in (25), which is the probability density of the solvency threshold $\hat{\theta}$. If the probability density $f(\cdot)$ around the point $\hat{\theta}$ is large, then this means that a draw of the random variable θ is likely to be close to $\hat{\theta}$. Second, the denominator is the probability that the bank is solvent, i.e. the probability $Pr(\theta \geq \hat{\theta}) = 1 - F(\hat{\theta})$. Therefore the hazard function tells us how likely it is that θ will be close to the solvency threshold – relative to the probability of being solvent. Loosely speaking, the hazard function tells us something like "*In the case of solvency, how close is the bank to being insolvent?*" which is a notion of insolvency-hazard in this model. The next figure depicts the intuition behind the impact of the hazard function:



(a) Positive Risk Effect

(b) Negative Risk Effect

Figure 7: The Hazard Function $h(\hat{\theta})$

Notes: This figure qualitatively illustrates the hazard function $h(\hat{\theta}) = \frac{f(\hat{\theta})}{1-F(\hat{\theta})}$ for a generic distribution of θ. We denote a particular realization of the random variable θ by $\tilde{\theta}$ and the blue area under the PDF is the probability of bank solvency $1 - F(\hat{\theta})$.

In Figure 7a the probability density $f(\hat{\theta})$ is small relative to the probability $1 - F(\hat{\theta})$. Therefore it is likely that the realization $\tilde{\theta}$ is not close to the solvency threshold $\hat{\theta}$ and thus in case of solvency, the bank is likely to be "highly solvent". In this sense the hazard of bank insolvency is small. In equilibrium a positive change $\Delta\kappa_R > 0$ therefore increases a bank's risk exposure by increasing $\hat{\theta}$ (indicated by the arrow pointing to the right) and therefore the risk effect is positive, that is $\frac{d\hat{\theta}^*}{d\kappa_R} > 0$, which will cause the probability of bank failure $F(\hat{\theta}^*)$ to increase. In Figure 7b the opposite case with a negative risk effect is depicted.

Next I present some formal results related to the risk effect that we will use to proof Proposition 2.

Lemma 2. *The risk effect exists and is well-defined if and only if* $1 \neq \kappa_R \times h(\hat{\theta}^*) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) \Leftrightarrow 1 \neq \kappa_R S(\hat{\theta}^*)$.

Proof. The risk effect in (23) is only defined if its denominator as given by (41) is non-zero, which is the case if $\kappa_R \times h(\hat{\theta}^*) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) \neq 1$. □

Lemma 3.

(i) *The risk effect is positive if* $1 > \kappa_R \times h(\hat{\theta}^*) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) \Leftrightarrow S(\hat{\theta}^*) < \frac{1}{\kappa_R}$.

(ii) *The risk effect is negative if* $1 < \kappa_R \times h(\hat{\theta}^*) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) \Leftrightarrow S(\hat{\theta}^*) > \frac{1}{\kappa_R}$.

Proof. From statement (i) of Lemma 1 it follows that the conditional expectation $\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]$ is strictly positive. Therefore the numerator of the risk effect in (23) is strictly positive and hence the sign of the risk effect is determined by the sign of the denominator given in (41), which is determined by $1 \gtrless \kappa_R \times h(\hat{\theta}^*) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right)$. □

Lemma 4. $\kappa_R \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) \in [0, 1]$.

Proof. This is implied by Proposition 1 and by statement (ii) of Lemma 1. □

Lemma 5. *If* $\kappa_R = 0$ *or* $\kappa_R = 1$*, then* $\kappa_R \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) = 0$.

Proof. If $\kappa_R = 0$, then the product is zero. If $\kappa_R = 1$, then equation (20) implies $\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] = \hat{\theta}^* \Leftrightarrow \mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^* = 0$. Therefore $\kappa_R \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) = 0$. □

A.3.1 Proof of Proposition 2.

Suppose $\kappa_R \neq 0$ and $\kappa_R \neq 1$. From Lemma 4 and 5 it follows

$$0 < \kappa_R \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) \leq 1 \Leftrightarrow \frac{1}{\kappa_R \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right)} \geq 1. \tag{42}$$

(i) Lemma 3 implies that the risk effect is positive if $\kappa_R \times h(\hat{\theta}^*) \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) < 1$, which rewrites as

$$h(\hat{\theta}^*) < \frac{1}{\kappa_R \left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right)}.$$

If the hazard function is sufficiently small such this inequality holds, then the risk effect is positive. There are several possibilities for the hazard function to satisfy this inequality. From the expression in (42) we realize that we have to distinguish two cases:

(1) If $\frac{1}{\kappa_R\left(\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]-\hat{\theta}^*\right)}=1$, then $h(\hat{\theta}^*)<1$ satisfies the inequality.

(2) If $\frac{1}{\kappa_R\left(\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]-\hat{\theta}^*\right)}>1$, then both

$$h(\hat{\theta}^*)\leq 1 \quad \text{and} \quad 1\leq h(\hat{\theta}^*)<\frac{1}{\kappa_R\left(\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]-\hat{\theta}^*\right)}$$

satisfy the inequality.

Therefore $h(\hat{\theta}^*)<1$ is a sufficient, but not a necessary condition for a positive risk effect.[33] This concludes the proof of (i) in Proposition 2.

(ii) Lemma 3 implies that the risk effect is negative if $\kappa_R\times h(\hat{\theta}^*)\left(\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]-\hat{\theta}^*\right)>1$. Together with the result in (42) this implies $h(\hat{\theta}^*)>\frac{1}{\kappa_R\left(\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]-\hat{\theta}^*\right)}\geq 1$. If the hazard function is sufficiently large such this inequality holds, then the risk effect is negative. Therefore $h(\hat{\theta}^*)>1$ is a necessary condition for a negative risk effect. This concludes the proof of (ii) in Proposition 2

Bank Lending Channel of Monetary Policy. In the model, there is only the bank lending channel of monetary policy at work. The existence of this channel for the transmission of monetary policy crucially depends on market power on the lending side, because it operates through the loan spread. On page 3, Ulate (2021) explains that "*The policy rate also differs from the rate that borrowers pay commercial banks for loans. This friction is essential for the bank lending channel, since banks are only able to lower their lending rate (despite the fact that their funding costs are constant, i.e., stuck at the ZLB) because of the existence of a profit margin.*"

Given Assumption 2, the bank lending channel in my model corresponds to the notion that since the loan demand function (5) is decreasing in the loan rate, lower loan rates

[33] A example is the case of the uniform distribution: There the hazard function is larger than one, but the risk effect is still positive.

are stimulative to the economy, because banks expand their lending volume. As established, due to the risk effect, the solvency threshold can increase or decrease in response to a change in the policy rate. Therefore, since the optimal loan rate, characterized by (12), depends on the solvency threshold through the conditional expectation, the risk effect matters for the bank lending channel. In equilibrium, the effect of the policy rate on the loan rate is given by

$$\frac{dl}{di} = \frac{\sigma_L}{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]} \left(1 - \frac{(1+i)}{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]} \times \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di} \right), \tag{43}$$

where

$$\begin{aligned} \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di} &= \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}}\Big|_{\hat{\theta}=\hat{\theta}^*} \times \frac{d\hat{\theta}^*}{d\kappa_R} \times \frac{d\kappa_R}{di} \\ &= S(\hat{\theta}^*) \times \underbrace{\frac{d\hat{\theta}^*}{d\kappa_R}}_{\text{Risk Effect} \gtrless 0} \times \underbrace{\frac{d\kappa_R}{di}}_{\text{Direct Effect}} . \end{aligned} \tag{44}$$

The second equality follows from the definition of the slope function $S(\hat{\theta}^*)$ in (40). We see in equation (43) that the differential $\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di}$ governs the sign of the effect, because all other objects in that equation are non-negative. As can be seen in (44), this differential depends on the risk effect.

Since in regime 1 the direct effect of the policy rate is zero, i.e. $\frac{d\kappa_1}{di} = 0$, the differential in (44) is zero. Hence the loan rate depends positively on the policy rate and equation (43) reduces to

$$\frac{dl}{di} = \frac{\sigma_L}{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]} > 0.$$

Result 6 (Bank Lending Channel in Regime 1). *If the zero lower bound on the deposit rate is not binding, then the lending rate depends positively on the reserve rate in equilibrium, i.e.* $\frac{dl}{di} > 0$. *A decrease in the policy rate (i.e. expansionary monetary policy) decreases the loan rate.*

In regime 2 the interaction is more complex, because the direct effect of the policy rate is negative, i.e. $\frac{d\kappa_2}{di} < 0$. Since $S(\hat{\theta}^*) \geq 0$ the differential $\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di}$, given by (44), can be either positive or negative, depending on the sign of the risk effect. If the risk effect is positive, then $\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di} < 0$ and thus the effect of the policy rate on the loan rate in (43) is unambiguously positive, because of the negative sign in front of the second term in brackets.

Result 7 (Bank Lending Channel in Regime 2). *If the zero lower bound on the deposit rate is binding, then* $\frac{d\kappa_2}{di} < 0$*. If the risk effect is positive, then the lending rate depends positively on the reserve rate in equilibrium, i.e.* $\frac{dl}{di} > 0$*. A decrease in the policy rate (i.e. expansionary monetary policy) decreases the loan rate.*

However, if the risk effect is negative such that $\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di} > 0$, then one can show that the term in brackets in equation (43) can be negative. As postulated in Proposition 3, in combination with Assumption 2 this can give raise to a reversal effect of the policy rate.

A.3.2 Proof of Proposition 3.

First let us rewrite the risk effect in (23) by using the definition of the slope function in (40):

$$\frac{d\hat{\theta}^*}{d\kappa_R} = \frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{1 - \left(\kappa_R \times S(\hat{\theta}^*)\right)} = \frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{S(\hat{\theta}^*)\left(\frac{1}{S(\hat{\theta}^*)} - \kappa_R\right)}.$$

We use this formulation of the risk effect to rewrite equation (44):

$$\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di} = \frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{\left(\frac{1}{S(\hat{\theta}^*)} - \kappa_R\right)} \times \frac{d\kappa_R}{di}. \tag{45}$$

Now consider the term in brackets in equation (43) for $R = 2$. We state the following conjecture. Suppose

$$1 > \frac{(1+i)}{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]} \times \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di}. \tag{46}$$

Using the expression in (45) for $\frac{d\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]}{di}$, this inequality rewrites as

$$1 > \frac{(1+i)\frac{d\kappa_2}{di}}{\left(\frac{1}{S(\hat{\theta}^*)} - \kappa_2\right)}. \tag{47}$$

From (27) we know that $\frac{d\kappa_2}{di} < 0$ and hence the numerator on the right-hand side of the inequality is negative.

1.) Assume that the risk effect is negative. Lemma 3 implies

$$S(\hat{\theta}^*) > \frac{1}{\kappa_2} \;\Leftrightarrow\; \frac{1}{S(\hat{\theta}^*)} - \kappa_2 < 0,$$

and hence also the denominator in (47) is negative. Therefore the inequality in (47) writes as

$$\frac{1}{S(\hat{\theta}^*)} < (1+i)\frac{d\kappa_2}{di} + \kappa_2.$$

We plug in for $\frac{d\kappa_2}{di} = -\frac{1}{\sigma_L}\frac{D^A}{L^A}\frac{1}{(1+i)^2}$ and for $\kappa_2 = \frac{1}{\sigma_L}\left(\frac{L^A-E}{L^A} - \frac{i}{(1+i)}\frac{D^A}{L^A}\right)$ to get

$$\frac{1}{S(\hat{\theta}^*)} < \frac{1}{\sigma_L}\left(1 - \frac{(D^A+E)}{L^A}\right). \tag{48}$$

2.) Assume that central bank reserves holdings are strictly positive, i.e. $H = D^A + E - L^A > 0$, such that $D^A + E > L^A$. Then $1 - \frac{(D^A+E)}{L^A} < 0$ and since $\sigma_L > 0$, the right-hand side in (48) is negative. However, since $S(\hat{\theta}^*) \geq 0$, the inequality in (48) cannot be true because it is a contradiction. Thus our initial conjecture in (46) and (47) respectively, cannot be true and therefore

$$1 \leq \frac{(1+i)}{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]} \times \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di}. \tag{49}$$

3.) Assume that the bank lending channel is operative such that the effect $\frac{dl}{di}$ in (43) exists and is non-zero, i.e. $\frac{dl}{di} \neq 0$. This is the case if $1 \neq \frac{(1+i)}{\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]} \times \frac{d\mathbb{E}[\theta|\theta\geq\hat{\theta}^*]}{di}$, which implies that the inequality in (49) is strict. Thus

$$1 < \frac{(1+i)}{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]} \times \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di} \Leftrightarrow 1 - \frac{(1+i)}{\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]} \times \frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}^*]}{di} < 0.$$

Hence the term in brackets in equation (43) is negative, which implies $\frac{dl}{di} < 0$. This concludes the proof of Proposition 3.

B Distribution of Credit Risk

B.1 Uniform Distribution

In this section, we solve the model for the particular case of uniformly distributed credit risk. Given this distributional assumption, there exist a unique equilibrium and we can explicitly solve the key equilibrium equation in (20) and derive the solution for the solvency threshold and the loan rate.[34] Suppose $f(\theta) \sim U[0,1]$ such that

$$f(\theta) = \begin{cases} 1, & \theta \in [0,1] \\ 0, & \text{else} \end{cases} \quad \text{and} \quad F(\theta) = \begin{cases} 0, & \theta < 0 \\ \theta, & \theta \in [0,1] \\ 1, & \theta > 1. \end{cases}$$

Then the conditional expectation is given by

$$\mathbb{E}[\theta|\theta \geq \hat{\theta}] = 0.5(1+\hat{\theta}).$$

In this case, the key equation in (20) writes as

$$\hat{\theta} = 0.5(1+\hat{\theta})\kappa_R.$$

[34] For the deposit side of the model, credit risk and thus its distribution does not matter: Due to the deposit insurance, deposits are fully covered for any realization of credit risk. Hence the solution of the deposit side of the model is unaffected.

Solving this equation for $\hat{\theta}$ yields the unique solution for the solvency threshold (as a function of κ_R):

$$\hat{\theta}^* = \frac{0.5\kappa_R}{1 - 0.5\kappa_R}.$$

The hazard function is given by

$$h(\hat{\theta}^*) = \frac{1}{1 - \hat{\theta}^*} = \frac{1 - 0.5\kappa_R}{1 - \kappa_R} > 1.$$

Under uniform credit risk, the risk effect is positive, because the solution for the solvency threshold depends positively on κ_R:

$$\frac{d\hat{\theta}^*}{d\kappa_R} = \frac{0.5}{(1 - 0.5\kappa_R)^2} > 0. \tag{50}$$

Let us verify this risk effect using formula (23). The relevant ingredients of this formula in the case of a uniform distribution are

$$\begin{aligned}
&\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] = \frac{0.5}{1 - 0.5\kappa_R}, \\
&\frac{d\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{d\hat{\theta}}|_{\hat{\theta}=\hat{\theta}^*} = h(\hat{\theta}^*)\left(\mathbb{E}[\theta|\theta \geq \hat{\theta}^*] - \hat{\theta}^*\right) = \frac{1 - 0.5\kappa_R}{1 - \kappa_R}\frac{0.5(1 - \kappa_R)}{1 - 0.5\kappa_R} = 0.5.
\end{aligned}$$

Therefore formula (23) yields

$$\frac{d\hat{\theta}^*}{d\kappa_R} = \frac{\frac{0.5}{1-0.5\kappa_R}}{1 - 0.5\kappa_R} = \frac{0.5}{(1 - 0.5\kappa_R)^2},$$

which is identical to the result in (50).

Result 8 (Uniform Distribution: Positive Risk Effect). *If the stochastic loan repayments are uniformly distributed, then the risk effect is positive in each regime, i.e.* $\frac{d\hat{\theta}^*}{d\kappa_R} > 0$.

Combining the solution for the solvency threshold with the conditional expectation and plugging into the optimality condition for the gross loan rate given by (12), we get the solution for the gross loan rate, denoted by $(1 + l_R)$, as

$$(1 + l_R) = \frac{\varepsilon^l}{\varepsilon^l - 1}(2 - \kappa_R)(1 + i).$$

This loan rate depends positively on the policy rate:

$$\frac{dl_R}{di} = \frac{\varepsilon^l}{\varepsilon^l - 1}(2 - \kappa_R) > 0.$$

Result 9 (Uniform Distribution: Unique Symmetric Equilibrium). *If the stochastic loan repayments are uniformly distributed, then for regime* $R = \{1, 2\}$ *there exist a unique solution for the solvency threshold given by* $\hat{\theta}^* = \frac{0.5\kappa_R}{1-0.5\kappa_R}$. *The unique solution for the gross loan rate is given by* $(1 + l_R) = \frac{\varepsilon^l}{\varepsilon^l-1}(2 - \kappa_R)(1 + i)$. *Therefore the equilibrium is unique.*

B.2 Other Distributions

Since the random variable θ is a share, this restricts the family of distribution to ones with support $(0, 1)$. But there still exist many distributions we can assume – examples of such continuous probability distributions are the Triangular distribution, the Truncated Normal distribution or the Beta distribution. However not all of these distributions have (convenient) closed form expressions for their PDF and CDF, which makes it difficult to use them analytically.

Technically, a possible family of distributions is the Kumaraswamy distribution $X \sim Kumaraswamy(a, b)$, first described in Kumaraswamy (1980). It is a two-parameter distribution defined on the interval $(0, 1)$ and similar to the Beta distribution, but its PDF and CDF can be expressed in closed form. It can take a variety of shapes and is therefore very flexible. For example, it can be used to model prevalence and probabilities of fractions and ratios. Its probability density function is

$$f(x) = abx^{a-1}(1 - x^a)^{b-1},$$

for $x \in (0, 1)$ and where $a > 0$ and $b > 0$ are shape parameters. The cumulative distribution function is

$$F(x) = 1 - (1 - x^a)^b.$$

Its hazard function is given by

$$h(x) = \frac{f(x)}{1 - F(x)} = \frac{abx^{a-1}(1 - x^a)^{b-1}}{(1 - x^a)^b}.$$

The hazard function is larger than one if

$$abx^{a-1}(1 - x^a)^{b-1} > (1 - x^a)^b \quad \Leftrightarrow \quad ab + x > x^{1-a}.$$

For instance $a = 1$ and $b > 1$ satisfy this inequality for any value of x. On the other hand, the hazard function is smaller than one if $ab + x < x^{1-a}$. A necessary condition for the latter inequality to hold is $x < 1$. This is not restrictive, as we only consider interior solutions for $\hat{\theta}$. Finally, notice that the Kumaraswamy distribution is related to other named distributions. Some examples of these relations are as follows:

(i) If $X \sim Kumaraswamy(1, 1)$, then $X \sim U(0, 1)$.

(ii) If $X \sim Beta(a, 1)$, then $X \sim Kumaraswamy(a, 1)$.

(iii) If $X \sim Beta(1, b)$, then $X \sim Kumaraswamy(1, b)$.

From an economic point of view, even if there exist a mathematical convenient distribution that satisfies the necessary properties to model the stochastic share of loan repayments, the relevant follow up question is of course if such a distribution is indeed appropriate to model credit risk.

C Risk-Independent Insurance

Suppose a bank has to pay a insurance premium for its deposit insurance and that this premium payment P_j is proportional to total deposits:

$$P_j = cD_j,$$

with $c \in (0,1)$. This captures a risk-independent deposit insurance scheme, because the premium does not depend on a bank's expected losses. The balance sheet identity in this case is $H_j + L_j + P_j = D_j + E$ and the objective function becomes

$$\mathbb{E}[NV_j] = \int_{\hat{\theta}_j}^{1} \left[(\theta(1+l_j) - (1+i)) L_j + (i - d_j)D_j - (1+i)P_j + (1+i)E \right] f(\theta)\, d\theta.$$

Since $\frac{\partial P_j}{\partial l_j} = 0$, the first-order condition for the loan rate is unaffected by the premium payment. On the other hand, the first-order condition for the deposit rate is given by

$$(1 + d_j) = \frac{\varepsilon^d}{\varepsilon^d - 1}(1-c)(1+i).$$

Compared to the optimality condition in (11), the insurance premium payment just scales down the optimal gross deposit rate. Therefore in its effect, the premium is similar to more market power in the deposit markets (which implies a lower deposit rate). The deposit spread is

$$(i - d_j) = \frac{1 - c\varepsilon^d}{1 - \varepsilon^d}(1+i),$$

which for a given policy rate is now larger as the deposit rate is smaller. The premium also affects the upper threshold $\bar{i}$:

$$\bar{i} \equiv -\frac{1 - c\varepsilon^d}{\varepsilon^d(1-c)}.$$

Note that this threshold is larger compared to the case $c = 0$ in the main section of this paper. So in Figure 3 this risk-independent premium would shift $\bar{i}$ to the right and thus the regime switch occurs at a higher policy rate. In regime 1, the coefficient κ_1 is now given by

$$\kappa_1 \equiv \frac{\varepsilon^l - 1}{\varepsilon^l} \left(\frac{L^A - E}{L^A} - \frac{1-c}{1-\varepsilon^d} \frac{D^A}{L^A} \right)$$

and in regime 2 as

$$\kappa_2 \equiv \frac{\varepsilon^l - 1}{\varepsilon^l} \left(\frac{L^A - E}{L^A} + \left(c - \frac{i}{(1+i)} \right) \frac{D^A}{L^A} \right).$$

We see that the insurance premium parameter c does not alter the effect of the policy rate on κ_R, because in regime 1, there is still no effect of the policy rate on κ_1 and in regime 2, the parameter c and the policy rate i are additive and do not interact. Therefore a risk-independent deposit insurance only affect the levels of the variables, but not the main causal channel of the zero lower bound constraint and of the policy rate on the probability of bank insolvency.

D Model Extensions

Reserve Requirement. Suppose there is a reserve requirement of the form $H_j = \phi D_j$, with $\phi \in (0, 1)$. Plugging into the balance sheet identity yields

$$L_j = (1 - \phi) D_j + E.$$

We see that such a reserve requirement distinctly connects the loan and deposit position in the balance sheet. Therefore the *independence* result of the canonical Monti-Klein model will no longer hold. It is important to realize that the collapse of the independence result occurs even without introducing credit risk. Indeed, adding credit risk to a setup with an reserve requirement makes solving the model less tractable. To see this, note that in this case the first-order conditions for the loan rate and the deposit rate respectively, can be written as follows:[35]

$$-\frac{\mathbb{E}[\theta|\theta \geq \hat{\theta}]}{\hat{\theta}} = \frac{h(\hat{\theta}) i (1+\phi) D}{(1+l)L},$$

$$\big((i-d) - (1-\phi)\big) \frac{D'}{D} - \frac{D}{D} = \frac{h(\hat{\theta}) i (1+\phi) D}{(1+l)L} \left(\big((1+d) - (1+i)\phi\big) \frac{D'}{D} + \frac{D}{D} \right).$$

[35] I omit the index j for the purpose of clarity. This should not give the impression that there is necessarily a symmetric equilibrium.

Solving this system of equations for $(1 + l)$ and $(1 + d)$ is relatively involved. But we can see that both optimality conditions depend on the loan as well as the deposit rate through the solvency threshold. A last interesting observation is revealed when rewriting the solvency constraint:

$$\hat{\theta} = \frac{\big((1+d) - (1+i)\phi\big)}{(1+l)} \frac{D}{L}.$$

Regarding balance sheet positions, notice that only the deposit-to-loan ratio $\frac{D}{L}$ remains relevant for the solvency threshold.

Adverse Selection in Credit Markets. Suppose we want to account for incentive problems similar to Stiglitz and Weiss (1981). The idea is that the repayments of loans may depend on the loan rate: A higher loan rate may cause more borrowers to default. One way to capture such a setup is by making the stochastic share of loan repayments a function of the loan rate: $\theta(l)$ with $\theta'(l) < 0$, such that the higher the loan rate, the smaller the share of loan repayments. Suppose further that a proper inverse function $l^{-1}(\theta)$ exists. The implications for the model in the main section of the paper are not straight forward: For instance, the loan demand function in (5) is no longer consistent, because in the final loan producer's problem, the loan rate can no longer be taken out of the integral (when integrating over θ) and therefore the equilibrium will certainly be affected.

Towards General Equilibrium. The results in the paper are based on a partial equilibrium analysis. This partial equilibrium setup is useful in order to clearly understand the causal mechanism. However a valid objection is that in general equilibrium, there may be additional channels at work. Let us for example focus on the policy rate. Its mechanism works through the deposit spread, which is a price effect. But the aggregate quantities D^A and L^A also may depend on the policy rate i, which is a quantity effect of balance sheet positions. It is a shortfall of the partial equilibrium approach that the model cannot account for such endogenous quantity effects.

To get an idea how general equilibrium effects could play a role, let us assume that in the aggregate, we have a loan demand function that depends on the policy rate $L^A(i)$, with $\frac{dL^A}{di} < 0$, and a deposit supply function that depends on the policy rate $D^A(i)$, with $\frac{dD^A}{di} > 0$. Suppose further that the aggregate quantities are no function of $\hat{\theta}$ such that the only new element in the key equation (20) is the fact that aggregate quantities are functions of the policy rate. This does not affect the risk effect, but the effect of the policy rate is now given by

$$\frac{d\kappa_R}{di} = \frac{1}{\sigma_L}\left[\frac{E}{(L^A(i))^2}\frac{dL^A(i)}{di} - \frac{d\varphi_R}{di}\frac{D^A(i)}{L^A(i)} - \varphi_R\frac{1}{L^A(i)}\left(\frac{dD^A(i)}{di} - \frac{D^A(i)}{L^A(i)}\frac{dL^A(i)}{di}\right)\right]. \tag{51}$$

This differential is a complicated expression and depends on many interactions; without a fully fledged general equilibrium model, conclusions about the sign are nebulous. To get a quantitative feeling for the economic relevance of these effects, let us apply the annual calibration in Ulate (2021). The author estimates for his baseline calibration that $\varepsilon^d = -\frac{268}{4} = -67$ and thus $\bar{i} \approx 0.015$. Furthermore the author argues $\underline{i} \approx -0.022$. Of course, this is only one particular calibration but may give a feeling for the quantitative relevance. The relevant objects across regimes are then as follows:

Regime	φ_R	$\frac{d\varphi_R}{di}$
1	$\frac{1}{1-\varepsilon^d} \approx 0.0147$	0
2	$\frac{i}{(1+i)} \in [-0.022, 0.015]$	$\frac{1}{(1+i)^2} \in [0.971, 1.045]$

Table 2: Quantitative Comparison across Regimes

Notes: This table shows the numerical values of φ_R and $\frac{d\varphi_R}{di}$ using the baseline calibration in Ulate (2021), where $\varepsilon^d = -67$ and thus $\bar{i} \approx 0.015$. The author argues $\underline{i} \approx -0.022$ and therefore in regime 2 we consider the interval $-0.022 \leq i \leq 0.015$ for the policy rate.

Since φ_R is reasonably small in both regimes, let us hypothesize that the fraction $\frac{\varphi_R}{L^A(i)}$ is close to zero such that it does not matter quantitatively. The third term in (51) is then zero. In regime 1, we get

$$\frac{d\kappa_1}{di} \approx \frac{1}{\sigma_L}\left[\frac{E}{(L^A(i))^2}\frac{dL^A(i)}{di}\right] < 0.$$

We can see that in this case, a *portfolio effect* for loans matters: Lower policy rates make risky asset more attractive and thus the bank increases its risk exposure (in the sense of increasing κ_1) by holding more loans. Therefore Result 2, where the spread effect is zero, would no longer hold. In regime 2, the differential is given by

$$\frac{d\kappa_2}{di} \approx \frac{1}{\sigma_L}\left[\frac{E}{(L^A(i))^2}\frac{dL^A(i)}{di} - \frac{d\varphi_2(i)}{di}\frac{D^A(i)}{L^A(i)}\right] < 0.$$

We see that in the second regime, the zero lower bound-effect $\frac{d\varphi_2(i)}{di}$ described in the partial equilibrium model still plays a role in addition to a portfolio effect.[36] Therefore the partial equilibrium results can be interpreted as extreme cases that emphasize one causal mechanism, namely the price effect or spread effect of the policy rate, and abstract from quantity effects.

E The Independence Result of the Monti-Klein Model and Credit Risk: A Commentary

Although not stated explicitly, the static banking model in Ulate (2021) is in fact just a slightly modified version of the canonical Monti-Klein (M-K) model with monopolistic competition rather than one monopolistic bank and an exogenous zero lower bound constraint on the deposit rate. The canonical M-K model captures the industrial organization (IO) approach to banking: In this one period model, there is one monopolistic bank that has market power in both the lending as well as the deposit market and therefore the bank is able to set the loan and the deposit rate. It does so by choosing a point on the downward-sloping loan demand and on the upward sloping-deposit supply function. It observes both functions and the deposits attracted by the bank (they are

[36] Evaluating the size of the effects is challenging, because regimes 1 and 2 apply at different policy rates such that $D^A(i), L^A(i)$ and their corresponding differentials are evaluated at different values of i.

on its liability side of the balance sheet) are invested in either loans or an safe asset.[37] The safe asset pays a risk-free interest rate (in my model central bank reserves play the role of safe assets). There is no uncertainty or risk regarding loans and deposits; loans are always fully repaid, the bank never defaults and depositors always get all of their funds back. The main outcome of the model is the well-known *independence result*: The optimal choice of the loan rate and the optimal choice of the deposit rate are completely independent from each other. That is, in the first-order condition for the loan rate, the deposit rate does not show up and vice versa. These two rates are set by a standard monopolistic behavior: For the optimal loan rate, the bank charges a markup over the safe interest rate and for the optimal deposit rate, the bank charges a markdown.

One may argue that the optimal behavior in the M-K model is too simplistic and it is not an appropriate description of bank behavior, because it predicts that a bank consists of two entities, a lending division and a deposit division, that act completely independently from each other. It seems reasonable to think that a bank takes into account its behavior on one market, when setting rates on the other market and that these two decision are interconnected and not independent. From a modeling perspective however, the independence result is extremely useful: Because once we expand the model by adding different characteristics of the banking sector and if those features cause the independence result to no longer hold, then we can clearly understand and identify the mechanism and causal links that make the loan and the deposit rate interdependent. Therefore from a scientific standpoint, this simplistic and maybe unrealistic feature of the M-K model is actually one of its advantages.

While market power of commercial banks is well documented in the literature, due to its IO nature, the M-K model reduces banks to ordinary firms and does not take into account the particular role a bank plays for the financial system. Financial intermediaries are characterized by very specific peculiarities which do not apply to ordinary firms such as maturity transformation, solvency- and liquidity-risk, monitoring

[37]Hence the model captures the *loanable funds* approach of banking rather than the money creation view. The former states that banks attract deposits to fund their assets whereas the later states that banks create deposits whenever they grant loans.

and bank specific regulations. Additionally, they have access to central bank reserves which makes them a key player in the transmission of monetary policy.

Hence a logical next step is to extend the M-K model by taking into account the special role of commercial banks in modern economies. This is partially what Dermine (1986) does: The author introduces bankruptcy risk and a deposit insurance into the canonical M-K model. For the insurance premium, the author chooses a very general formulation, where the premium depends on expected losses. Bankruptcy risk is introduced by making the end of period loan repayments stochastic. This affects the independence result: With a fair deposit insurance scheme, the loan and deposit rate are still independent. But when the deposit insurance is such that the insurance premium is independent of a bank's expected loss, or the premium is over- or under priced, then the loan rate will depend on the deposit rate but the reverse is not true: Deposit rates are still independent of loan rates. Intuitively, it is not surprising that the presence of a deposit insurance does not affect the independence of the deposit rate from the loan rate: A deposit insurance guarantees that deposit contracts are safe and hence non-random (i.e. they are not not affected by the stochastic element in the model) – therefore these contracts preserve their characteristic from the baseline M-K model. Incorporating risk into the M-K model is in general not tied to a deposit insurance scheme and abstracting from insurance may a priori appear to be the cleanest way to study risk in this model environment.[38] For the purpose of my research question however, the implications of a deposit insurance are advantageous: Because my model incorporates a zero lower bound on the deposit rate, at some point the lower bound binds and the deposit rate is zero. To examine such a regime, we can start with the choice of a deposit rate of zero and then track what this means for the loan rate, but we do not have to worry about a potentially feedback loop from the loan rate back to

[38]In Dermine (1986) it is also described what happens when there is no deposit insurance. The author argues that in this case, there are two deposit rates, namely the *expected* and the *posted* deposit rate. The former matters for depositors and thus the supply function of deposits depends on the expected deposit rate, while the latter is the one a bank sets at the beginning of the period. However, the form of the deposit supply function is not affected – it is now just a function of the expected rather than the posted deposit rate. But one may argue that with rational expectations, a bank takes uncertainty into account and this may change also the deposit supply function and therefore the posted deposit rate. Or stated differently: With rational expectations, there is still only one deposit rate that matters and not two. Hence there may be a consistency problem with the argument in the paper for the case without deposit insurance.

the deposit rate, as the deposit rate is still independent of the loan rate.[39]

To introduce credit risk into my model, I follow Taggart and Greenbaum (1978). At first glance, this modeling approach differs from Dermine (1986): In the former paper, the stochastic element is the *share* of performing loans whereas in the later, the *size* of loan repayment is stochastic. From a reduced form perspective, both approaches capture the same phenomena (and the implications are similar), but the approach in Taggart and Greenbaum (1978) turns out to be more convenient in my case due to is multiplicative nature.[40]

[39]Therefore this (partial) breakdown of the independence result creates a sequential decision process for a bank: Since the loan rate depends on the deposit rate, but the reverse is not true, a bank has first to decide upon the optimal deposit rate, because this variable must be known in order to choose the optimal loan rate.

[40]It is much easier to derive a loan demand function that is consistent with monopolistic competition in this case, because in essence the random share θ just acts as a scaling factor to credit contract.