Credit and Anonymity

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Abstract

It is commonly believed that borrowers cannot be anonymous in unsecured credit relations because anonymity heavily reduces the scope for punishment and therefore makes credit unfeasible except for very special circumstances. However, we demonstrate that credit is generally feasible even if borrowers are anonymous. In particular, we construct equilibria where borrowers use potentially multiple pseudonyms (such as usernames or wallet addresses) to interact with lenders. We assume that the complete history of past actions committed by a pseudonym is public but not the identity behind that pseudonym. While borrowers cannot be directly punished due to their anonymity, there is still scope for punishment. One possibility is based on the loss of reputation accumulated by a pseudonym over time. Another involves charging a fee to create pseudonyms. Although credit and anonymity are not mutually exclusive, we also show that maintaining a borrower's anonymity is costly.

Keywords: Reputation, Credit, Anonymity, Pseudonymity, Decentralised Finance
JEL Codes: D82, E51, L14, G19

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1 Introduction

There is general agreement that a) credit plays a crucial role in modern financial and monetary systems, and b) there is a strong desire for anonymity among its users. However, if we want to facilitate a credit contract between two parties, it is also widely believed that anonymity cannot be maintained. The idea is that absent any collateral, the borrowing party has to reveal its identity which allows the lender party to punish the borrower in case of default. This in turn provides the necessary incentives for repayment on the borrower side. The absence of anonymity in uncollaterlised credit is problematic for many different applications, in particular for the development of decentralised finance (DeFi[1]) credit applications, as one of the aspiring goals of DeFi is to maintain the anonymity of it's users. In this paper we demonstrate that credit and anonymity are, contrary to popular belief, compatible.

At the heart of the matter lies the question of what qualifies as anonymity. We distinguish two concepts of anonymity that we refer to as *strict anonymity* and *pseudonymity*. Under the former, a user is deemed anonymous if and only if there is no public record on actions committed by an agent. For instance, a cash transaction can be considered strictly anonymous, as there is generally no public record of cash transactions. In contrast, a user is pseudonymous if and only if the identity of agents responsible for certain actions is unknown even if the entire history of actions of those agents is publicly known. The scenario we have in mind is one in which agents use *accounts* (or pseudonyms) when transacting with each other such as a wallet address.[2] These accounts are then used to negotiate and record credit contracts. The history of accounts is public information but it is private information to whom the accounts belong. Only the account owner knows which ones are his accounts. Practically, many blockchain related applications are pseudonymous, for example, lending and borrowing via smart contracts on Ethereum-based protocols like Compound, Aave, and Uniswap.

Except for some special cases (see literature review), credit and strict anonymity are not compatible as the lack of a public record (in particular of defaults) makes the punishment of defaulting borrowers impossible. Whether or not credit is feasible under pseudonymity is still largely unexplored.[3] The lack of research on this question is surprising since many

[1] A comprehensive overview of DeFi and how it compares to traditional (centralised) finance is provided in Qin et al. (2021).

[2] Other examples are email accounts, user names or gamer tags just to name a few.

[3] We focus on economic feasibility, not technical feasibility. For credit to attain technical feasibility under pseudonymity, it requires the technology to a) record all debt obligations between accounts, b) validate the fulfillment of these obligations, and c) establish a transparent and easily accessible public record of these

areas of the internet operate under a pseudonymous regime where users interact with each other through website accounts, blockchain wallets, or virtual avatars, while a large part of the user's activity is recorded and observable. We will therefore focus our attention on the economic feasibility and efficiency of pseudonymous credit systems. Our approach represents a middle ground between the two extremes often considered in the literature: strict anonymity (which excludes credit) and full information (where anonymity is absent).

Our model, which is populated by two types of agents called borrowers and lenders, has a simple structure. Each period is divided into two subperiods: the borrowing and the repayment stage. During the first, borrowers and lenders meet bilaterally and it is assumed that lenders can produce a particular good, the credit good, of which the borrowers derive instantaneous utility from consuming it. The key challenge arises from the fact that borrowers cannot produce anything of value for the lender on the spot. Instead, the borrower can only promise to repay the lender in the second stage, the repayment stage, by producing a different good, the settlement good, to reimburse the lender. This structure captures in a very simple way the main economic challenges in credit relations which is the lack of intertemporal commitment from the borrower's side.[4]

Different to standard models of credit, borrowers and lenders interact with each other using accounts (or pseudonyms).[5] More precisely, borrowers and lenders have access to a record-keeping technology that perfectly tracks debt relations and repayment histories across different accounts, which are owned by borrowers. Borrowers can always create new accounts at zero cost and choose each period on which account to record actions. This record-keeping technology aligns with the previously mentioned notion of pseudonymity. Finally, to make the problem interesting, we assume that borrowers exit the economy stochastically and are replaced by new borrowers. Thus, at any given time, an account lacking any history could indicate either a "young" borrower, opening an account for the first time, or an "older" borrower who has opened a new account. The key challenge of establishing a credit system in such an environment is: How to punish borrowers who have defaulted on their debt if

obligations. With the rapid advancements in blockchain technology, the prospect of achieving this feasibility within the upcoming years appears promising, despite some challenges.

[4]What makes this formulation so attractive is the fact that the lenders in our model perform two roles simultaneously which, in reality, are usually performed by two different agents. To take the example of mortgages, the bank usually lends funds to a borrower who uses them to buy a house. In our model, the lender lends the goods directly and performs in this sense the role of the banker and the house seller at the same time.

[5]We take the preference for anonymity (more precisely, pseudonymity) as a given. That is, our model does not explain why agents desire anonymity. However, it is not difficult to imagine potential advantages of remaining anonymous in reality. For example, consider racially motivated lending behavior, which could result in some groups receiving loans under worse conditions (or no loans at all). More formally, anonymity could prevent the emergence of equilibria where irrelevant but observable characteristics are used to determine lending conditions (see, for example, Carapella and Williamson (2015)).

they can always "clear their history" by creating and using new accounts?

Throughout the paper we consider two cases: the first where accounts are costless to create and the second where agents need to pay a fee to open a new account.

Our main finding is that there always exists an equilibrium where credit is feasible even if agents are pseudoynmous and accounts are costless to create. Intuitively, those equilibria work in the following way: accounts, rather than borrowers, earn *reputation*. Reputation is a mapping from the account's history. Consistently repaying debt increases reputation. A higher reputation implies a higher credit limit and therefore a higher level of feasible consumption. Accounts that have a history of defaulting on their debts are barred from borrowing in the future. Even though borrowers can always create new accounts and borrow again after defaulting, this is costly to do because newly created accounts have no reputation and can therefore borrow only little. In these equilibria, borrowers never endogenously default on their debt because the *value of reputation*, i.e. the difference in continuation values between an account with a given level of reputation compared to an account with no reputation, is sufficiently high. An important, and somewhat surprising, implication is that borrowers optimally use only one account despite having the option to use a second account in parallel with the first.[6] The rationale behind this is that using a second account incurs an opportunity cost of forfeiting the chance to use the primary account which allows both, to gain more reputation on that account and to consume more. The underlying assumption generating this effect is that matching with lenders is time-consuming.[7]

We also show what is required to construct such equilibria. First, accounts that have defaulted have to be punished by reducing the amount they can borrow in the future (in our case, we assume that no amount can ever be borrowed again). Second, lenders need to reject any off-equilibrium offers. The reason is that absent such a punishment, borrowers and lenders bilaterally agree to exchange the highest amount of credit such that the borrower is indifferent between repaying and defaulting. However, we show that those "not-too-tight" debt limits, using the language of Alvarez and Jermann (2000), cannot be part of an equilibrium. The reason is the following: when bargaining borrowers and lenders do not internalise how their choices affect the value of reputation in equilibrium. We then show that if debt limits are not-too-tight, then the value of reputation collapses to zero and therefore also debt limits. Therefore, by punishing deviation from the equilibrium amount, we can

[6] The fact that borrowers can always create new accounts if another account is flagged as a defaulter still has an important effect on the equilibrium as it lowers the cost of defaulting.

[7] Two accounts held by a borrower cannot be used to accumulate reputation by pretending to trade with each other. To gain reputation goods have to be produced and exchanged. Only lenders can produce these goods (this assumption ultimately introduces gains from trade into the model).

enlarge the set of incentive-feasible equilibria and construct equilibria where the value of reputation and debt limits are positive.[8] This suggests that lenders need to "artificially" lower the amount they lend so that, in equilibrium, reputation is sufficiently valuable to provide the incentives for borrowers to repay.

While our model shows that credit is always economically feasible, we show that maintaining credit in a pseudonymous environment is costly. This is due to a trade-off between the consumption of "older" borrowers (those with a lot of reputation) and of "younger" borrowers (those with little or no reputation). Intuitively, supporting a large volume of trade requires a high value of reputation to prohibit borrowers from defaulting on their debts. But for reputation to be highly valued, it must be that consumption is restricted for those agents with little reputation, such as "young" borrowers. Generally, pseudonymity is costly because it is impossible to differentiate between "young" borrowers and those who have defaulted, and, as a result, any punishment scheme affects borrowers not only off-equilibrium (if they default) but also on-equilibrium (when they are "young").

Finally, we discuss the case where agents need to pay a fee in order to open an account. The idea is that there is an authority that manages those accounts and charges a fee whenever an agent wants to open an account. We differentiate between two subcases: a first where those fees are a real cost and a second where the fees consist of collected goods and are redistributed. We show that the incentives for repayment are now directly related to those fees. To be precise, the debt limit is now a linear function of those fees and in the second case, also of the transfers the borrowers receive. In equilibrium, borrowers do not default because this would mean that they would have to pay the fee for a new account and, in the second case, they also forgo the transfer. We show that in both subcases we can always find equilibria for some traded quantities that exist. In a second step we analyse the optimal equilibrium for the case when the authority can choose the fee and redistribute it. We find that the quantity traded in the optimal costly accounts equilibrium is always higher than in the optimal reputation equilibrium in all reputation stages.

This second approach with costly accounts provides a high technological feasibility while the case with costless accounts speaks to the idea of making accounts on platforms accessible for everyone (independent of their budget).

Literature review Our model builds on the long tradition of search models. In particular, our model is based on Lagos and Wright (2005) which has been used for many different

[8] See Bethune et al. (2018) for a detailed discussion.

applications, such as monetary economics, banking and finance.[9] Examples of models of credit using that framework are Gu et al. (2013), Lotz and Zhang (2015), Carapella and Williamson (2015) and Gu et al. (2016) just to name a few. A feature of many of these models is that credit is not feasible under anonymity. As mentioned above, we innovate by using a different notion of anonymity.

A notable exception is Araujo (2004) in which he shows that a credit equilibrium is still feasible even with strict anonymity. Credit in his model is maintained by a trigger-strategy propagated through "word-of-mouth". However, this only works if agents are sufficiently patient and the population is sufficiently small. Our equilibrium exists for infinitely many agents with arbitrary discount factors.

The paper closest to ours is Wang and Li (2023) who also study credit equilibria in a weakly anonymous environment. We share some similar results but, and importantly, there are also some interesting and significant differences leading to complementary insights. Firstly, our environment is different. For example, borrowers only sometimes match with lenders in our model. This is important as it rules out some of the mechanisms used in their paper to prevent the use of multiple accounts. We present a mechanism that is robust in this respect.[10] Furthermore, we construct a different equilibrium in the sense that ours is a finite reputation equilibrium while theirs is an infinite reputation equilibrium[11] This allows us to use backwards induction and derive more analytical results. For example, we show that such an equilibrium always exists and that these equilibria cannot be not-too-tight.

Friedman and Resnick (2004) discuss pseudonyms in a game theoretic context. They study an infinitely repeated prisoner's dilemma where, similar to our model, agents are pseudonymous. They conclude, as we do, that maintaining cooperation is costly as building up reputation is costly. Nevertheless, they miss many of the specifics of credit in their treatment that we believe are important to highlight.

The work of Kehoe and Levine (1993) and Alvarez and Jermann (2000) share with our work the feature that endogenous debt limits arise in equilibrium. Agents lack commitment, and they can thus only credibly promise to repay a certain amount of debt.

Our construction of reputation systems and the costs they bear is also reminiscent of the "starting small" literature. For example in Watson (1999) agents play repeatedly a game

[9] A comprehensive review of this literature can be found in Lagos et al. (2017).

[10] A key question in both papers is how to prevent borrowers from using a second account on the equilibrium path. The approaches taken are altogether different.

[11] Although we show that our equilibrium exists even if "reputation goes to infinity". In this sense, one might consider our approach more general. However, as already mentioned, the constructed equilibria are different and therefore it is not more general strictly speaking. Additionally, the terminology in Wang and Li (2023) is different. For example, their "increasing-credit-limit schemes depending on account age" is equivalent to our notion of reputation.

similar to a prisoner's dilemma where they cooperate or betray each other. Agents decide before the first game how payoffs evolve over time. There are high and low type agents with incomplete information about each other. He shows that cooperation between high types can be maintained regardless of initial beliefs about each other, as long as the relationship starts small enough in terms of what is at stake in the relationship. Hua and Watson (2022) study a similar game. Each period the first of two players chooses a so called trust level which determines what is at stake. Player two says if he wants to betray or corporate. They also show that in equilibrium, due to trade-offs, relationships start small in terms of payoffs and then increase in level until a maximum. This notion of "starting small" in both examples is similar to our set-up.

Another strand of literature studies the impact of credit information sharing and reporting. Elul and Gottardi (2015) study the effects and welfare impact of information restrictions on credit equilibria. Similar to our set-up, an entrepreneur's reputation is determined by his credit record on repayments and defaults. Brown and Zehnder (2007) demonstrate that information sharing in credit markets has especially strong effects on repayment in the absence of third-party enforcement and repeated trading relationships. Lastly, our work is related to Kocherlakota and Wallace (1998) who do not focus on pseudonymous record keeping, but on another form of limited record keeping, namely delayed record keeping.

The paper is structured as follows: Section 2 sets out the general environment, preferences and technology. Section 3 describes the equilibrium recursively and proves the existence of anonymous credit. Section 4 analyses an alternative approach with costly accounts.

2 The environment

fTime is discrete and indexed by $t = 0, 1, \ldots \infty$. There are two types of agents called *borrowers* and *lenders*. There is a measure one of each type.

Each period consists of two subperiods. During the first subperiod, agents enter the *borrowing stage* (BS) where each borrower matches bilaterally with a random lender with probability $\sigma \in (0, 1]$. Lenders can produce and sell a *credit good*, $q_t \in \mathbb{R}_0^+$, which the borrowers desire to consume.[12] At the end of the BS borrowers exit the economy stochastically with probability p and are immediately replaced by new borrowers entering the economy so that there is always a mass one of borrowers. In the second subperiod, agents enter the *repayment stage* (RS), in which agents stay matched until the end of the period. In this

[12] q_t can also be interpreted as quality instead of quantity which might be more appropriate for some applications.

stage, borrowers can produce the *settlement good*, $y_t \in \mathbb{R}$, at a linear cost.[13] Lenders on the other hand receive linear utility from consuming the settlement good. Both the credit and settlement goods are non-storable across subperiods. The borrower's and lender's expected lifetime utility are given by

$$U_t^b = \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[u(q_{t+j}) - y_{t+j}],$$

$$U_t^l = \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[-c(q_{t+j}) + y_{t+j}],$$

where $\beta \in (0,1)$ is the *discount factor*. Moreover, $u(q)$ is $\mathcal{C}^2$ and satisfies $u'(q) > 0$, $u''(q) < 0$, $u(0) = 0$, $\lim_{q\to\infty} u'(q) = 0$ and $\lim_{q\to 0} u'(q) = \infty$.[14] Similarly, $c(q)$ is $\mathcal{C}^2$ and satisfies $c'(q) > 0$, $c''(q) > 0$, $c(0) = 0$, $\lim_{q\to\infty} c'(q) = \infty$ and $\lim_{q\to 0} c'(q) = 0$. Therefore, it is socially optimal to produce q as $u(q) - c(q) > 0$ for some $q > 0$. In particular, let us define the *first best quantity* by q^* where $u'(q^*) = c'(q^*)$.[15]

When borrowers and lenders are matched during the BS, they have the opportunity to conduct the following trade: the lender produces q_t credit goods for the borrower on the spot and, as compensation, the borrower promises to produce b_t units of settlement goods for the lender in the subsequent RS. That is, borrowers finance their consumption of q_t by borrowing b_t from lenders, which is repayable by the end of the period. The terms of trade are determined by the borrower making a take-it-or-leave-it offer to the lender which the lender can either accept or reject. If the lender accepts, the credit good is produced. Otherwise the match is dissolved, no credit goods are produced and both parties proceed to the next RS.

We make the following assumptions on the information structure of the economy: all current and past actions are perfectly observable by anyone but there is limited knowledge about who committed those actions. To be precise, borrowers use an *account*[16], $a \in \mathcal{A}$ where $\mathcal{A}$ is any set with $|\mathcal{A}| = \infty$, when interacting with a lender and all actions undertaken by a given account are perfectly observable.[17] We denote any action x by x^a to denote that the action was committed with account a. Let us denote by $\xi_t^a = \{m_j^a, q_j^a, b_j^a, y_j^a\}_{j=0}^t$ the *history of past actions* which records all actions undertaken by account a up to period t where m_j^a

[13] We interpret y_t as net production of a borrower.

[14] The expectations operator captures the randomness implied by the matching process and the stochastic life expectancy.

[15] Because marginal cost and marginal utility are always equalised at every level of production for the settlement good, the social planner only cares about the production of the credit good.

[16] Equivalently, one could use the term *pseudonym*.

[17] We assume that accounts are unique. Hence, it is not possible to imitate another agent by taking on her history.

is an indicator variable equal to one if the account was matched with a lender in period j and equal to zero if not. Importantly, however, the ownership of these accounts is private information and borrowers may create as many accounts as they wish. Borrowers can create new accounts during each RS at zero cost (we will modify this in section 4).

3 Costless accounts

We will now construct a particular equilibrium in recursive form. Let us first define *reputation* as a mapping from the account's history to a natural number. The reputation of account a at time t is denoted by n_t^a. We can then define reputation recursively: if an account has no history (i.e. $m_j^a = 0$ for all $j \leq t$), then $n_t^a = 0$ and

$$n_{t+1}^a = \begin{cases} \min\{n_t^a + 1, N\} & \text{if } m_t^a = 1 \text{ and } b_t^a \leq y_t^a \\ n_t^a & \text{if } m_t^a = 0, \\ -1 & \text{else.} \end{cases}$$

We say that an account is a *deviator* if $n_t^a = -1$ (more on that below). Intuitively, all accounts start with zero reputation and gain reputation by matching with lenders and repaying their debt. As the definition makes clear, we assume that there is some maximum level of reputation $N \in \mathbb{N}$ that can be achieved, so that $n_t^a \in [-1, 0, 1, \ldots, N]$. If accounts do not repay their debt, their account is marked as a deviator (i.e. $n_{t+1}^a = -1$).

Going forward, we want to construct an equilibrium with the following particular properties. First, agent's actions are conditioned only on reputation and not the entire history of actions of their accounts. Another way of saying this is that, in equilibrium, reputation is a sufficient statistic for the history of actions. We can therefore proceed without referring to the history of actions explicitly. Second, only pure-strategy equilibria are considered. Third, we assume that lenders believe that other lenders will not trade with accounts which are marked as deviators. Given that lenders believe that other lenders refuse to trade with deviators, it is in fact optimal for them to do so as well.[18] This behaviour by lenders makes defaulting costly for borrowers. Fourth, we restrict ourselves to equilibria where $b_{t,n} \geq b_{t,n-1}\ \forall n > 0$.[19] That is, in equilibrium a higher level of reputation will never

[18] The reason is straightforward: the borrower's incentive to honour their debt is to avoid being marked as a deviator. As a result, accounts already marked as deviators have "nothing to lose" and will therefore default on any promise. Lenders would anticipate this and therefore refuse to lend to a deviator.

[19] Even though it might seem that an equilibrium necessarily needs to satisfy this condition, we conjecture that there could be equilibria that do not. However, given that this assumption simplifies some of the proofs, we will focus on equilibria that do satisfy this condition.

decrease the amount that borrowers borrow. Moreover, we need to make the following assumption:

Assumption 1 *Borrowers maximally hold two accounts with $n > 0$.*

Let us briefly discuss the meaning, necessity and importance of this assumption. To be clear, we are not restricting how many accounts borrowers can use over their lifetime. Rather the restriction is on the simultaneous holding of more than two accounts with positive reputation. Therefore, we still allow borrowers to actively use multiple accounts. Using this assumption we are able to prove that in the equilibrium we study, borrowers have no incentive to use a second account even though they could. Naturally, it would be preferable to prove this more generally, but it also seems reasonable that if a borrower has no incentives to use a second account if she can use up to two, it would be very surprising to learn that this would change as soon as the borrower could use three or more accounts.[20]

Finally, a couple of words on presentation. A borrower's individual state variables include the reputation of all her accounts. However, given the assumption that borrowers can hold only up to two accounts with $n > 0$ and the fact that borrowers can always replace deviator accounts with new accounts[21], we will omit the reference to any deviator accounts and simply write a borrower's state space consisting of the reputation of two accounts (all other accounts are either deviators or have zero reputation). Also, we will suppress time indexes unless unclear and denote any generic variable x by x_n to denote it being conditional on reputation n.

We proceed by describing the model in the following order: first, we start from the second subperiod, the RS, then, second, move on to describe the bargaining problem. Third, we discuss the BS-value function and finally, we define the equilibrium formally.

3.1 The repayment stage

Consider the problem of a borrower entering the RS with two accounts. Let us generally call these accounts 1 and 2, where each account has reputation (n^1, n^2) respectively. Let us adopt the convention that if we write the borrower's state variables as (n^1, n^2) we implicitly assume that the first entry corresponds to the account used in the last subperiod (so that we do not need to introduce an additional variable in the state space). Let us now denote

[20] The proof relies on exhausting all possibilities and showing that all options involving opening a second account are dominated by a strategy that focuses on using only one account. Of course, by enlarging the space of accounts that borrowers could hold simultaneously, the proof becomes exponentially more complicated and lengthy. We will leave the generalization of the proof for future work.

[21] It is easy to see that since accounts are free to create, it is a (weakly) optimal strategy to create a new account as soon as an account has been marked as a deviator.

the value function of a borrower who has matched with a lender in the previous subperiod by:

$$W_1^b(b, n^1, n^2) = \max_{\eta} -\eta b + \beta\Big[\eta V^b(\min\{N, n^1+1\}, n^2) + (1-\eta)V^b(0, n^2)\Big] \tag{1}$$

where $\eta \in \{0,1\}$ is the optimal decision of a borrower with accounts (n^1, n^2) to repay its debt b and let us denote it's solution by $\eta(b, n^1, n^2)$. If they repay ($\eta = 1$), they incur linear costs to produce for lenders and their first account will gain reputation (unless reputation is already at maximum reputation). If they decide to default ($\eta = 0$), then the borrower does not suffer any disutility from producing the settlement good but the agent's first account will be marked as a deviator and, as explained in the previous section, the borrower then optimally replaces the account with a new one with no reputation such that $n^1 = 0$ in the next period. It is straightforward to see that $\eta(b, n^1, n^2) = 1$ if and only if the following *no-default* (ND) constraint is satisfied:

$$B(n^1, n^2) \equiv \beta\Big(V^b(\min\{N, n^1+1\}, n^2) - V^b(0, n^2)\Big) \geq b \tag{2}$$

where $V^b(n^1, n^2)$ is the BS value function for a borrower with accounts (n^1, n^2). We define the *debt-limit* as the left-hand side of (2) which is the value above which a borrower with accounts (n^1, n^2) will default (i.e. $\eta = 0$). Hence, borrowers only repay their debt if the amount of debt is less or equal than the cost of losing the account's reputation.

Next, let us consider the case where borrowers did not match with any lenders in the previous subperiod. In that case, borrowers did not issue any debt and the value function can be written as:

$$W_0^b(n^1, n^2) = \beta V^b(n^1, n^2). \tag{3}$$

Similarly, we denote the RS-value function of a lender who enters the RS with claims on debts b issued by a borrower with an account with reputation n^1 by:

$$W^l(b, n^1) = b(1-p)\rho(b, n^1) + \beta V^l, \tag{4}$$

where $\rho(b, n^1)$ is the lender's belief that a borrower using an account with reputation n^1 decides to pay back its debt b. Note that because a borrower's reputation on its second account is unobservable to the lender, lenders cannot perfectly anticipate a borrowers default. They must therefore form expectations about the probability of default. In addition, lenders can perfectly anticipate that p borrowers will exit the economy and therefore not pay back their debt.

3.2 Terms of trade

The terms of trade are determined by the borrower making a take-it-or-leave-it offer to the lender. The offer takes the form (q, b), that is it specifies the production of credit goods by the lender, q, and the amount borrowed in terms of settlement goods by the borrower, b. The lender can either accept or reject. If the lender accepts, the credit good is produced and debt is recorded. If the lender rejects, the match is dissolved. A borrower with accounts (n_1, n_2) solves:

$$\max_{q,b} u(q) + (1-p)\left[W_1^b(b, n^1, n^2) - W_1^b(0, n^1, n^2)\right]$$
$$\text{s.t. } W^l(b, n^1) - W^l(0, n^1) = c(q),$$

such that the borrower maximises her surplus taking into account the lender's participation constraint. Using the RS value functions (1) and (4), the problem can be rewritten as:

$$\begin{aligned} &\max_{q,b} u(q) - \eta(b, n^1, n^2)(1-p)b \\ &\text{s.t. } (1-p)\rho(q, n^1)b = c(q). \end{aligned} \tag{5}$$

Let us denote the resulting equilibrium values as q_n and b_n.

3.3 The borrowing stage

The value function for a borrower entering the BS with accounts (n^1, n^2) can be written as

$$\begin{aligned} V^b(n^1, n^2) =& \sigma \max\left\{u(q_{n^1}) + (1-p)W_1^b(b, n^1, n^2), u(q_{n^2}) + (1-p)W_1^b(b, n^2, n^1)\right\} \\ &+ (1-\sigma)(1-p)W_0^b(0, n^1, n^2). \end{aligned} \tag{6}$$

When agents enter the BS they either match with a lender or no match occurs. If they meet a lender they choose which account they use. As seen from the bargaining problem (5), the choice of account influences the terms of trade as different accounts may have different levels of reputation. Finally, observe that the borrower only proceeds to next period's RS if she does not exit the economy which occurs with probability $(1-p)$.

From equation (6) one can also infer that a borrower never uses a second account given the first account has positive reputation if and only if:

$$u(q_n) + (1-p)W_1^b(b_n, n, 0) > u(q_0) + (1-p)W_1^b(b_0, 0, n) \quad \forall n > 0. \tag{7}$$

In simple terms, if using an account with positive reputation yields more instantaneous utility from consuming and a higher continuation value compared to using an account with zero reputation, a second account is never used.[22]

The BS function for a lender can be written similarly:

$$V^l = \sigma \int_0^N \Big(- c(q_n) + W^l(b_n, n)\Big) dF(n) + (1-\sigma)W^l(0,0)$$

where $F(n) \in [0,1]$ is the *distribution of reputation*, i.e. $F(n)$ is the probability of matching with an account with reputation less or equal to n. Intuitively, if lenders match they encounter a random borrower with some given reputation n. Since borrowers search with accounts of varying reputation levels, the lender's counterparty possesses a stochastic reputation level with a known distribution function, $F(n)$. More generally, let us also define $G(n^1, n^2) \in [0,1]^2$ as the *distribution of accounts* where $G(n^1, n^2)$ is the probability that a random borrower has accounts with reputation equal or less to (n^1, n^2) respectively.[23] Let us also define their corresponding probability mass functions by $f(n)$ and $g(n^1, n^2)$ such that $F(n) = \sum_{\hat{n}=0}^{n} f(\hat{n})$ and $G(n^1, n^2) = \sum_{\hat{n}^1=0}^{n^1} \sum_{\hat{n}^2=0}^{n^2} g(\hat{n}^1, \hat{n}^2)$.

3.4 Equilibrium

Now we define the equilibrium. Specifically, we examine a *stationary*, *symmetric*, and *no-voluntary-default* equilibrium. Stationarity here means that $F(n)$ and $G(n^1, n^2)$ remain constant over time. Symmetry indicates that agents with identical state variables act in the same manner. No-voluntary default implies that borrowers always choose to repay when given the chance to do so on the equilibrium path (if borrowers randomly exit, they cannot repay).[24]

[22] At first, this may seem to be obvious since we assumed that more reputation allows to trade more. However, the issue is more subtle as borrowers could use the second account to eventually default on her promise while using the primary account as a "back-up". As we will see below however, in our equilibria this cannot occur.

[23] One can characterise $F(n)$ and $G(n^1, n^2)$ quite easily as they can be derived from an underlying law of motion based on the definition of reputation and the fact that agents stochastically exit the economy. However, as will soon become clear, knowing $F(n)$ is not necessary to characterise the equilibrium. Hence, we do not derive it explicitly.

[24] Our equilibrium definition is based on the concept of a perfect Bayesian equilibrium. That is, our equilibrium will be defined by strategies and beliefs such that strategies are sequentially rational and beliefs are derived, if possible, by Bayes rule. See Mas-Colell et al. (1995) for a formal definition.

Definition 1 (Reputation equilibrium) *A stationary, symmetric and no-voluntary default equilibrium with a monotone reputation system and given maximum reputation N is given by a list of credit good consumption $\{q_n\}_{n=0}^{N}$, debts $\{b_n\}_{n=0}^{N}$ and debt limits $\{B_n\}_{n=0}^{N}$ such that*

1. *the RS-value functions are given by (1), and (3),*
2. *debt limits are determined by (2),*
3. *(q_n, b_n) solve the bargaining problem (5),*
4. *the borrower's BS-value function is given by (6),*
5. *borrowers always decide to repay, i.e. $\eta = 1$,*
6. *lender's believe that borrowers default on any off-equilibrium offers made, i.e. $\rho(b, n) = 1$ only if $b = b_n$.*

We proceed by characterising the equilibrium. First, given our assumption on the beliefs of lenders, we can rewrite the lender's participation constraint in the bargaining problem (5) to

$$b_n = \frac{c(q_n)}{(1-p)}. \tag{8}$$

That is, given lender's belief that borrowers will always repay if given the chance to do so, the lender's expected utility from receiving debt b is $c(q)/(1-p)$ where $1/(1-p)$ is the default premium which arises from the fact that p borrowers exit exogenously. Therefore, the borrower's offer equalises the lender's cost with their expected benefit.

Second, since lenders belief that any-off equilibrium offer will not be repaid, the bargaining problem reduces to:

$$\max \Big\{ u(q_n) - c(q_n), 0 \Big\}.$$

Therefore, it must be that $q_n \le \bar{q}$ for all n where $\bar{q}$ is given by $u(\bar{q}) = c(\bar{q})$. Since the surplus is increasing for any $q \le q^*$, let us from now on assume that this is the case which implies that $q_n < \bar{q}$ for any n.

Third, from this we can use condition (2) and derive the following necessary condition for repayment to be always optimal:

$$\frac{c(q_n^1)}{1-p} \le B(n^1, n^2) \tag{9}$$

for any (n_1, n_2) satisfying $g(n_1, n_2) > 0$. That is, for every distribution of accounts $g(n_1, n_2)$ that occurs in equilibrium borrowers need to be willing to repay their debt.

Observe that since lenders do not know the reputation of the borrower's second account, the following condition ensures repayment:

$$\frac{c(q_n^1)}{1-p} \leq \bar{B}(n^1) = \min_{n^2}\{B(n^1, n^2)\}. \tag{10}$$

This is a sufficient condition for (9) to hold. If condition (9) holds then any possible borrower with (n^1, n^2) will repay even if $g(n^1, n^2) = 0$ in equilibrium.

We can then show the following:

Lemma 1 *(7) is always satisfied if (10) holds.*

Proof. Given (10) borrowers never default. Given that we have assumed a monotone reputation system, we know that $b_0 \leq b_1, ..., \leq b_N$. Using (8) this implies $q_0 \leq q_1, ..., \leq q_N$. If $q_N \leq q^*$ then $S(n) = u(q_n) - c(q_n)$ is weakly increasing in n. The borrower's utility is then simply the discounted sum of $\sigma S(\hat{n})$ where $\hat{n}$ is the reputation of any account used. But then since $S(n)$ is weakly increasing in n and because the best strategy to increase n as fast as possible is to use only one account, it implies (7). ■

That is to say, there is no incentive for borrowers to open a second account if debt limits are sufficiently tight so that default is not beneficial. If defaulting is never an option, then the borrower's best strategy is to use one account to accumulate reputation as fast as possible to increase the sum of discounted future surpluses $u(q) - c(q)$.

But now observe that if borrowers never hold a second account, i.e. $g(n^1, n^2) = 0$ for any $n^1 > 0$ and $n^2 > 0$, then (9) reduces to:

$$\frac{c(q_n)}{1-p} \leq B(n, 0). \tag{11}$$

Of course, so far we have only shown that (10) implies (7). Whether or not (11) also implies (7) is not a priori clear. However we can show that this is in fact the case.

Proposition 1 *If debt limits are given by (11) then borrowers only use one account, i.e. (11) implies (7).*

Proof. See appendix A.1.

The proposition implies that ensuring that borrowers with only one account are willing to repay, as specified in condition (11), also implies that borrowers never resort to use a second account. This may not be immediately apparent, as (11) does not imply that borrowers with multiple accounts are inclined to repay their debts. This raises the question of whether there exists a profitable deviation, where a borrower creates a second account and potentially defaults in the future. However, proposition 1 demonstrates that, although it might be advantageous for a borrower to default if they end up with multiple accounts, it is not beneficial to create a second account initially. The intuition behind this result is that regardless of whether a borrower intends to default or not, concentrating efforts on one account is the optimal strategy, leading to the accumulation of the most reputation and thus yielding the greatest benefits.[25]

Going forward, we can therefore simplify the notation by dropping any reference to the second account's reputation as, according to proposition 1, it is always zero (for example, $V^b(n^1, 0) = V^b(n)$).

Moreover, by combining (1), (3) and (6) and the fact that only one account is ever used, the BS-value function takes the following form:

$$V^b(n) = \sum_{j=0}^{\infty} \frac{(\beta(1-p)\sigma)^j}{(1-(1-\sigma)\beta(1-p))^{j+1}} \sigma S_{n+j}, \tag{12}$$

where $S_n \equiv u(q_n) - c(q_n)$ for all $n \leq N$ and $S_n = S_N$ for all $n > N$.

Lemma 2 *Any sequence $\{q_n\}_{n=0}^N$ which satisfies (11) and (12) is a reputation equilibrium.*

Proof. Proposition 1 indicates that (11) implies (7). Furthermore, (7) implies $g(n^1, n^2) = 0$ for any $n^1 > 0$ and $n^2 > 0$. Given this, we immediately see that (9) implies (11). Finally, the value function (1), (3) and (6) can be collapsed into equation (12). ■

Proposition 2 *In a reputation equilibrium $q^* = q_n$ for all n cannot be an equilibrium.*

Proof. Suppose $q^* = q_n$ for all n is an equilibrium. Then by (12) it follows that $V^b(n) = V^b$ for all n. But then according to (11) $c(q^*) = 0$ which is a contradiction. ■

This is not so surprising as trading the first best quantity q^* at every level of reputation implies that reputation conveys no benefits and therefore the loss of reputation, the punishment for defaulting, has no effect. As a result, the equilibrium cannot be sustained.

Going forward, it is useful to differentiate between two types of equilibria.

[25] The mechanism employed in Wang and Li (2023) to prevent multiple accounts being used in equilibrium is different. They assume that accounts are punished if the account is observed to be inactive for one period. In their environment borrowers and lenders always match and therefore observing that an account was not used implies that another account was used. Such a mechanism would not work here as borrowers and lenders do not always match. Therefore, observing that an account did not trade could either be because a borrower did not match with a lender or because a different account was used.

Definition 2 (Not-too-tight and too-tight equilibria) *A reputation equilibrium is called not-too-tight if (11) holds with equality for all n. Otherwise, we call the equilibrium too-tight.*

The "not-too-tight" terminology originates from Alvarez and Jermann (2000) and implies that $b_n = B_n$ for all n. That is, if an equilibrium is not-too-tight then the borrower is indifferent between repaying and defaulting for each level of reputation. Therefore, in each match, lenders lend the maximum amount of debt such that the borrower is willing to repay. This is the most natural equilibrium to study as it maximises the gains from trade between a borrower and a lender while ensuring that borrowers always repay.

3.4.1 Not-too-tight equilibria

Let us for now assume that $b_n = B_n$ for all n. We can show the following:

Proposition 3 *In a not-too-tight equilibrium it must be that $q_n = 0$ for all n.*

For illustrative purposes we assume $\sigma = 1$ and $N = 1$. **Proof.** Consider the no-default constraints (2) for $n = 0$ and $n = 1$

$$B_0 = \beta[V^b(1) - V^b(0)] \geq \frac{c(q_0)}{(1-p)}, \tag{13}$$

$$B_1 = \beta[V^b(1) - V^b(0)] \geq \frac{c(q_1)}{(1-p)}, \tag{14}$$

which imply that $B_1 = B_0$. Given we study a not-too-tight equilibrium, this implies $b_0 = b_1$ and from (8) it follows that $q_0 = q_1$. But then $V^b(1) = V^b(0)$ and $B_1 = B_0 = 0$ which implies $q_0 = q_1 = 0$. ■

A general proof can be found in the appendix A.2. The intuition for this result can be understood in the following way: in the simplified case where $N = 1$ borrowers either have reputation ($n = 1$) or they have none ($n = 0$). In this case, a borrower's debt limit depends on the value of reputation, $V^b(1) - V^b(0)$, which is independent of the borrower's current level of reputation (see equations (13) and (14)). Therefore, a borrower with and a borrower without reputation has the same debt limit. Since in a not-too-tight equilibrium borrowers borrow up to the maximum level of debt such that they are willing to repay, it then implies that borrowers borrow the same amount irrespective whether they have reputation or not. But this is problematic because this makes reputation not valuable in the first place since the amount of trade for a borrower with $n = 0$ is the same as for a borrower with $n = 1$. If reputation has no value, there is no incentive to repay and lenders are thus not willing to accept any amount of debt.

How should we interpret this result? A valid interpretation of the not-too-tight equilibrium is that it is the only surviving equilibrium if we apply the intuitive criterion by Cho and Kreps (1987). That is to say, as long as an offer satisfies (11), the lender should not believe that the borrower will default. But then as long as $q < q^*$, the borrower would optimally borrow as much as possible, and the lender should believe that borrowers repay. Thus, condition (11) must hold with equality (which is to say that the equilibrium is not-too-tight). But, as this section has shown, (11) holding with equality implies that debt limits have to be zero.

Intuitively, borrowers and lenders have an incentive to lend "too much" to accounts with little reputation and therefore dilute the value of reputation (which they take as given). Again, one arrives at this conclusion only if one assumes the intuitive criterion in which case there has to be some "mechanism" to prevent lenders from lending too much and diluting the value of reputation.

In our approach going forward, we will be agnostic about how this mechanism looks like and stick to definition 1 (and hence the concept of a Perfect-Bayesian equilibrium). However, it is not hard to see that a slightly modified equilibrium definition would provide such a mechanism. For example, if we mark borrowers as deviators not only for defaulting but also for making off-equilibrium offers, lenders would reject such offers even given the intuitive criterion. This is because borrowers would default on any off-equilibrium offer since they will be marked as deviators anyway.

3.4.2 Too-tight equilibria

From now on, we will focus on too-tight equilibria. Thus, it must be that (11) is not holding with equality for at least one n. We proceed by first defining an optimal reputation equilibrium, then derive sufficient conditions for such an optimal reputation equilibrium to exist and then finally show that this optimal reputation equilibrium always exists (naturally, this also shows that reputation equilibria always exist).

We start by defining an optimal reputation equilibrium:

Definition 3 (Optimal reputation equilibrium) *An optimal reputation equilibrium solves*

$$\max_{\{q_n\}_{n=0}^{N}} V^b(0) \ s.t. \ V^b(\min\{N, n+1\}) - V^b(0) \geq \frac{c(q_n)}{\beta(1-p)} \quad \forall n \text{ where } V^b(n) \text{ is given by } (12). \tag{15}$$

Observe that an optimal reputation equilibrium is a reputation equilibrium which maximises a borrower's lifetime utility so that the borrower repays.[26] We can then derive the following proposition:

Proposition 4 *A sufficient condition for an optimal reputation equilibrium is* $V^b(0) = \bar{V}(\hat{q})$ *where* $\hat{q}$ *and* $\bar{V}(\hat{q})$ *are determined by*

$$\frac{u'(\hat{q})}{c'(\hat{q})} = 1 + \frac{1 - \beta(1-p)}{\sigma\beta(1-p)}, \tag{16}$$

$$\bar{V}(\hat{q}) = \frac{\sigma[u(\hat{q}) - c(\hat{q})]}{1 - \beta(1-p)} - \frac{c(\hat{q})}{\beta(1-p)}. \tag{17}$$

Proof. Using the ND-constraint for $n = N$, inserting (12) for $n = N$ and solving for $V^b(0)$ yields $V^b(0) \leq \bar{V}(q_N)$ where $\bar{V}(q_N)$ is given by (17). The upper bound, $\bar{V}(q_N)$, depends solely on q_N. The first order condition of (17) with respect to q_N yields (16). If $V^b(0) = \bar{V}(\hat{q})$ then the equilibrium must solve (15) because any $V^b(0) > \bar{V}(\hat{q})$ can never satisfy (12) for $n = N$ given that $\bar{V}(\hat{q})$ is maximal at $q_N = \hat{q}$. ∎

Hence, $\bar{V}(\hat{q})$ is the highest value of $V^b(0)$ that can be achieved and if we find a sequence $\{q_n\}_{n=0}^N$ that achieves $\bar{V}(\hat{q})$ we have found an optimal reputation equilibrium. Of course, it is not a priori clear whether the upper bound $\bar{V}(\hat{q})$ can be attained throughout the entire parameter space. Let us now consider the kind of sequences $\{q_n\}_{n=0}^N$ that achieve $\bar{V}(\hat{q})$. To gain some intuition, it is useful to consider the simplified case where $N = 1$ and $\sigma = 1$ (again, accounts either have reputation or they have none). The ND-constraints for $n = 1$ simplifies to:

$$V_1^b - V_0^b = S_1 - S_0 = \frac{c(q_1)}{\beta(1-p)}, \tag{18}$$

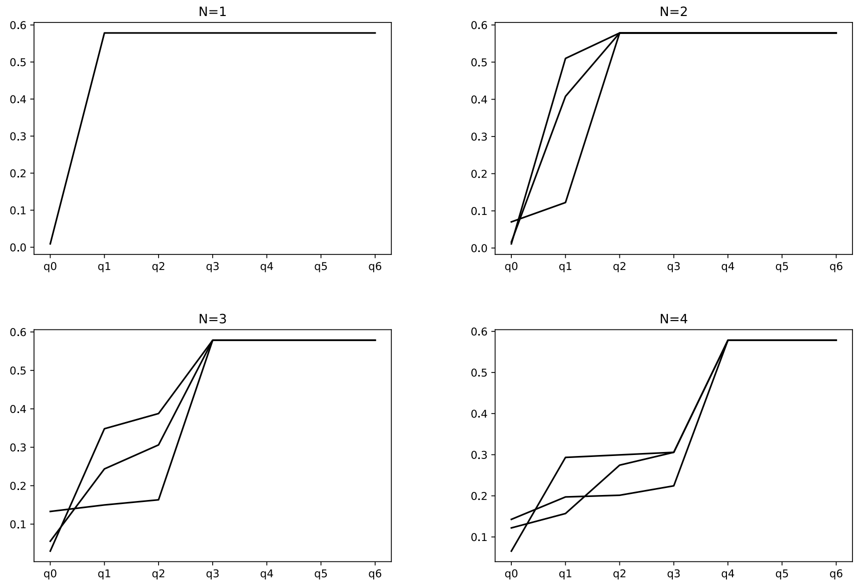
while the ND-constraint for $n = 0$ is satisfied if $q_0 \leq q_1$. The sufficient condition derived in proposition 4 implies that optimally $q_1 = \hat{q}$ and q_0 is set as such that (18) holds. If the implied q_0 is between zero and q_1 the equilibrium exists. From (18) we can easily see that $q_0 \leq q_1$ is always satisfied as $S_1 > S_0$ requires $q_1 > q_0$. On the other hand, $q_0 \geq 0$ holds if and only if $S_0 \geq 0$ which, according to (18), is the case if:

$$u(\hat{q}) \geq c(\hat{q}) \frac{1 + \beta(1-p)}{\beta(1-p)}. \tag{19}$$

These equations have a clear interpretation: equation (18) tells us that the value of having reputation, $V_1^b - V_0^b$, is given by higher surpluses which can be obtained with reputation, $S_1 - S_0 > 0$, and this value has to optimally equal the gain of defaulting, $\frac{c(q_1)}{\beta(1-p)}$. Moreover,

[26] The lender's expected lifetime value is always zero because the borrower, by making a take-it-or-leave-it offer, appropriates all the surplus.

Figure 1



Notes: This figure shows different sequences that do not violate the no-default constraints and all reach the maximum lifetime utility, i.e. $V^b(0) = \bar{V}(\hat{q})$.

even though $q_1 = q^*$ might be feasible, it is in general not optimal. The reason is that trading q^* would require a high level of reputation to incentivise borrowers to not default. This in turn is only achievable if q_0 is sufficiently small. However, due to the concavity of $u(q)$ it is optimal to set $q_1 < q^*$ in order to increase q_0. Finally, the upper bound can only be achieved if (19) is satisfied. Intuitively, the highest value of reputation is attained if $q_0 = 0$. For some parameter values however, even this value is not sufficient to incentivise agents to repay $\hat{q}$. However, as we show next, one can increase N to ensure that the upper bound can be achieved.

In the more general case where $N > 1$ we find that an optimal reputation equilibrium can be implemented with infinitely many sequences. This indeterminacy follows from the fact that in order to achieve the upper limit $\bar{V}(\hat{q})$ only two equations need to hold with equality: $q_N = \hat{q}$ and the ND-constraint for $n = N$. The other ND-constraints do not need to hold with equality. Thus, there are $N + 1$ variables to determine and two equations to pin them down. One way to find possible solutions is to compute them numerically. Figure 1 plots several sequences with different N which all achieve $\bar{V}(\hat{q})$.[27]

There are however also closed form solutions. Consider the following: set $q_N = \hat{q}$ and $q_n = 0$ for all $n < N - 1$. Notice, that the ND-constraint for all $n < N - 1$ are in that

[27] We assumed the following functional forms: $u(q) = \frac{q^{1-\eta}}{1-\eta}$ and $c(q) = q$. The parameters that were used are: $\beta = 0.96$, $\eta = 0.5$, $\sigma = 0.5$ and $p = 0.1$.

case automatically satisfied. The implied solution for q_{N-1} must however satisfy $0 \leq S_{N-1}$ (no negative surplus possible) and $S_{N-1} < S_N$ (otherwise the ND-constraint for $n = N-1$ would be violated). We can show the following:

Proposition 5 *There always exists a reputation equilibrium such that* $V^b(0) = \bar{V}(\hat{q})$ *for a particular* $N = \hat{N}$.

Proof. See appendix A.3.

In the proof we derive $\hat{N}$ analytically. As we have seen in the special case where $N = 1$ and $\sigma = 1$, there was the possibility that the harshest possible punishment (setting $q_0 = 0$) was not sufficient to guarantee that agents repay their debt associated with trading $\hat{q}$. The proposition indicates that we can always increase N and thereby increase the scope for punishment in order to guarantee that an equilibrium always exists.

In conclusion, we have shown that credit can, in principle, be anonymous and not be reliant on any form of collateral. According to proposition 5 such credit equilibria do always exist provided that N is chosen appropriately. However, while such credit equilibria always exist proposition 4 indicates that they are costly in the sense that the punishment for default implicitly is imposed on young agents too. There is therefore a trade-off between making defaulting costly and letting young agents consume.

4 Costly accounts

So far we have assumed that borrowers can create accounts for free. We now modify our original model by making accounts costly to create. We believe this modification to be highly practically relevant as its implementation is simple. In fact, there are already many providers of blockchain wallets which ask for a fee in order to set up a new wallet.[28]

4.1 Modification to the baseline model

We make three modifications to the environment. First, each time an account is opened up in the RS, the borrower is required to pay a *fee* $\kappa \in \mathbb{R}^+$ in terms of settlement goods. We consider two cases: In the first case the fee is considered a real cost and therefore a dead-weight loss. In the second case, the fee is viewed to be a transfer to some benevolent authority managing the accounts and which redistributes those fees (for example, this could be the government or a private platform). Let us call the redistributed fees *transfers* and

[28] For example to open up the wallets Trezor Model T and Ledger Nano X a user pays $100-$300 (source: CNET Money).

denote the per-capita transfer by τ. In order to nest both of these cases into one, let us assume that only a share $\epsilon \in [0,1]$ of these costs can be redistributed as transfers and we will study the two polar cases.

Second, we modify the recording technology. To be specific, let us redefine the history of actions to be $\xi_t^a = \{m_j^a, q_j^a, b_j^a, y_j^a, k_j^a\}_{j=0}^t$ where k_j^a is an indicator variable which equals one if the fee was paid by account a in period j.

Third, we assume that accounts are deleted if the owner exits the economy. One could easily modify the model to make this an outcome. For example, assume that accounts need to pay a tiny fee each period in order to keep the account active. However, to keep notation to a minimum we introduce this as a restriction in the environment.

In addition to these changes in the environment, we will construct a different equilibrium compared to the case without fees. Let us therefore re-define the reputation n_t^a to be:

$$n_t^a = \begin{cases} 1 & \text{if } k_j^a = 1 \text{ for some } j < t, \text{ and } b_t^a \leq y_t^a, \\ -1 & \text{else.} \end{cases}$$

Let us call accounts with $n = 1$ to be *active accounts* and, as before, accounts with $n = -1$ to be deviator accounts. That is, an account is active if the account has paid the fee sometime in the past and the account has always repaid it's debts. Otherwise, an account is considered a deviator. We maintain the same assumption concerning the lender's beliefs: lenders will never trade with deviator accounts (i.e $n = -1$) and reject any off-equilibrium offers, i.e. $\rho(b, n) = 1$ only if $n = 1$ and $b = b_1$. But different to before, there is no longer any notion of increasing one's reputation over time. Moreover, let us assume that all active accounts receive transfers τ.

We will study an equilibrium where borrowers will only ever use one account and always repay endogenously. While we do not prove this formally, it should be clear that the reasoning behind proposition 1 should apply to this case too. Therefore, we do not need to keep track of multiple accounts as we did in the previous section.

4.2 Equilibrium

We first look at the RS-value function of a borrower who owns an active account ($n = 1$) who has issued b debt:

$$W_1^b(b) = \max_{\eta_b, \eta_k} \eta_b(\tau - b) - \eta_k \kappa + \beta \Big(\mathbb{I} V_1^b + (1 - \mathbb{I}) V_{-1}^b \Big) \tag{20}$$

where $\eta_b \in \{0,1\}$ and $\eta_k \in \{0,1\}$ are both discrete choices indicating whether the borrower repays its debt or creates a new account respectively. Let us denote the solutions by $\eta(b)$ and $\eta(k)$ respectively. Furthermore, let us denote $\mathbb{I} = \mathbb{1}(\eta_b = 1 \vee \eta_k = 1)$ which indicates whether the borrower will enter the next period with an active account. Similarly, for a borrower with an inactive account:

$$W^b_{-1}(0) = \max_{\eta_k} -\eta_k \kappa + \beta\Big(\mathbb{I} V^b_1 + (1-\mathbb{I})V^b_{-1}\Big). \tag{21}$$

From (21) one can easily see that $\eta_k = 1$ if and only if:

$$\kappa \leq \beta(V^b_1 - V^b_{-1}). \tag{22}$$

Clearly, this is a necessary condition for an equilibrium to exist because if this condition is not satisfied borrowers would never hold an active account. Furthermore, from equation (20), we can conclude that borrowers will repay their debt ($\eta(b) = 1$) if and only if the following no-default constraint is satisfied (conditional on 22 holding):

$$B(\kappa, \tau) \equiv \kappa + \tau \geq b \tag{23}$$

where, similar to the previous section, $B(\kappa, \tau)$ is the debt limit which corresponds to the maximum amount of debt which the borrower is willing to repay. We observe from the no-default constraint (23) that borrowers only repay their debt if the amount of debt minus transfers is less or equal than the cost of setting up a new account. Notice, that both a higher fee κ and a higher transfer τ increases the debt limit (the latter because transfer are only received by active accounts). Naturally, in equilibrium, the no-default constraint has to be satisfied in order that borrowers always repay their debt, i.e. $\eta(b) = 1$.

Next, we write the bargaining problem between a lender and a borrower. The same way as before, the borrower makes a take-it-or-leave-it offer:

$$\begin{aligned} &\max_{q,b} u(q) - (1-p)b \\ &\text{s.t. } b(1-p)\rho(b,n) = c(q). \end{aligned}$$

Given our assumption on lender's belief we again find that:

$$b(1-p) = c(q) \tag{24}$$

for any equilibrium offer $b = b_1$.

Furthermore, the BS-value function of the borrowers with an active and deviator account are, respectively, given by:

$$V_1^b = \sigma[u(q_1) + (1-p)W_1^b(b_1)] + (1-\sigma)(1-p)W_1^b(0), \tag{25}$$

$$V_{-1}^b = (1-p)W_{-1}^b(0). \tag{26}$$

The same holds for the RS- and BS-value functions of the lenders which are given by $W^l(b_1) = (1-p)b_1 + \beta V^l$ and $V^l = \sigma[-c(q_1) + W^l(b_1)] + (1-\sigma)W^l(0)$.

In equilibrium the amount of paid fees must equal the amount of redistributed fees in each period (as settlement goods cannot be stored). As the mass of agents paying the fee is p (new entrants) and the one receiving it is $(1-p)$ (all agents without new entrants) it follows that

$$\epsilon \cdot p \cdot \kappa = (1-p)\tau. \tag{27}$$

We can now define the equilibrium formally.

Definition 4 (Costly accounts equilibrium) *A stationary, symmetric and no-voluntary-default equilibrium with costly accounts is given by credit good consumption q_1, debts b_1, debt limits B_1 and transfers τ such that*

1. *the value functions are given by (20), (21), (25) and (26),*
2. *borrowers are willing to open accounts, (22),*
3. *borrowers always decide to repay, (23),*
4. *q_1 and b_1 satisfy (24),*
5. *the size of the transfer is determined by (27)*

for any κ.

Next, we can insert (24) and (21) into (25):

$$V_1^b = \frac{\sigma S_1 + (1-p)\tau}{1-\beta(1-p)}$$

where $S_1 \equiv u(q_1) - c(q_1)$. Similarly, we insert (24) and (20) into (26):

$$V_{-1}^b = -\kappa(1-p) + \beta(1-p)\frac{\sigma S_1 + (1-p)\tau}{1-\beta(1-p)}.$$

Using these two equations we find that:

$$V_1^b - V_{-1}^b = \sigma S_1 + (1-p)(\kappa + \tau) \tag{28}$$

which tells us that the value of having an active account is the ability to borrow with some probability, σS_1, the value of transfers received with probability $(1-p)$ in the next superiod, $(1-p)\tau$, and since fees only have to be paid once, the value of the fee paid with probability $(1-p)$ in the next subperiod, $(1-p)\kappa$. In a next step we prove existence of the costly accounts equilibrium.

Proposition 6 *A sufficient condition for a costly account to exist is:*

$$\bar{\kappa}(q_1) \equiv \frac{\beta\sigma(u(q_1) - c(q_1))}{1 - \beta(1 - p(1-\epsilon))} \geq \kappa \geq \frac{c(q_1)}{1 - p(1-\epsilon)} \equiv \underline{\kappa}(q_1) \tag{29}$$

for any κ and q_1. Furthermore, there always exists some $q_1 > 0$ such that $\bar{\kappa}(q_1) > \underline{\kappa}(q_1)$.

Proof. We can derive the lower bound by combining (23) with (24) and (27):

$$\kappa \geq \frac{c(q_1)}{1 - p(1-\epsilon)} \equiv \underline{\kappa}(q_1).$$

Furthermore, the upper bound is found by inserting (27) and (28) into (22):

$$\bar{\kappa}(q_1) \equiv \frac{\beta\sigma(u(q_1) - c(q_1))}{1 - \beta(1 - p(1-\epsilon))} \geq \kappa.$$

Next, we want to show that there always exists some q_1 for which $\bar{\kappa}(q_1) > \underline{\kappa}(q_1)$. Let us first define $\Omega(q_1) \equiv \bar{\kappa}(q_1) - \underline{\kappa}(q_1)$. Then observe that $\Omega(0) = 0$ and

$$\Omega'(q_1) = \frac{\beta\sigma(u'(q_1) - c'(q_1))}{1 - \beta(1 - p(1-\epsilon))} - \frac{c'(q_1)}{1 - p(1-\epsilon)}.$$

But then observe that $\lim_{q_1 \to 0} \Omega'(q_1) = \infty$ because $\lim_{q \to 0} u'(q) = \infty$ and $\lim_{q \to 0} c'(q) = 0$. This implies that there exists some $q_1 > 0$ such that $\bar{\kappa}(q_1) > \underline{\kappa}(q_1)$. ∎

The proposition tells us that the fee κ can neither be too large or too small. If the fee is too low then borrowers have an incentive to default on their debt. However, if the fee is too large then borrowers never create an account in the first place. Of course, the feasible range for κ depends on q_1 and, as we show, there always exists some q_1 so that there exists a feasible fee to incentivise borrowers to both create an account and repay their debts.

It is also easy to see that the equilibrium set is larger in the case where we interpret the fee not as a real cost, $\epsilon > 0$. In particular, going from $\epsilon = 0$ to $\epsilon = 1$ increases the upper bound and decreases the lower bound. Intuitively, being able to redistribute the fees increases the lifetime value of having an account (upper bound) and decreases the incentive to default (lower bound).

Finally, and again assuming that the fee can be redistributed, the result implies that a benevolent authority can always find some q_1 and associated κ that incentivises borrowers to participate and repay their debts. In the next section, we will ask: what is the optimal fee that a benevolent authority should charge?

4.3 Optimal costly accounts equilibrium

So far, we have shown that there are many combinations of q_1 and κ that constitute an equilibrium. We now want to show that there exists a uniquely optimal combination of q_1 and κ. As a result, from now on we will stick with the interpretation that the fee is not a real cost (i.e. $\epsilon > 0$) and can therefore be set by the benevolent authority. Let us then define the following:

Definition 5 (Optimal costly accounts equilibrium) *An optimal costly accounts equilibrium is given by q_1 and κ that maximises V_1^b subject to (27) and (29).*

That is, an optimal costly accounts equilibrium maximises the lifetime value of a borrower that newly enters the economy. The associated program is given by:

$$\begin{aligned}
&\max_{\kappa,q_1} -\kappa(1-p) + \beta(1-p)\frac{\sigma(u(q_1)-c(q_1)) + (1-p)\tau}{1-\beta(1-p)}, \\
&\text{s.t.} \;\; \tau = \frac{p}{1-p}\epsilon\kappa, \\
&\frac{\beta\sigma(u(q_1)-c(q_1))}{1-\beta(1-p(1-\epsilon))} \geq \kappa \geq \frac{c(q_1)}{1-p(1-\epsilon)}.
\end{aligned}$$

Let us denote the optimal quantity of credit good by $\tilde{q}$ and the optimal fee by $\tilde{\kappa}$. We assume that the benevolent authority redistributes all fees, i.e. $\epsilon = 1$.

Proposition 7 *The optimal costly accounts equilibrium is given by:*

$$\tilde{\kappa} = \frac{c(\tilde{q})}{1-p(1-\epsilon)},$$

$$\frac{u'(\tilde{q})}{c'(\tilde{q})} = 1 + \frac{1-\beta}{\sigma\beta}. \tag{30}$$

Proof. See appendix A.4.

One immediately observes that the optimal costly accounts equilibrium implies a credit good quantity which is below the first-best quantity, i.e. $\tilde{q} < q^*$. This is true even though q^* might be attainable. The intuition is similar to the reputation equilibrium: because the fee has to be financed in advance to borrowing and since agents are impatient, it is optimal to reduce the amount of borrowing by agents which allows account fees to be lowered. Another way of putting it is that at $q = q^*$ reducing q has a first order effect on κ but only a second order effect on $u(q) - c(q)$.

Furthermore, by comparing the optimal quantity of the costly accounts equilibrium (30) with the quantity consumed by a full reputation borrower ($n = N$) from the previous section, (16), one can immediately see that the amount consumed is strictly higher in the optimal costly accounts equilibrium.

5 Closing remarks

In this paper, we present a novel perspective on credit and anonymity, departing from the existing body of literature which either neglects agents' anonymity concerns entirely or imposes strict anonymity prerequisites that effectively rule out the existence of credit systems. Instead, our focus lies on pseudonymity, a particularly prevalent form of anonymity currently observed in many areas of the internet and, in particular, blockchains.

What can we learn from this exercise? First and foremost, there is often an assumption that anonymity and credit cannot coexist. Our analysis, from an economic standpoint, challenges this notion. We believe our rationale extends to credit in a broader context. This holds particular significance for blockchain-related endeavours aiming to integrate credit within blockchain networks. Secondly, our analysis shows that there are also costs in maintaining anonymous credit. The root of the costs is the impossibility to distinguish between first time entrants and entrants due to former default. This implies that if credit is at entrance kept low to punish former defaulters this also applies to first time entrants.

Lastly, we wish to address some potential concerns arising from our framework. Firstly, as our study explores merely a subset of all possible credit equilibria, we refrain from making definitive assertions about the optimality of credit systems. However, we conjecture that our analysis did indeed capture the most efficient credit equilibria. Secondly, a question arises regarding the significance of the assumption that borrowers only engage with one lender per period. It is evident that if borrowers were free to interact with an unlimited number of lenders, a dominant strategy would involve creating an infinite number of accounts, thereby reducing the debt limit to zero. While our current framework does not explicitly demonstrate this, we hypothesise that any cost of contacting additional lenders would suffice to prevent such a scenario. Thirdly, it is worth noting that our model simplifies many real-world complexities associated with credit. For example, we ignore potential heterogeneity in agents' ability to repay their debt. We leave all these considerations for future research.

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A Appendix

A.1 Proof of proposition 1

Proof. We show that (7) holds. In other words, we want to prove that it is never advantageous for a borrower to open up and gain reputation on a second account. To establish a contradiction, assume that the borrower opens a second account at time t_0, while already possessing a first account with reputation n^1. For brevity, we denote $\tilde{\beta} \equiv \beta(1-p)$. There are two primary scenarios concerning the reputation on the first account when the borrower opens the second account:

1. There exists an $n < n^1$ such that $q_{n^1} > q_n$. There are four feasible strategies which cover all possibilities:

   (a) Consider the case where the borrower opens a second account and accumulates reputation until it reaches $n^2 = n^1 + 1$. At this point, the borrower's state variables can be expressed as (n^1, n^1+1), assuming that this occurs in some period t_1. The *deviation value* can be formulated as follows:

   $$\tilde{V} = \mathcal{S} + \tilde{\beta}^{t_1-t_0-1}[u(q_{n^2-1}) - c(q_{n^2-1})] + \tilde{\beta}^{t_1-t_0} V(n^1, n^1+1)$$

   where $\mathcal{S}$ is the discounted payoff of using the second account between t_0 and $t_1 - 1$.[29] In t_0 there is another feasible strategy: alternatively, the agent could use the first account one more time in t_0 and then start using the second account so that in t_1 the agent's state variables are (n^1+1, n^1). This *alternative value* is then given by:

   $$V^* = [u(q_{n^1}) - c(q_{n^1})] + \tilde{\beta}\mathcal{S} + \tilde{\beta}^{t_1-t_0} V(n^1+1, n^1).$$

   The deviation value can only be optimal if it weakly exceeds the alternative value: $V^* \leq \tilde{V}$. Inserting the values and recognizing that $q_{n^1} = q_{n^2-1}$ and $V(n^1+1, n^1) = V(n^1, n^1+1)$ implies

   $$[u(q_{n^1}) - c(q_{n^1})](1 - \tilde{\beta}^{t_1-t_0-1}) \leq \mathcal{S}(1-\tilde{\beta}).$$

   Since there exists an $n^2 \leq n^1$ such that $q_{n^1} > q_{n^2}$ it must be that $\mathcal{S} < [u(q_{n^1}) - c(q_{n^1})] \sum_{j=0}^{t_1-t_0-1} \tilde{\beta}^j = [u(q_{n^1}) - c(q_{n^1})] \frac{1-\tilde{\beta}^{t_1-t_0}}{1-\tilde{\beta}}$ and therefore the previous inequality implies: $u(q_{n^1}) - c(q_{n^1}) < u(q_{n^1}) - c(q_{n^1})$. This is a contradiction.

   (b) Suppose the borrower opens a second account and builds up reputation until $n^2 \leq n^1$. He then switches accounts, borrows and repays with the first account so that the agent's state variables are (n^1+1, n^2) at this point. Without loss of generality, let us suppose the switch occurs in period t_1. The deviation value can be written as

   $$\tilde{V} = \mathcal{S} + \tilde{\beta}^{t_1-t_0}[u(q_{n^1}) - c(q_{n^1})] + \tilde{\beta}^{t_1-t_0+1} V(n^1+1, n^2) \tag{31}$$

   where $\mathcal{S}$ is the discounted payoff of using the second account between t_0 and $t_1 - 1$.[30] In t_0 there is another feasible strategy: alternatively, the agent could use the first account one more time in t_0 and then start using the second account so that the borrower's state variables are the same in $t_1 + 1$. This alternative value is then given by:

   $$V^* = [u(q_{n^1}) - c(q_{n^1})] + \tilde{\beta}\mathcal{S} + \tilde{\beta}^{t_1-t_0+1} V(n^1+1, n^2).$$

[29] The explicit expression is given by $\mathcal{S} = \sum_{n=0}^{n^2-2} \tilde{\beta}^{\hat{t}_n - t_0}[u(q_n) - c(q_n)]$ where $\hat{t}_n$ is the time where the borrower trades with the second account with reputation n.

[30] The explicit expression is given by $\mathcal{S} = \sum_{n=0}^{n^2-1} \tilde{\beta}^{\hat{t}_n - t_0}[u(q_n) - c(q_n)]$ where $\hat{t}_n$ is the time where the borrower trades with the second account with reputation n.

The deviation value can only be strictly beneficial if it exceeds the alternative value: $V^* \leq \tilde{V}$. Inserting the values yields:

$$[u(q_{n^1}) - c(q_{n^1})](1 - \tilde{\beta}^{t_1 - t_0}) < \mathcal{S}(1 - \tilde{\beta}).$$

Since there exists an $n^2 \leq n^1$ such that $q_{n^1} > q_{n^2}$ it must be that $\mathcal{S} < [u(q_{n^1}) - c(q_{n^1})]\frac{1-\tilde{\beta}^{t_1-t_0-1}}{1-\tilde{\beta}}$ and therefore the previous inequality implies: $u(q_{n^1}) - c(q_{n^1}) < u(q_{n^1}) - c(q_{n^1})$. This is a contradiction.

(c) Suppose the borrower opens a second account and builds up reputation until $0 \leq n^2 \leq n^1$. At some point, the agent defaults on the second account, and without loss of generality, let us assume this default occurs in period t_1:

$$\tilde{V} = \mathcal{S} + \tilde{\beta}^{t_1 - t_0} u(q_{n^2}) + \tilde{\beta}^{t_1 - t_0 + 1} \tilde{V}.$$

Importantly, the borrower's state variables and therefore the continuation value are the same before using the second account and defaulting on the second account. However, this would imply that the first account would never be used again and $\tilde{V} = V(n^1, 0)$. But since $\tilde{V} = V(n^1, 0)$ is a possible strategy with state variables $(0, 0)$, it must be that $V(0, 0) \geq V(n^1, 0) = \tilde{V}$. But according to the ND-constraint (11) this implies $q_n = 0$ for all $n < n^1$ and therefore $V(0, 0) < V(n^1, 0) = \tilde{V}$. This is a contradiction.

(d) Suppose the borrower opens a second account and builds up reputation until $n^2 \leq n^1$. Then the agent switches back to the first account, borrows from it, and immediately defaults on that account. Without loss of generality, let us suppose the switch and default occur in period t_1. The deviation value can then be written as:

$$\tilde{V} = \mathcal{S} + \tilde{\beta}^{t_1 - t_0} u(q_{n^1}) + \tilde{\beta}^{t_1 - t_0 + 1} V(0, n^2).$$

Observe, that in t_0 there is another feasible strategy: the agent defaults in t_0 on the first account and then starts using the second account so that in t_1 the agent's state variables are the same as in $t_1 + 1$. The alternative value is then given by:

$$V^* = u(q_{n^1}) + \tilde{\beta}\mathcal{S} + \tilde{\beta}^{t_1 - t_0 + 1} V(0, n^2).$$

The deviation value can only be strictly beneficial if it weakly exceeds the alternative value: $V^* \leq \tilde{V}$. Inserting the values yields:

$$u(q_{n^1})(1 - \tilde{\beta}^{t_1 - t_0 - 1}) < \mathcal{S}(1 - \tilde{\beta}).$$

Since there exists an $n^2 \leq n^1$ such that $q_{n^1} > q_{n^2}$ it must be that $\mathcal{S} \leq [u(q_{n^1}) - c(q_{n^1})]\frac{1-\tilde{\beta}^{t_1-t_0-1}}{1-\tilde{\beta}}$ and therefore the previous inequality implies: $0 < -c(q_{n^1})$. This is a contradiction.

2. There exists no $n < n^1$ such that $q_{n^1} > q_n$. Let us therefore denote $\hat{n}$ the lowest level of reputation such that $q_{\hat{n}} > q_n$ for all $n < \hat{n}$.

(a) The borrower trades with both accounts until the reputation of both accounts is given by $(\hat{n}_1, \hat{n}_2)$ where $\hat{n}_1 < \hat{n}$ and $\hat{n}_2 < \hat{n}$. The borrower then defaults on either of these accounts. Without loss of generality, let us suppose the default occurs on the second account and the switch and default occur in period t_1. The deviation payoff can be written as

$$\tilde{V} = \mathcal{S} + \tilde{\beta}^{t_1 - t_0} u(q_{n^2}) + \tilde{\beta}^{t_1 - t_0 + 1} V(\hat{n}_1, 0)$$

where $\mathcal{S}$ is the discounted payoff of using the first and second account between t_0 and $t_1 - 1$.[31] In t_0 there is another feasible strategy: alternatively, the agent

[31] The explicit expression is given by $\mathcal{S} = \sum_{n=0}^{\hat{n}_1 - n^1 + n^2} \tilde{\beta}^{\hat{t}_n - t_0}[u(q_0) - c(q_0)]$ where $\hat{t}_n$ is the time where

could default on the second account with reputation zero instead of trading with the account. This alternative value is then given by:

$$V^* = \mathcal{S}^* + \tilde{\beta}^{t_1-t_0}u(q_{n^2}) + \tilde{\beta}^{t_1-t_0+1}V(\hat{n}_1, 0)$$

where $\mathcal{S}^*$ is the discounted payoff of defaulting on the second account between t_1 and t_0.[32] Since $q_n = q_{n^1} = q_{n^2}$ for all $n < n^1$ and $n < n^2$, this implies $\mathcal{S}^* > \mathcal{S}$ if $q_{n^1} = q_{n^2} > 0$. This is a contradiction.

(b) The borrower does not default before either $n^1 = \hat{n}$ or $n^2 = \hat{n}$ is reached in period t_1. Without loss of generality, let us suppose $n^1 = \hat{n}$. It is easy to verify that we can now apply exactly the same argument as in case 1 and conclude that the borrower has no incentive to increase the reputation on the account with lower levels of reputation.[33] If the agent will not use the second account to accumulate further reputation then he either defaults on the account or he will never use it again:

i. The borrower defaults on the second account. However, according to the same argument as in subcase a), there is a better strategy where the borrower always defaults on the second account instead of accumulating reputation. This is a contradiction.

ii. The borrower never uses the second account again. The deviation value is then given by:

$$\hat{V} = \mathcal{S} + \tilde{\beta}^{t_1-t_0}V(\hat{n}_1, \hat{n}_2)$$

where $\mathcal{S} = [u(q_0) - c(q_0)]\frac{1-\tilde{\beta}^{t_1-t_0}}{1-\tilde{\beta}}$ is the discounted payoff of using the second account between t_1 and t_0. In t_0 there is another feasible strategy: instead of using the second account, the borrower could use only the first account. Let us denote the time when the borrower achieves $n^1 = \hat{n}$ as $t_1^* < t_1$. The alternative value is then given by:

$$V^* = \mathcal{S}^* + \tilde{\beta}^{t_1^*-t_0}V(\hat{n}_1, 0)$$

where $\mathcal{S}^* = [u(q_0) - c(q_0)]\frac{1-\tilde{\beta}^{t_1^*-t_0}}{1-\tilde{\beta}}$ is the discounted payoff of using the first account between t_1^* and t_0. The deviation value can only be strictly beneficial if it exceeds the alternative value, $V^* \leq \hat{V}$:

$$\frac{u(q_0) - c(q_0)}{1 - \tilde{\beta}} \geq V(\hat{n}_1, 0)$$

where we used the fact that $V(\hat{n}_1, 0) = V(\hat{n}_1, \hat{n}_2)$ since the second account will never be used from time t_1 onwards. This implies that the deviation value is bounded from above. But then there exists a second alternative strategy where the agent always defaults on the first account from t_1 onwards which yields

$$V^{**} = \frac{u(q_0)}{1 - \tilde{\beta}} > \frac{u(q_0) - c(q_0)}{1 - \tilde{\beta}} \geq \hat{V}.$$

This is a contradiction.

We conclude that opening a second account cannot be optimal. ■

the borrower trades with the second account with reputation n.

[32] The explicit expression is given by $\mathcal{S}^* = \sum_{n=0}^{\hat{n}_1-n^1+n^2} \tilde{\beta}^{\hat{t}_n-t_0}u(q_0)$ where $\hat{t}_n$ is the time where the borrower trades with the second account with reputation n.

[33] Essentially, the argument in case 1 is independent of the level of reputation on the second account as long as the second account has lower reputation than the first.

A.2 Proof of proposition 3

Proof. Consider the following two ND-constraints for $n = N$ and $n = N - 1$:

$$B_N = \beta(1-p)[V^b(N) - V^b(0)] \geq c(q_N),$$

$$B_{N-1} = \beta(1-p)[V^b(N) - V^b(0)] \geq c(q_{N-1}),$$

which implies that $B_N = B_{N-1}$. Given we study a not-too-tight equilibrium, this implies $b_N = b_{N-1}$ and from (8) it follows that $q_N = q_{N-1}$. Therefore, by (12) it follows that $V^b(N) = V^b(N-1)$. Next, consider the following two ND-constraints for $n = N - 1$ and $n = N - 2$:

$$B_{N-1} = \beta(1-p)[V^b(N) - V^b(0)] \geq c(q_{N-1}),$$

$$B_{N-2} = \beta(1-p)[V^b(N-1) - V^b(0)] \geq c(q_{N-1}).$$

But since $V^b(N) = V^b(N-1)$ we find that $B_{N-1} = B_{N-2}$. Given we study a not-too-tight equilibrium, this implies $b_{N-1} = b_{N-2}$ and from (8) it follows that $q_{N-1} = q_{N-2}$. Therefore, by (12) it follows that $V^b(N-1) = V^b(N-2)$. Applying this argument recursively implies that $q_n = q$ for all n. But then $V(n) = V(0)$ for all n and thus $B_n = 0$ for all n. As a result, $b_n = 0$ for all n and by (8) $q_n = 0$ for all n. ■

A.3 Proof of proposition 5

Proof. Consider the following sequence: $q_N = \hat{q}$ and $q_n = 0$ for all $n < N - 1$. The ND-constraints for $n < N - 1$ are thus satisfied. The ND-constraint for $n = N - 1$ is satisfied if $0 < q_{N-1} < q_N$. q_{N-1} is determined as such that the ND-constraint for $n = N$ is satisfied:

$$\beta(1-p)\Big[V^b(N) - V^b(0)\Big] = c(q_N). \tag{32}$$

To simplify notation, let us define $(1 + r) \equiv \frac{1}{\beta(1-p)}$. We use $S_n = 0$ for all $n < N - 1$ and rearrange (12). For $n = 0$ this yields:

$$V^b(0) = \left(\frac{\sigma}{r+\sigma}\right)^{N-1} \frac{\sigma}{1-(1-\sigma)\frac{1}{1+r}} S_{N-1} + \sum_{j=N}^{\infty} \left(\frac{\sigma}{r+\sigma}\right)^j \frac{\sigma}{1-(1-\sigma)\frac{1}{1+r}} S_N$$

$$V^b(0) = \left(\frac{\sigma}{r+\sigma}\right)^{N-1} \frac{1+r}{r+\sigma}\sigma S_{N-1} + \left(\frac{\sigma}{r+\sigma}\right)^N \sum_{j=0}^{\infty} \left(\frac{\sigma}{r+\sigma}\right)^j \frac{1+r}{r+\sigma}\sigma S_N$$

$$V^b(0) = \left(\frac{\sigma}{r+\sigma}\right)^{N-1} \frac{1+r}{r+\sigma}\sigma S_{N-1} + \left(\frac{\sigma}{r+\sigma}\right)^N \frac{1+r}{r}\sigma S_N.$$

For $n = N$ this yields:

$$V^b(N) = \sum_{j=0}^{\infty} \left(\frac{\sigma}{r+\sigma}\right)^j \frac{\sigma}{1-(1-\sigma)\frac{1}{1+r}} S_N$$

$$V^b(N) = \frac{1+r}{r}\sigma S_N.$$

We insert $V^b(0)$ and $V^b(N)$ into (32):

$$\frac{1+r}{r}\sigma S_N - \left[\left(\frac{\sigma}{r+\sigma}\right)^{N-1} \frac{1+r}{r+\sigma}\sigma S_{N-1} + \left(\frac{\sigma}{r+\sigma}\right)^N \frac{1+r}{r}\sigma S_N\right] = (1+r)c(q_N).$$

We can solve for S_{N-1}:

$$S_{N-1} = \left[\left(\frac{r+\sigma}{\sigma}\right)^N - 1\right]\frac{\sigma}{r}S_N - \left(\frac{r+\sigma}{\sigma}\right)^N c(q_N). \tag{33}$$

Next, we derive conditions under which $S_{N-1} < S_N$ and therefore $q_{N-1} < q_N$. We insert (33) into $S_{N-1} < S_N$:

$$S_N > \left[\left(\frac{r+\sigma}{\sigma}\right)^N - 1\right]\frac{\sigma}{r}S_N - \left(\frac{r+\sigma}{\sigma}\right)^N c(q_N).$$

We can apply the natural logarithm and solve for N:

$$\bar{N} \equiv \frac{\ln\left(\frac{(1+\frac{\sigma}{r})S_N}{\frac{\sigma}{r}S_N - c(q_N)}\right)}{\ln\left(\frac{r+\sigma}{\sigma}\right)} > N.$$

Next, we derive conditions under which $S_{N-1} \geq 0$ and therefore $q_{N-1} \geq 0$. We insert (33) into $S_{N-1} \geq 0$:

$$\left[\left(\frac{r+\sigma}{\sigma}\right)^N - 1\right]\frac{\sigma}{r}S_N - \left(\frac{r+\sigma}{\sigma}\right)^N c(q_N) \geq 0.$$

We can apply the natural logarithm and solve for N:

$$N \geq \frac{\ln\left(\frac{\frac{\sigma}{r}S_N}{\frac{\sigma}{r}S_N - c(q_N)}\right)}{\ln\left(\frac{r+\sigma}{\sigma}\right)} \equiv \underline{N}.$$

One can show that $\bar{N} - \underline{N} = 1$. Given that N is an integer value this means there always exists a unique $\bar{N} > N \geq \underline{N}$ as long as $\frac{\sigma}{r}S_N - c(q_N) > 0$. We can rearrange this condition, use $q_N = \hat{q}$ and find $(1+r)\Big(\frac{\sigma}{r}[u(\hat{q}) - c(\hat{q})] - c(\hat{q})\Big) = \bar{V}(\hat{q})$. But we know that at $q_N = \hat{q}$ the function $\bar{V}(q_N)$ is maximised. Hence, as long as $\bar{V}(q_N) > 0$ for some q_N, it follows that $\bar{V}(\hat{q}) > 0$. To show this, let us define:

$$\mathcal{O}(q) \equiv \frac{\sigma}{r}\Big[u(q) - c(q)\Big] - c(q).$$

Observe that $\mathcal{O}(0) = 0$ and $\lim_{q\to 0}\mathcal{O}'(q) > 0$ implies that $\bar{V}(q_N) > 0$ for some q_N. Therefore, the optimal reputation equilibrium always exists. ■

A.4 Proof of proposition 7

Proof.
We can insert the first constraint and write the Lagrangian:

$$\begin{aligned}
\mathcal{L} = \max_{q,\kappa,\mu^l,\mu^u} & -\kappa(1-p)\left[1 - \frac{\beta p\epsilon}{1-\beta(1-p)}\right] + \beta(1-p)\frac{\sigma(u(q)-c(q))}{1-\beta(1-p)} \\
& -\bar{\mu}\left[\kappa - \frac{\beta\sigma(u(q)-c(q))}{1-\beta(1-p(1-\epsilon))}\right] \\
& +\underline{\mu}\left[\kappa - \frac{c(q)}{1-p(1-\epsilon)}\right]
\end{aligned}$$

where $\underline{\mu}$ and $\bar{\mu}$ are the Lagrangian multipliers associated with each constraint. The sufficient conditions for an optimum are given by:

$$-(1-p)\left[1-\frac{\beta p\epsilon}{1-\beta(1-p)}\right]+\underline{\mu}-\bar{\mu}=0, \tag{34}$$

$$\beta(1-p)\frac{\sigma(u'(q)-c'(q))}{1-\beta(1-p)}+\bar{\mu}\left[\frac{\beta\sigma(u'(q)-c'(q))}{1-\beta(1-p(1-\epsilon))}\right]-\underline{\mu}\left[\frac{c'(q)}{1-p(1-\epsilon)}\right]=0, \tag{35}$$

$$\frac{\beta\sigma(u(q)-c(q))}{1-\beta(1-p(1-\epsilon))}\geq\kappa, \tag{36}$$

$$\kappa\geq\frac{c(q)}{1-p(1-\epsilon)}, \tag{37}$$

$$\bar{\mu}\left[\kappa-\frac{\beta\sigma(u(q)-c(q))}{1-\beta(1-p(1-\epsilon))}\right]=0,$$

$$\underline{\mu}\left[\kappa-\frac{c(q)}{1-p(1-\epsilon)}\right]=0.$$

First, we want to argue that $\underline{\mu}>0$. Suppose not. Then $\underline{\mu}=0$. By (34)

$$-(1-p)\left[1-\frac{\beta p\epsilon}{1-\beta(1-p)}\right]-\bar{\mu}=0.$$

But this is a contradiction since $\left[1-\frac{\beta p\epsilon}{1-\beta(1-p)}\right]>0$ and $\bar{\mu}\geq 0$. Thus, $\underline{\mu}>0$ and

$$\kappa=\frac{c(q)}{1-p(1-\epsilon)}.$$

Next, suppose that $\bar{\mu}=0$. In that case, we solve (34) for $\underline{\mu}$ and insert into (35):

$$\frac{u'(q)}{c'(q)}=1+\frac{1-\beta(1-p(1-\epsilon))}{\sigma\beta(1-p(1-\epsilon))}.$$

Because the left-hand side spans every positive real number and is strictly decreasing in q while the right-hand-side is a positive real number, it follows that a unique q solves that equation. Let us denote this candidate solution by q_1. For this to be a solution, q_1 must satisfy (36). To see that (36) is indeed satisfied at q_1, suppose that $\bar{\mu}>0$. In that case (36) and (37) imply:

$$\frac{c(q)}{1-p(1-\epsilon)}=\frac{\beta\sigma(u(q)-c(q))}{1-\beta(1-p(1-\epsilon))}.$$

This equation has to be satisfied for some q. To study the implied q, let us define:

$$\mathcal{A}(q)\equiv\frac{\beta\sigma(u(q)-c(q))}{1-\beta(1-p(1-\epsilon))}-\frac{c(q)}{1-p(1-\epsilon)}. \tag{38}$$

One can easily see that $\mathcal{A}(0)=0$, $\lim_{q\to 0}\mathcal{A}'(q)=\infty$ and $\mathcal{A}(\bar{q})<0$. This has a couple of implications. First, there must be two solutions to (38): $q=0$ and some $\dot{q}>0$. Second, it follows that $0<\tilde{q}<\dot{q}$ where $\tilde{q}=\arg\max_q\mathcal{A}(q)$. Third, as we assume full redistribution of the fees, $\tilde{q}$ is uniquely pinned down by:

$$\frac{u'(\tilde{q})}{c'(\tilde{q})}=1+\frac{1-\beta}{\sigma\beta}.$$

But observe that $\hat{q}=\tilde{q}$. But then we know that $\hat{q}<\dot{q}$ and therefore $\bar{\mu}>0$. Hence, the optimal allocation is $q=\hat{q}$. ■