Puzzling Comovements between Output and Interest Rates? Multiple Shocks are the Answer.

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Puzzling Comovements between Output and Interest Rates?
Multiple Shocks are the Answer.*

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Abstract

Stylized facts on output and interest rates in the U.S. have so far proved hard to match with business cycle models. But these findings do not acknowledge that the economy might well be driven by different shocks, and by each in different ways. I estimate covariances of output, nominal and real interest rate conditional on three types of shocks: Technology, monetary policy and sources of inflation persistence.

Conditional and technology and monetary policy, the results square with standard models. However these two shocks explain only about 50% of persistent movements in inflation which are key for understanding the overall comovements. The puzzle lies in modeling the shocks and transmission channels behind inflation persistence, not in standard transmission channels for technology and monetary policy errors.

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1 Introduction

The relationship between output and interest rates has long been important to macroeconomists and policymakers alike. But basic stylized facts on their comovements in U.S. data have proved difficult to match within a variety of modern business cycle models. For instance, King and Watson (1996) study three models: a real business cycle model, a sticky price model, and a portfolio adjustment cost model. They report that this battery of modern dynamic models fails to match the business cycle comovements of real and nominal interest rates with output:

While the models have diverse successes and failures, none can account for the fact that real and nominal interest rates are “inverted leading indicators” of real economic activity.

Calling interest rates inverted leading indicators refers to their negative correlation with future output. These correlations are typically measured once the series have been passed through a business cycle filter. Amongst the diverse failures mentioned by King and Watson, RBC models generate mostly a pro-cyclical real rate.

But in the data, the real rate is clearly anti-cyclical, it is negatively correlated with current output. As mentioned already, it is also a negative leading indicator. This commonly found pattern of correlation between bc-filtered output and short-term interest rates is depicted in Figure 1.4

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4King and Watson (1996, p.35). The inverted leading indicator property has been the subject of various empirical studies, for example Sims (1992) and Bernanke and Blinder (1992). The expression “negative leading indicator” is synonymous.

2When it can be applied without confusion, I use the phrase “business cycle filter”, or short “bc-filter”, to describe the bandpass filters developed and applied in Baxter and King (1999) and Stock and Watson (1999) or the filter of Hodrick and Prescott (1997, “HP”) since each eliminates nonstationary and other low frequency components from a time series. These filters differ mainly in that the typical bandpass filters eliminates not only cycles longer than 32 quarters but also those shorter than 6 quarters, while this latter high-frequency component is retained in the HP filter.

3This evidence is broadly in line with previous studies, see for instance the stylized facts collected by Stock and Watson (1999, Table 2) for bandpass-filtered U.S. data. The facts are also significant as can be seen from the confidence intervals plotted in Figure 10 of Appendix A.
What is the correct conclusion from a mismatch between implications from a dynamic model and stylized facts? Modern dynamic models always involve a joint specification of fundamental economic structure and driving processes. Model outcomes, such as the output-interest rate correlation, involve the compound effect of these two features. Yet, when “puzzling” findings are taken as evidence against a particular structural feature – such as sticky prices or portfolio adjustment costs – it is typically not acknowledged that the economy might alternatively be driven by different types of shocks that yield different effects within the given structure. Yet, more carefully, it is simply unclear whether dynamic models fail (or succeed) because of their transmission mechanisms or because of the nature of their driving forces.
To shed more light on this important issue, I provide empirical evidence about output-interest rate comovement conditional on three types of shocks: Technology shocks, monetary shocks and sources of inflation persistence. The first two of these also drive the models of King and Watson (1996). There are striking results of my decomposition, which are reported in section 3 using plots analogous to Figure 1:

- After conditioning on technology, the real rate is pro-cyclical and a positive leading indicator – just the opposite of its unconditional behavior. In response to such permanent growth shocks, this is a common outcome for variants of the neoclassical growth model, be they of the RBC or the New Keynesian variety (King and Watson 1996; Gali 2003; Walsh 2003; Woodford 2003).

- Conditional on monetary shocks the real rate is counter-cyclical and a negative leading indicator, which squares with simple New-Keynesian models, too.

- Like monetary shocks, but even stronger, persistent shocks to inflation induce anti-cyclical behavior of the real rate and cause it to be a negative leading indicator. These shocks also account for the bulk of comovements between output and the nominal rate.

Thus, the “output-interest rate puzzle” is already defused by conditioning on two widely-studied shocks: Technology and monetary shocks, which counteract each other. Such opposing effects of shocks to “supply” and “demand” are a general theme in Keynesian models (Bénassy 1995). To explain the overall behavior, in particular for nominal movements, it is important to focus attention on sources of inflation persistence.


4Money is neutral in RBC models, so they have not much to say here. Conditional on monetary shocks, output remains in steady state and correlations are zero.
or hours. In applying this general idea to output and interest rates, my specific approach is motivated by the fact that the “puzzle” in this area is typically expressed in terms of bc-filtered data.

The backbone of my calculations is a VAR for the joint process of (unfiltered) output, nominal and real interest rate. The VAR serves both as a platform for identifying the structural shocks and to model the bc-filtered covariances and correlations. The identified shocks are shocks to the unfiltered data. For instance, the technology shock has a permanent effect on output but it might also have important effects on economic fluctuations. The point of bc-filtered statistics is to judge models solely on those cyclical properties, not on their implications for growth (Prescott 1986). In this vein, the VAR is used to trace out the effects of shocks to the bc-filtered components of output and interest rates. This is done analytically using a frequency domain representation for the VAR and the bc-filters.

This paper is structured as follows: Section 2 lays out my VAR framework for identification of the shocks as well as for decomposing the filtered covariances. Results are presented in Section 3. Related literature is briefly discussed in Section 4. Concluding comments are given in Section 5.

2 Empirical Methodology

The variables of interest to my study are the logs of per-capita output\(^5\), the nominal as well as the real interest rate:

\[
Y_t = \begin{bmatrix} y_t \\ i_t \\ r_t \end{bmatrix} \tag{1}
\]

\(^5\)All quantity variables shall be per-capita without further mention.
Let us call their bc-filtered component $\tilde{Y}_t$. The goal is to model and estimate how structural shocks induce comovements between the elements of $\tilde{Y}_t$.

The backbone of all my calculations is a VAR. Owing to the real interest rate, $Y_t$ is not fully observable. So the VAR is not run directly over $Y_t$ but rather over a vector of observables $X_t$. As a benchmark, I specify a simple four-variable system using output growth, inflation, the nominal rate and a monetary policy measure constructed by Romer and Romer (2004):

\[
X_t = \begin{bmatrix}
\Delta y_t \\
\pi_t \\
i_t \\
m_t
\end{bmatrix}
\]  

(2)

The dynamics of $X_t$ are captured by a $p$-th order VAR:

\[
A(L)X_t = e_t = Q \varepsilon_t
\]  

(3)

where

\[
A(L) = \sum_{k=0}^{p} A_k L^k, \quad A_0 = I
\]

and $E_{t-1} \varepsilon_t = 0$, $E_{t-1} \varepsilon_t \varepsilon_t' = I$

The coefficients $A_k$ and forecast errors $e_t$ can be estimated using OLS. Identification of the structural shocks $\varepsilon_t$ will be concerned with pinning down $Q$. Since fewer shocks are identified than the VAR has equations, there remains an unidentified component without

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6For convenience, I dropped the constants such that $X_t$ is mean zero. This is without loss of generality since estimating a VAR from demeaned data is equivalent to running a VAR with constants.
structural interpretation.

The real rate is computed from the Fisher equation $r_t = i_t - E_t \pi_{t+1}$ where inflation expectations are given by the VAR. So $Y_t$ can be constructed from $X_t$ by applying a linear filter:

$$Y_t = H(L)X_t$$

where

$$H(L) = \begin{bmatrix}
(1 - L)^{-1}h_{\Delta y} \\
h_i \\
h_i - h_\pi \left( \sum_{k=1}^{p} A_k L^{k-1} \right)
\end{bmatrix}$$

and $h_{\Delta y}$, $h_i$ and $h_\pi$ are selection vectors such that $\Delta y_t = h_{\Delta y} X_t$ and so on.

The remainder of this section describes the following: First, how the structural shocks are identified (Sections 2.1, 2.2 and 2.3). This gives us $Q$ and the conditional dynamics of the unfiltered variables can be computed from $Y_t = H(L)A(L)^{-1}Q \varepsilon_t$. Second, how to apply a bc-filter to the structural components of $Y_t$ to obtain the decomposition of their auto-covariances (Section 2.4).

### 2.1 Technology Shocks

Following Gali (1999), technology shocks are typically identified as the only innovation to the permanent component of labor-productivity (output per hour). While both the measurement of hours and the treatment of their stationarity have been found to be contentious issues, hours are of no direct interest to my study.

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7 However, Fisher (2006) and Greenwood, Hercowitz, and Krusell (1997) employ an alternative definition of technology as specifically improving investments.

Instead of looking at labor productivity, I label innovations to permanent output (per capita) as “technology shocks” for the following reason: The predictions of standard models – RBC or New Keynesian – for output and interest rates remain identical, even when non-technology shocks have permanent effects on output. Appendix D argues in more detail how non-technology candidates such as government spending or changes in the workforce composition (Francis and Ramey 2005b) pose the same output interest rate puzzle as technology shocks do. If hours are stationary, there are no non-technology influences on permanent output and my identification is actually equivalent to Gali’s definition. Unit root tests for quarterly hours data, even favor the view of stationary hours over my sample. Not using hours data makes it also possible to use a monthly instead of a quarterly VAR.

The identifying restriction is that the first row of \( A(1)^{-1}Q \) is full of zeros, except for a positive entry in its first column:

\[
A(1)^{-1}Q = \begin{bmatrix}
  a_{11} & 0 & 0 & 0 \\
  \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot \\
\end{bmatrix}
\]

where \( a_{11} > 0 \)

Together with the orthogonality of the structural shocks, this identifies the first column of \( Q \), which is then computed as in Blanchard and Quah (1989).

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9. This is easy to see from \( y_t = (y_t - l_t) + l_t \) where \( l_t \) are log hours (per capita). A stochastic trend in output will be identical to the one of labor productivity if \( l_t \sim I(0) \).

10. The results are not reported here, but can be found at [http://www.elmarmertens.ch/thesis](http://www.elmarmertens.ch/thesis).

11. This number corresponds to the square root of the zero-frequency spectral density of output growth. Dots represent otherwise unrestricted numbers.

12. An alternative method, yielding the same results, would be the instrumental variables regressions of Shapiro and Watson (1988). This framework is more amenable to include overidentifying restrictions, such as the orthogonality of technology and monetary shocks. See the Appendix C for a description.
2.2 Monetary Shocks

Again following standard conventions, monetary shocks are defined as unexpected deviations from endogenous policy. With the Fed Funds rate as policy instrument, they are unexpected Taylor-rule residuals, just as in Christiano, Eichenbaum, and Evans (1999) or Rotemberg and Woodford (1997). Strictly speaking, I do not identify such shocks myself. Rather, the measure of Romer and Romer (2004) is hooked up as a fourth variable \( m_t \) to my VAR. This allows to keep the VAR small, whilst the measure of the Romers takes care of the Fed’s information about future activity and inflation – variables which typically influence endogenous policy. Without that information, my small VAR would likely produce the “price puzzle” by confounding an anticipatory increase in interest rates with an exogenous policy move.

Romer and Romer have recently constructed a measure from minutes of the Federal Open Market Committee (FOMC) and the Greenbook of the Federal Reserve Board of Governors (Fed) which explicitly accounts for the Fed’s policy intentions and for the Fed’s anticipation of future inflation and activity. The series has been constructed for each FOMC meeting from 1964 to 1997. It is based on a series of “Intended Fed Funds rates” for each FOMC meeting. Their policy measure is the residual from a regression of these policy intentions on Greenbook forecasts of activity and inflation. The details are described by Romer and Romer (2004) and the insightful discussion by Cochrane (2004).

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13 \( m_t \) in corresponds to \( \varepsilon_t \) in equation (14) of Romer and Romer (2004). The data is available from their website.

14 The VAR would spuriously document inflation to rise in response to a monetary tightening. That is the price puzzle. A partial but classic response would be to include inflation-forecasting variables, like commodity prices (Christiano, Eichenbaum, and Evans 1999). See Hanson (2004) and Giordani (2004) for a critical discussion.

15 The Greenbook publishes forecasts by the Fed’s staff for future real activity and inflation.

16 In constructing the series from FOMC minutes (prior to official targets) Romer and Romer (2004) found that even when the Funds rate was not the official policy instrument, policy makers’ thinking was fairly well shaped around informal fund rate targets. This supports the identification of policy shocks from interest rates over a period which featured different schemes of official monetary policy making (Bernanke and Blinder 1992; Bernanke and Mihov 1998).
Comparing the Romer series against technology shocks estimated from a three-variable VAR using only output growth, inflation and nominal rate shows that the two series are virtually uncorrelated. This squares nicely with the survey of McCandless and Weber (1995) who find no long-term effects of monetary policy on the real economy. Likewise the estimated technology shocks are very similar whether they are estimated from a VAR with or without the Romer series. Appendix C lays out a test strategy following Shapiro and Watson (1988) and cannot reject that the Romer measure is orthogonal to the technology shocks. The two identification strategies barely interfere with each other. My technology shocks would be estimated to be practically the same when disregarding the Romer measure and vice versa. For convenience only, I impose orthogonality in sample by projecting the Romer measure off the technology shocks as explained below. The Romer measure series is not iid and contains some persistence which is pruned by including it in the VAR similarly as it is done by Romer and Romer (2004) themselves.

Formally, the normalized monetary shock is the standardized residual $\varepsilon^m_t$ obtained from projecting the forecast error of the VAR’s Romer equation, $\varepsilon^m_t$ the fourth element of $\varepsilon_t$ in (3), off the technology shocks (denoted $\varepsilon_z^t$):

$$
\varepsilon^m_t = \beta_z \varepsilon^z_t + \tilde{\varepsilon}^m_t
$$

$$
\varepsilon^m_t \equiv \tilde{\varepsilon}^m_t / \sqrt{\text{Var}(\tilde{\varepsilon}^m_t)}
$$

The second column of $Q$ is then filled up with the slopes of regressing the forecast errors $e_t$ onto the time series of monetary shocks $\varepsilon^m_t$.

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17 Romer and Romer (2004) use the cumulated series of $m_t$ instead which is by construction a unit root process. This would however interfere with the long-run identification scheme of the technology shocks.

18 This regression follows from $e_t = Q\varepsilon_t$ and the orthogonality of the structural shocks. Even though the regression of $e_t$ on $\varepsilon_t$ is multivariate, it follows from the Frisch-Waugh-Lowell Theorem, that the regression slopes can be computed from separate, univariate regressions as well.
2.3 Inflation Persistence Shocks

Inflation persistence is a pertinent feature of the data ([Fuhrer and Moore 1995a]) but the nature of shocks and transmission channels behind this phenomenon are subject of an ongoing debate. For example, Chari, Kehoe, and McGrattan (2000) and King and Dotsey (2006) try to link inflation persistence to particular forms of nominal stickiness in the economy. A different approach is taken by the flexible price model of Dittmar, Gavin, and Kydland (2005), where inflation persistence is a function of the central bank’s interest rate policy.

Inflation persistence is also an important characteristic of fluctuations in my VAR which are not captured by technology and monetary shocks. Looking at the forecast error variance of inflation between the 2 and 20 years, half of these persistent fluctuations are unexplained. I am concerned with the associated comovements in output and interest rates and given the uncertain sources of inflation persistence an agnostic scheme has been chosen for the identification. Building on Uhlig (2004a, 2004b), the shock to “inflation persistence” is constructed to explain the most of inflation’s forecast error variance over a horizon of 2 to 20 years and to be orthogonal to technology and monetary policy errors. The sign of the shock is determined by making it raise inflation on impact. Details of the computations behind this procedure are described in Appendix [B]. In implementing the aforementioned orthogonality constraint, I extend the eigenvector computations used by Uhlig (2004b).

The scheme is agnostic is that it allows for the various sources of inflation persistence listed above. In a modest interpretation, it merely groups together a large part of fluctuations unexplained by technology and Romer shocks based on their effect on inflation persistence. This “shock” captures very well the persistent fluctuations not driven by technology and monetary policy. What is more, it turns out to be an important source for output interest rate comovements, too.

Related ideas have been expounded also by Dotsey (1999) and King and Lin (2005).
2.4 Decomposition of BC-Filtered Covariances

Summarizing the previous discussion, the impulse responses of the unfiltered variables in $Y_t$ are given by $H(L)A(L)^{-1}Q$. These do not only trace out the business cycle responses of $Y_t$ to the structural shocks $\varepsilon_t$, but also how the shocks induce growth as well as high-frequency variations. The motivation for bc-filtering is now to focus only on the business cycle effects. Formally, it remains to apply a bc-filter and to decompose the filtered lead-lag covariances into the contributions of the structural shocks. The computations are straightforward to perform in the frequency domain. A classic reference for the necessary tools is Priestley (1981). Similar techniques are employed by Altig et al. (2004) and Chari, Kehoe, and McGrattan (2006).

The analysis is applicable to a wide class of bc-filters, including the HP-Filter, the approximate bandpass filter of Baxter and King (1999) as well as the exact bandpass filter. For the computations it is key that the bc-filter can be written as a linear, two-sided, infinite horizon moving average whose coefficients sum to zero:

$$\tilde{Y}_t \equiv B(L)Y_t$$

$$\text{where } B(L) = \sum_{k=-\infty}^{\infty} B_k L^k$$

Business cycle filters have also been criticized for creating spurious cycles, originally by Harvey and Jaeger (1993) and followed by Cogley and Nason (1995) as well as in the discussion between Canova (1998a, 1998b) and Burnside (1998). Whilst most of these papers focused on the HP filter, their analysis also applies to the bandpass filter. But the bc-filtered statistics employed here can be perfectly justified from the perspective of model evaluation in the frequency domain: The goal is not to match data and model over all spectral frequencies, but only over a subset which is associated with “business cycles”. For the U.S. this is typically taken to be 6 to 32 quarters following the NBER definitions of Burns and Mitchell (Baxter and King 1999; Stock and Watson 1999). Formal concepts of model evaluation in this vein have been advanced by Watson (1993), Diebold, Ohanian, and Berkowitz (1998), as well as Christiano and Vigfusson (2003). Using the concept of the pseudo-spectrum this extends also to nonstationary variables, notwithstanding the analysis of Harvey and Jaeger.

Of course, some coefficients $B_k$ can be zero. So $B(L)$ could also be the first-difference filter. But meaningful bc-filters should also be symmetric, such that they have a zero phase shift. Otherwise, comovements over one frequency band, say business cycles, could be attributed by the filter to other frequencies, like growth.
and \( B(1) = 0 \)

The bandpass-filter is a such a symmetric moving average. It is explicitly defined in the frequency domain and most of my calculations are carried out in the frequency domain. For frequencies \( \omega \in [-\pi, \pi] \), evaluate the filter at the complex number \( e^{-i\omega} \) instead of the lag operator \( L \). This is also known as the Fourier transform of the filter which represents it as a series of complex numbers (one for each frequency \( \omega \)). Requiring \( B(1) = 0 \) sets the zero-frequency component of the filtered time series to zero. For instance, the bandpass filter\(^\text{22}\) passes only cycles between two and a half and eight years. For monthly data, it is specified as follows:

\[
B(e^{-i\omega}) = \begin{cases} 
1 & \forall |\omega| \in \left[\frac{2\pi}{8}, \frac{2\pi}{2.5}\right] \\
0 & \text{otherwise}
\end{cases}
\]

Since the bc-filtered variables in \( \tilde{Y}_t \) are covariance-stationary, their lead-lag covariances exist and so does their spectrum. They can be computed from the VAR parameters and the filters \( H(L) \) and \( B(L) \). To ease notation, the impulse responses of \( Y \) after applying the bc-filter are written as

\[
\tilde{C}(L) \equiv B(L)H(L)A(L)^{-1}Q
\]

\(^\text{22}\)Alternatively, the HP filter approximates a high-pass filter \(^\text{[King and Rebelo 1993]}\) and its Fourier transform is

\[
B(e^{-i\omega}) = \frac{4 \lambda (1 - \cos(\omega))^2}{1 + 4 \lambda (1 - \cos(\omega))^2}
\]

where \( \lambda \) is a smoothing parameter, conventionally set to 1600 for quarterly data. Likewise, the approximate Bandpass-Filter can be implemented by computing the Fourier transform of the (truncated) lag polynomial \( B(L) \) described by \(^\text{[Baxter and King (1999)].}\)
so that the bc-filtered spectrum can be expressed as

\[ S_{\tilde{Y}}(\omega) = \tilde{C}(e^{-i\omega})\tilde{C}'(e^{-i\omega})' \]  

(4)

For each frequency \( \omega \), this is simply a product of complex-numbered matrices\(^{23}\). The lead-lag covariance matrices of \( \tilde{Y}_t \) can be recovered from the spectrum in what is known as an inverse Fourier transformation

\[ \Gamma_k^{\tilde{Y}} = E\tilde{Y}_t\tilde{Y}_{t-k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\tilde{Y}}(\omega) e^{i\omega k} d\omega \]  

(5)

which can be accurately and efficiently computed using well-established algorithms\(^{23}\).\(^{24}\)

Since the structural shocks are orthogonal to each other, the decomposition of the covariances \( \Gamma_k^{\tilde{Y}} \) is straightforward. First, the spectrum is computed conditional on each shock. Then, the conditional lead-lag covariances follow from an inverse Fourier transformation, analogously to equation (5). To fix notation, the shocks are indexed by \( s \) and \( J_s \) is a square matrix, full of zeros except for a unit entry in its \( s \)'th diagonal element. The spectrum conditioned on shock \( s \) is

\[ S_{\tilde{Y}|s}(\omega) = \tilde{C}(e^{-i\omega}) J_s \tilde{C}'(e^{-i\omega})' \]  

(6)

\(^{23}\)The transposes are conjugate transpose, i.e. they flip also the sign of the imaginary components.

\(^{24}\)In Matlab for instance, fast Fourier algorithms are encoded in \texttt{fft} and \texttt{ifft}. For \( \omega \) I use an evenly spaced grid over the unit circle with 1024 respectively 512 elements depending on the persistence of the VAR (the discrete fast Fourier algorithms behind \texttt{fft} and \texttt{ifft} work best for powers of 2). Here is a rule of thumb for the accuracy of the discretized Fourier transformation: Call \( n \) the number of grid points and \( \lambda \) the largest eigenvalue (in absolute terms) of the VAR’s companion matrix. \( \lambda^n \) should be numerically close to zero to ensure accuracy over the entire range of frequencies. The reason is that discretizing the frequencies over \([-\pi;\pi]\) is analogous to approximating the complete dynamics by a finite number of impulse responses. For stationary variables, the impulse responses ultimately converge to zero. The rule of thumb picks \( n \) large enough to capture this. Computations can be sped up dramatically using \( S_\tilde{Y}(\omega) = S_\tilde{Y}(\omega)' = S_\tilde{Y}(-\omega)^T \) (where \( T \) is the simple, non-conjugate transpose) and by computing the spectrum only for frequencies where \( B(e^{-i\omega}) \neq 0 \).
Since $\sum_s J_s = I$ the conditional spectra add up to $S_Y(\omega)$. This carries over to the coefficients $\Gamma^k_{Y|s} = E(\tilde{Y}_t \tilde{Y}_{t-k}|s)$ from the inverse Fourier transformation of $S_{Y|s}(\omega)$.

$$\sum_s \Gamma^k_{Y|s} = \Gamma^k_Y$$

This VAR framework is also capable of handling unit roots in $Y_t$. By construction, $H(L)$ and thus $Y_t$ has a unit root such that $H(1)$ is infinite. For computing the bc-filtered spectrum $S_Y(\omega)$ in (4), $B(1) = 0$ takes precedence over this unit root. It is straightforward to check that $H(e^{i\omega})$ is well defined everywhere, except at frequency zero. So we can think of the nonstationary vector $Y_t$ as having a pseudo-spectrum $S_Y(\omega) = C(e^{-i\omega}) C(e^{-i\omega})'$ where $C(L) = H(L)A(L)^{-1}Q$ and which exists for every frequency on the unit circle except zero. Similar remarks apply to potential unit roots in the VAR such that some element(s) of $A(1)$ would be zero. As long as the solutions to the characteristic equation $A(z) = 0$ are on or outside, but not inside the unit circle, the computations above run through. Higher orders of integration, i.e. powers of unit roots $(1 - L)^d$ where $d$ is an integer, thus fit in this framework as well.\textsuperscript{25}

### 3 Results

This section presents the results for the VAR described in the previous section. Monthly data is taken from FRED\textsuperscript{26} on per-capita output (real Industrial Production), CPI inflation and the average nominal yield on three-month T-Bills for the U.S. from 1966 to 1996\textsuperscript{27}.

\textsuperscript{25} Even for a nonstationary VAR, the OLS estimates of its coefficients in $A(L)$ are consistent, they would even be super-consistent. The roots of $A(z)$ are the inverse of the eigenvalues to the VAR’s companion form matrix. The only computational issue is that this cannot handle VARs whose point estimates imply companion eigenvalues outside the unit circle.

\textsuperscript{26} Federal Reserve Economic Data, maintained by the Federal Reserve Bank of St. Louis \url{http://research.stlouisfed.org/fred2/}.

\textsuperscript{27} Output growth, inflation and interest are all expressed in annualized log-percentage rates. $i_t = \log(1 + I_t/100)$, $\Delta_y_t = 12 \cdot (\log Y_t - \log Y_{t-1})$ and $\pi_t = 12 \cdot (\log P_t - \log P_{t-1})$ where $I_t$ is the annualized nominal yield in percent, $Y_t$ and $P_t$ are the levels of real per-capita Industrial Production, respectively the PCE.
This sample is determined by the availability of the Romer shocks. Industrial production data is used in order to have a monthly data set, similar results are obtained from quarterly data using GDP data, albeit with reduced statistical significance.\footnote{Again, these additional results are available from the author upon request respectively at \url{http://www.elmarmertens.ch/thesis}.} Except for the interest rates all data is seasonally adjusted in this study. The VAR is estimated with 12 lags to ensure uncorrelated residuals. (See Appendix A for lag-length selection.) After accounting for initial values, the sample covers the period from 1967 to 1996.

To assess the statistical significance of the results, bootstrapped confidence intervals are computed for each shock. As discussed by (Sims and Zha 1999) these are best interpreted as the posterior distributions from a Bayesian estimation with flat prior. The small sample adjustment of Kilian (1998) is used to handle the strong persistence of the VAR\footnote{The largest root equals 0.979, see Table 2 in Appendix A. Furthermore, a rejectance sampling is applied considering only stable VARs such that the long run restrictions can be applied.}. In a first round, the small sample bias of the VAR coefficients is estimated from 1,000 Monte Carlo draws. In the second round, the posterior distribution is constructed from 2,000 draws using the VAR adjusted for the small sample bias. The procedure follows exactly Kilian (1998) where further details are given.

For the long-run identification it is important that all elements of $X_t$ are stationary. The critical elements of $X_t$ are here inflation and the nominal rate. Given their low power it is no wonder that standard Dickey-Fuller tests cannot reject the presence of a unit root in these variables. In the VAR context it is however more appropriate to use the covariate-augmented Dickey Fuller test of Hansen (1995), which has more power. It tests for the presence of unit roots directly in the context of the VAR equations and resoundingly rejects the unit root hypothesis \footnote{The $t$-statistics are $-11.87$ for inflation, respectively $-2.89$ for the nominal rate and the associated $\rho^{2}$ statistics are 0.43 and 0.54 which makes both $t$-statistics significant at the 1%, respectively 5% level.}.\footnote{As a robustness check, an alternative VAR has been specified allowing for a common trend in inflation and nominal rate. This specification yields qualitatively similar results to those obtained here. (The price deflator, as reported by FRED.)}
3.1 Real Interest Rate and Output

A key result of this paper is that technology shocks induce a strongly pro-cyclical real rate which is also a positive leading indicator for up to one year. This is depicted in Figure 2 which decomposes the filtered covariances between output and the real rate. Covariances add linearly, so they are a natural measure for the decomposition. The total covariances in Figure 2 are just a rescaling of the correlations reported in Figure 1 above.

Figure 2 shows further that monetary shocks induce negative covariances at leads between zero and a year and a half. Overall, monetary shocks appear to play a much smaller role in terms of explaining the overall covariation. Since they are essentially defined as the Fed rolling dice, it is no wonder that their impact is comparably small. Qualitatively similar, but quantitatively stronger are the comovements due to the inflation persistence shock. It causes the real rate to be strongly anti-cyclical and to be a negative leading indicator.

It is ambiguous to put a number like “percentage explained” on the decompositions, since covariances can be negative as well as positive. For a large part, the technology effects are offset by the covariances conditional on monetary shocks so that the shock to inflation persistence tracks the overall autocovariance function pretty well. But clearly, substantial comovements are induced by technology and monetary shocks, too. (Variance decompositions are discussed further in Section 3.3 below.)

Both the monetary autocovariances as well as those caused by the inflation persistence shock are highly significant, see Figure 3 respectively 4. But there is much larger uncertainty associated with the technology shocks (see Figure 5). This is a general feature of inference on long run effects in VARs (Christiano, Eichenbaum, and Vigfusson 2006). The 90% confidence interval covers zero practically at all leads and lags. Since the early inflation persistence shock has been replaced there with a permanent shock to inflation.). The results are available on http://www.elmarmertens.ch/thesis

16
Figure 2: Covariance Decomposition for Real Rate and Output

$cov(y_t, r_{t-k})$

Total
MP
Tech
InfP
Rest

Note: Bandpass-filtered moments, $cov(\tilde{y}_t, \tilde{r}_{t-k})$, computed from VAR described in Section 2. Monthly lags on the x-axis.
Figure 3: BC-Moments conditional on Monetary Shock

Note: Covariances $\tilde{E}_t \tilde{Y}_{t-k}$ conditional on monetary shocks for VAR described in Section 2. Correlations on the upper diagonal. Bandpass-filtered moments. Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light), 90% (middle), 68% (dark) and 38% (very dark). White line is median of bootstraps. Black line is point estimate of VAR. Monthly lags on the x-axis.
days of VARs, Sims (1987) has already advocated studying the shape and location of the posterior distribution instead of critical values alone. Indeed, the posterior distribution of $\text{cov} (\hat{y}_t, \hat{r}_{t-k})$ is highly skewed and clearly concentrated in the region of positive values for $k$ between zero and one year.

### 3.2 Nominal Interest Rate and Output

Turning to the nominal rate, it is striking that only monetary and inflation persistence shocks give rise to sizeable comovements with output, see Figure 6. The autocovariance functions associated with the two shocks are qualitatively similar to the overall behavior of a pro-cyclical nominal rate which is an inverted leading indicator for output after half a year and more. As for the real rate, these conditional autocovariances are clearly significant, see Figure 3 and 4. Qualitatively, technology shocks lead to a positively leading nominal rate, but these comovements are very small.

### 3.3 Impulse Responses and Forecast Error Variances

So far the cyclical behavior of output and interest rates has been described in terms of bandpass-filtered covariances. This subsection reports results on impulse response and variance decompositions for the unfiltered variables which corroborate the preceding analysis.

The first column of Figure 7 plots the response of $Y_t$ to a monetary policy shock. The shock leads to a contractionary increase in nominal and real interest rates for about a year which is followed by contractionary effects on output and inflation, which is similar to the results of Romer and Romer (2004). (Cochrane (2004) discusses the initial price puzzle evident in his and mine calculations.) This is consistent with counter-cyclical, negatively leading interest rates as found above for the bandpass filtered data.
Figure 4: BC-Moments conditional on Inflation Persistence Shock

Note: Covariances $E\tilde{Y}_t \tilde{Y}_t'_{t-k}$ conditional on inflation persistence shock for VAR described in Section 2. Correlations on the upper diagonal. Bandpass-filtered moments. Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light), 90% (middle), 68% (dark) and 38% (very dark). White line is median of bootstraps. Black line is point estimate of VAR. Monthly lags on the x-axis.
Figure 5: BC-Moments conditional on Technology Shock

Note: Covariances $E \tilde{Y}_t \tilde{Y}_{t-k}$ conditional on technology shocks for VAR described in Section 2. Correlations on the upper diagonal. Bandpass-filtered moments. Bootstrapped standard-errors bands with Kilian (1998)’s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light), 90% (middle), 68% (dark) and 38% (very dark). White line is median of bootstraps. Black line is point estimate of VAR. Monthly lags on the x-axis.
Figure 6: Covariance Decomposition for Nominal Rate and Output

Note: Bandpass-filtered moments, $\text{cov}(\tilde{y}_t, \tilde{i}_{t-k})$, computed from VAR described in Section 2. Monthly lags on the x-axis.
Note: Estimates from VAR, equation (2) described in Section 2. Responses of unfiltered variables to a one-standard deviation shock ($H(L)A(L)^{-1}Q$). Bootstrapped standard-errors bands with Kilian (1998)'s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light), 90% (middle), 68% (dark) and 38% (very dark). White line is median of bootstraps. Black line is point estimate of VAR. Monthly lags on the x-axis.
By construction, the technology shock raises output permanently. The second column of Figure 7 shows that this growth is accompanied by a significantly increased real rate for one to two years, which again matches the evidence discussed earlier. Because of the non-trivial effects of bc-filtering, it is not a foregone conclusion, that the picture emerging from the impulse responses should mirror the results for the bc-filtered comovements as it does here.

In order to better understand the shocks and transmission channels behind the inflation persistence shock, it is useful to study the associated impulse responses in the third column of Figure 7. Due to the sign restriction inflation increases on impact and – not necessarily but neither surprisingly – stays consistently positive for more than two years. This persistent rise in inflation is met by a very persistent increase in nominal rates, which is however not sufficiently commensurate to keep real rates from falling below steady state for up to a year. Real activity is accordingly stimulated for slightly more than two years. In my interpretation, these are instances of the Fed responding (at least initially) with an insufficient interest rate policy to expansionary shocks, like government spending. Adverse supply shocks are hardly compatible with this situation since both activity and inflation are increased.

As argued earlier, it is hard to measure “shares explained” for the covariance decompositions. Looking at the variance decompositions reported in Table 1, it is however clear that all three shocks are important for explaining movements in the VAR and that the unexplained remainder is very small. Looking at the bc-filtered variances in the bottom panel, technology is the key driver behind real rate fluctuations. It explains almost half of their bc-filtered variance. Monetary policy shocks and the inflation persistence shock explain each about half the fluctuations in the nominal interest rate, whereas the explanatory power of technology shocks is close to zero. Interestingly, at the bc-frequencies, technol-

---

32 For a critical discussion see for instance Canova (1998a) or King and Rebelo (1993).
33 Except for a brief positive spike in the seventh month.
Table 1: Variance Decompositions

<table>
<thead>
<tr>
<th>Shocks</th>
<th>From 1 to 12 lags</th>
<th>From 1 to 60 lags</th>
<th>From 1 to 120 lags</th>
<th>From 24 to 240 lags</th>
<th>Total Variance</th>
<th>Total BC-Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>y</td>
<td>i</td>
<td>r</td>
<td>π</td>
<td>y</td>
<td>i</td>
</tr>
<tr>
<td>MP</td>
<td>3.80%</td>
<td>41.10%</td>
<td>27.98%</td>
<td>5.66%</td>
<td>3.13%</td>
<td>18.17%</td>
</tr>
<tr>
<td>Tech</td>
<td>39.49%</td>
<td>0.99%</td>
<td>50.02%</td>
<td>51.41%</td>
<td>88.17%</td>
<td>45.57%</td>
</tr>
<tr>
<td>InfP</td>
<td>48.85%</td>
<td>48.92%</td>
<td>14.70%</td>
<td>32.08%</td>
<td>7.44%</td>
<td>4.55%</td>
</tr>
<tr>
<td>Rest</td>
<td>7.86%</td>
<td>8.99%</td>
<td>7.31%</td>
<td>10.85%</td>
<td>1.26%</td>
<td>4.85%</td>
</tr>
<tr>
<td>y</td>
<td>7.62%</td>
<td>17.67%</td>
<td>25.83%</td>
<td>9.94%</td>
<td>75.73%</td>
<td>25.31%</td>
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<tr>
<td>i</td>
<td>75.73%</td>
<td>3.66%</td>
<td>32.18%</td>
<td>43.19%</td>
<td>13.59%</td>
<td>3.66%</td>
</tr>
<tr>
<td>r</td>
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<td>32.50%</td>
<td>37.88%</td>
<td>3.06%</td>
<td>4.69%</td>
</tr>
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<td>8.99%</td>
<td>3.06%</td>
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<td>45.57%</td>
</tr>
<tr>
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<td>29.35%</td>
<td>41.13%</td>
<td>7.44%</td>
<td>4.55%</td>
</tr>
<tr>
<td>r</td>
<td>7.44%</td>
<td>72.44%</td>
<td>39.06%</td>
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<td>1.26%</td>
<td>4.85%</td>
</tr>
<tr>
<td>π</td>
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<td>4.85%</td>
<td>8.41%</td>
<td>8.70%</td>
<td>1.26%</td>
<td>4.85%</td>
</tr>
<tr>
<td>y</td>
<td>0.82%</td>
<td>6.04%</td>
<td>15.03%</td>
<td>26.05%</td>
<td>96.96%</td>
<td>21.92%</td>
</tr>
<tr>
<td>i</td>
<td>96.96%</td>
<td>10.37%</td>
<td>69.54%</td>
<td>43.42%</td>
<td>1.92%</td>
<td>6.19%</td>
</tr>
<tr>
<td>r</td>
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<td>82.13%</td>
<td>69.54%</td>
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<td>1.46%</td>
</tr>
<tr>
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<td>0.30%</td>
<td>1.46%</td>
</tr>
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<td>y</td>
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<td>18.09%</td>
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<td>100.00%</td>
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</tr>
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<td>18.09%</td>
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<td>0.00%</td>
<td>6.19%</td>
</tr>
<tr>
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<td>39.31%</td>
<td>40.40%</td>
<td>0.00%</td>
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</tr>
<tr>
<td>π</td>
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<td>8.70%</td>
<td>0.00%</td>
<td>6.19%</td>
</tr>
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<td>54.57%</td>
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<tr>
<td>r</td>
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<tr>
<td>π</td>
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<td>6.12%</td>
<td>1.93%</td>
<td>3.61%</td>
<td>6.30%</td>
</tr>
</tbody>
</table>

Note: Variance decompositions computed from the VAR described in Section 2. “MP” is the monetary policy shock and “InfP” the inflation persistence shock. The latter is computed as discussed in Section 2.3 based on the forecast error window from 24 to 240 lags. “BC-Variance” is bandpass filtered variance. Monthly lags.
ogy explains only about 17% of output fluctuations, the bulk being accounted for by the inflation persistence shock with 57%. The unfiltered variations in the VAR’s forecast errors yield qualitatively similar decompositions, except that technology shocks account for an ever increasing share in output variations. Please recall that the technology shock is constructed to completely account for output movements in the long run.

4 Related Literature

To overcome the output-interest rate puzzle, Beaudry and Guay (1996) and Boldrin, Christiano, and Fisher (2001) propose models with habit preferences and frictions to capital accumulation respectively sectoral factor immobility. This matches the real rate evidence by tweaking the transmission mechanism for a single kind of shock, namely technology. But the evidence presented in this study, suggests that the standard RBC mechanism for technology works fine. It is rather the interaction of several shocks leading to the “puzzling” evidence.

In this spirit, Rotemberg and Woodford (1997) report success with decision lags in a sticky price model. The only structural shock they identify are disturbances to monetary policy. But their solution to the output-interest rate puzzle is based on the interaction with other shocks, which are left unidentified. This is revealed by their impulse response functions (Rotemberg and Woodford 1997, Figure 1). Following a monetary shock, their model’s output responses are negative (respectively zero) at all lags whilst they are positive

Footnotes:
34 Beaudry and Guay (1996) recognize the importance of conditioning on technology, too. They use cointegrating properties between output, consumption and investment derived by King et al. (1991), which are similar in spirit to my specification described in Section 2.1. When conditioning on these permanent shocks, they report negative correlations between output growth and the unfiltered real rate. Since growth rates amplify high-frequency fluctuations instead of focusing on business cycle characteristics, these results are hardly comparable to my approach and the puzzle framed by King and Watson (1996).
35 Rotemberg and Woodford (1997) look only at output and the nominal rate. They use linear detrending instead of the stochastic procedures considered here. Still they find similar patterns of covariation and juxtapose their results to the puzzle posed by King and Watson (1996).
for the nominal rate. Since conditional lead-lag covariances are just convoluted impulse responses, they are negative (respectively zero) at all leads and lags. This contrasts with the changing signs in the unconditional covariances depicted in my Figure 1 respectively their Figure 2.

Likewise, Fuhrer and Moore (1995b) model the inverted leading indicator property of interest rates with multiple, non-structural shocks and couch their analysis just in terms of unconditional statistics. My paper is an empirical attempt to disentangle the underlying interaction of the various structural shocks.

5 Conclusions

An economic model specifies restrictions on how the economy responds to exogenous forces. Data may not conform to these predictions, either because the specified responses are wrong, or because the set of forces considered in the model does not sufficiently capture those impinging on the real world (or both).\footnote{A case in point is how Christiano and Eichenbaum (1992) add government spending shocks to RBC theory to resolve the Dunlop/Tarshis/Keynes debate on the overall cyclicality of real wages. See between Dunlop (1938), Tarshis (1939) and Keynes (1939), the issue is also summarized by Sargent (1987, p. 487).} King and Watson (1996) report an output-interest rate puzzle, because of discrepancies in the \textit{unconditional} correlations of output and interest rates in U.S. data and a variety of calibrated models. But it appears in a different light, once the bc-statistics are conditioned on structural shocks. At the root of the “puzzle” are not so much the transmission mechanisms of their models, but rather the interaction of several shocks.\footnote{Another line of attack in this area has been opened by Dotsey, Lantz, and Scholl (2003) by pointing out that the real rate evidence is sensitive to the choice of price deflator used for constructing the real rate. The widely reported anti-cyclicality of the real rate is particularly strong when deflating with the CPI which has been used in this paper, too. It is a-cyclical or weakly pro-cyclical using the deflator for personal consumption expenditures (PCE). I can replicate this with my VAR, too. However, the basic results for \textit{conditional} comovements between output and real rate remain valid. These alternative results for the PCE deflation are available from the author upon request or can be found at \url{http://www.elmarmertens.ch/}} Three points stand out:

\cite{Christiano1992, King1996}
Conditional on technology shocks, the comovements between output and real rate lines up fairly well with standard models, be it the standard RBC model or the technology channel of textbook New-Keynesian models as studied by Gali (2003), Walsh (2003) or Woodford (2003). For all specifications considered, the contemporaneous correlation between (bc-filtered) real rate and output is positive. Likewise, the real rate is a positive leading indicator of output for almost two years. Unconditionally, the real rate is widely reported to be just the opposite – namely counter- or a-cyclical and a negative leading indicator. Attempts to match this only with technology shocks appear to be going in the wrong direction.\textsuperscript{38} The overall behavior must be the outcome of an interaction of several shocks. Indeed:

When conditioning on monetary shocks, the real rate is counter-cyclical and a negative leading indicator as predicted by the simple New-Keynesian models. Such opposing responses to “supply” and “demand” shocks are a general theme in Keynesian models (Bénassy 1995).

Sources of inflation persistence make up for the bulk of comovements not explained by technology and monetary shocks. In particular, they explain most of the comovements between output and the nominal rate. They are also responsible for the overall anti-cyclical real rate. Models need to include other shocks than technology and monetary policy errors to explain these effects.

\textsuperscript{38}See for instance the RBC modifications of Beaudry and Guay (1996) and Boldrin, Christiano, and Fisher (2001) with habit preferences and frictions like capital accumulation and sectoral factor immobility. Beaudry and Guay (1996) recognize the importance of conditioning on technology. But since they use a quite different detrending method their results of a counter-cyclical real rate even after conditioning on technology are hard to compare with the results in this study. See also Footnote 34.
Appendix

This appendix contains details of the VAR lag-length selection (A), identification of “inflation persistence” shocks (B), tests for long run effects of monetary policy on output (C) and further arguments for conditioning on permanent shocks to output instead of labor productivity (D).

A Lag-Length Selection

Specification of the VAR’s lag-length is based on various criteria. As whiteness of the residuals is key for the auto-covariances, I focus in particular on Portmanteau tests. Another aspect is how well the unconditional, filtered covariances of the data are matched by their VAR analogues. This is an indirect measure for how well the VAR estimates the relevant frequency bands of the data’s spectrum.

For the benchmark VAR with four variables, equation (2) of Section 2, the portmanteau tests require a lag-length of at least $p = 12$ (one year) as reported in Table 2 which coincides with the AIC’s recommendation but is much higher than the one lag, advocated by SIC or HQIC. With 30 years of monthly data, 12 lags leave sufficiently many degrees of freedom to the VAR. Figure 8 shows how well this fits the sample bc-moments of the data.

Based on these results, a lag-length of $p = 12$ is used. The estimated VAR coefficients are reported in Table 3. Figure 9 plots the autocorrelations of the forecast errors, which are practically zero. Figure 10 shows the VAR’s autocovariance function and the associated confidence intervals.

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39 Since their large sample distribution tends to over-reject dramatically, I follow Altig et al. (2004) and bootstrap critical values.
## Table 2: Lag-Length Selection Criteria for Benchmark VAR

<table>
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<tr>
<th>lag</th>
<th>T-K</th>
<th>lll/T</th>
<th>max root</th>
<th><strong>VAR Information Criteria</strong></th>
<th>Portmanteau Tests for lags . . .</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>AIC</td>
<td>HQIC</td>
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<tr>
<td>2</td>
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<tr>
<td>24</td>
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<td>28</td>
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<td>0.9946</td>
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<td>−4.42</td>
<td>0.9963</td>
<td>13.62</td>
<td>17.23</td>
</tr>
</tbody>
</table>

Note: Model chosen with lag-length 12. † denotes minimum IC. Q-statistics for Portmanteau test. * denotes significance at the 10%, ** at the 5%, *** at the 1% level of bootstrapped distribution (2000 draws).
Figure 8: BC-Moments: Filtered VAR versus Filtered Data

Note: Bandpass-filtered moments, $E\tilde{Y}_t\tilde{Y}'_{t-k}$ for benchmark VAR. The thick lines plot unconditional correlations computed from filtering VAR-Spectrum (see Section 2). The thin lines are their analogues computed from filtering the data first and then taking sample correlations. Data for the ex-ante real rate is constructed from fitted values of the VAR for expected inflation. The exact bandpass was used for the VAR-based correlations, and the Baxter-King approximation for the data (Baxter and King (1999) recommend a lag truncation of 12 in quarterly data, with monthly data, $12 \times 4 = 48$ is used here). (The thin lines plot mostly underneath the thick ones). Correlations on upper diagonal.
Figure 9: Autocorrelation Function of Forecast Errors

Note: Autocorrelation of forecast errors for benchmark VAR (see Section 2)
Table 3: Estimated VAR Coefficients

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<thead>
<tr>
<th>Variable</th>
<th>$\Delta p$</th>
<th>$\pi$</th>
<th>$t$</th>
<th>$m$</th>
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<td>$i_{t-1}$</td>
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<td>0.6748*</td>
<td>1.2228***</td>
<td>0.0932*</td>
</tr>
<tr>
<td>$m_{t-1}$</td>
<td>0.6108</td>
<td>−0.2873</td>
<td>0.3825***</td>
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<tr>
<td>$\Delta \pi_{t-2}$</td>
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<td>−0.0323*</td>
<td>0.0058***</td>
<td>0.0021</td>
</tr>
<tr>
<td>$\pi_{t-2}$</td>
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<td>−0.0120</td>
<td>0.0003</td>
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<tr>
<td>$i_{t-2}$</td>
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<td>0.0618</td>
<td>−0.6004***</td>
<td>−0.2039***</td>
</tr>
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<td>$m_{t-2}$</td>
<td>−2.3511</td>
<td>−0.2213</td>
<td>0.1672**</td>
<td>0.0151</td>
</tr>
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<td>0.0863</td>
<td>0.0332**</td>
<td>−0.0010</td>
<td>−0.0003</td>
</tr>
<tr>
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<td>0.0428</td>
<td>0.0888</td>
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<tr>
<td>$i_{t-3}$</td>
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<td>−0.1213</td>
<td>0.4643***</td>
<td>0.1988***</td>
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<tr>
<td>$m_{t-3}$</td>
<td>−2.8398*</td>
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<td>−0.0211</td>
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<td>$\Delta \pi_{t-4}$</td>
<td>0.0819</td>
<td>0.0137</td>
<td>−0.0012</td>
<td>0.0006</td>
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<tr>
<td>$\pi_{t-4}$</td>
<td>−0.2152</td>
<td>0.0271</td>
<td>−0.0196**</td>
<td>−0.0180***</td>
</tr>
<tr>
<td>$i_{t-4}$</td>
<td>−2.5844</td>
<td>0.2581</td>
<td>−0.3229***</td>
<td>−0.1333*</td>
</tr>
<tr>
<td>$m_{t-4}$</td>
<td>−0.7082</td>
<td>−1.2555***</td>
<td>0.1530*</td>
<td>0.0294</td>
</tr>
<tr>
<td>$\Delta \pi_{t-5}$</td>
<td>−0.1286**</td>
<td>−0.0145</td>
<td>0.0063***</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\pi_{t-5}$</td>
<td>0.0577</td>
<td>0.0625</td>
<td>0.0393***</td>
<td>0.0158**</td>
</tr>
<tr>
<td>$i_{t-5}$</td>
<td>2.2792*</td>
<td>0.1689</td>
<td>0.5012***</td>
<td>0.0433</td>
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<tr>
<td>$m_{t-5}$</td>
<td>−2.3493</td>
<td>−0.7171</td>
<td>−0.2111***</td>
<td>−0.1477**</td>
</tr>
<tr>
<td>$\Delta \pi_{t-6}$</td>
<td>0.0053</td>
<td>0.0053</td>
<td>−0.0018</td>
<td>−0.0016</td>
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<td>$\pi_{t-6}$</td>
<td>−0.1400</td>
<td>0.0670</td>
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<td>0.0008</td>
</tr>
<tr>
<td>$i_{t-6}$</td>
<td>−0.9947</td>
<td>−1.3757***</td>
<td>−0.6228***</td>
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</tr>
<tr>
<td>$m_{t-6}$</td>
<td>−0.9146</td>
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<td>0.0260</td>
<td>0.0562</td>
</tr>
<tr>
<td>$\Delta \pi_{t-7}$</td>
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<td>−0.0483***</td>
<td>0.0001</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\pi_{t-7}$</td>
<td>0.0138</td>
<td>0.0087</td>
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<td>0.0022</td>
</tr>
<tr>
<td>$i_{t-7}$</td>
<td>1.3623</td>
<td>0.9634</td>
<td>0.2419***</td>
<td>1.3212***</td>
</tr>
<tr>
<td>$m_{t-7}$</td>
<td>−1.5928</td>
<td>−0.0915</td>
<td>0.1287</td>
<td>0.0376</td>
</tr>
<tr>
<td>$\Delta \pi_{t-8}$</td>
<td>0.0136</td>
<td>0.0004</td>
<td>−0.0001</td>
<td>0.0023</td>
</tr>
<tr>
<td>$\pi_{t-8}$</td>
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<td>−0.0396</td>
<td>−0.0055</td>
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</tr>
<tr>
<td>$i_{t-8}$</td>
<td>−1.6328</td>
<td>−0.5389</td>
<td>0.0447</td>
<td>0.0187</td>
</tr>
<tr>
<td>$m_{t-8}$</td>
<td>−3.0411*</td>
<td>0.9633*</td>
<td>0.1150</td>
<td>0.0770</td>
</tr>
<tr>
<td>$\Delta \pi_{t-9}$</td>
<td>0.1075</td>
<td>−0.0140</td>
<td>0.0061**</td>
<td>0.0041</td>
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<tr>
<td>$\pi_{t-9}$</td>
<td>−0.1457</td>
<td>0.2525***</td>
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<td>0.0069</td>
</tr>
<tr>
<td>$i_{t-9}$</td>
<td>4.0078**</td>
<td>0.7068</td>
<td>0.1857***</td>
<td>0.0758</td>
</tr>
<tr>
<td>$m_{t-9}$</td>
<td>−3.8653**</td>
<td>−0.485*</td>
<td>0.0012</td>
<td>−0.1754***</td>
</tr>
<tr>
<td>$\Delta \pi_{t-10}$</td>
<td>0.0090</td>
<td>0.0102</td>
<td>−0.0004</td>
<td>0.0024</td>
</tr>
<tr>
<td>$\pi_{t-10}$</td>
<td>−0.3089*</td>
<td>−0.0079</td>
<td>−0.0199**</td>
<td>−0.0075</td>
</tr>
<tr>
<td>$i_{t-10}$</td>
<td>−5.4376***</td>
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<td>−0.2352**</td>
<td>−0.0029</td>
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<tr>
<td>$m_{t-10}$</td>
<td>−5.1382***</td>
<td>−0.9265*</td>
<td>−0.2128***</td>
<td>−0.0855</td>
</tr>
<tr>
<td>$\Delta \pi_{t-11}$</td>
<td>0.0102</td>
<td>0.0139</td>
<td>0.0019</td>
<td>0.0037</td>
</tr>
<tr>
<td>$\pi_{t-11}$</td>
<td>0.0491</td>
<td>0.0668</td>
<td>0.0171*</td>
<td>0.0034</td>
</tr>
<tr>
<td>$i_{t-11}$</td>
<td>0.8793</td>
<td>0.1212</td>
<td>0.1904**</td>
<td>0.0479</td>
</tr>
<tr>
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<td>−0.8658</td>
<td>−1.1959***</td>
<td>−0.0440</td>
<td>−0.0402</td>
</tr>
<tr>
<td>$\Delta \pi_{t-12}$</td>
<td>−0.1066**</td>
<td>0.0063</td>
<td>0.0006</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\pi_{t-12}$</td>
<td>−0.0731</td>
<td>−0.0844*</td>
<td>0.0025</td>
<td>0.0092</td>
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<tr>
<td>$i_{t-12}$</td>
<td>0.5625</td>
<td>−0.5413</td>
<td>−0.1164**</td>
<td>−0.0112</td>
</tr>
<tr>
<td>$m_{t-12}$</td>
<td>−0.7876</td>
<td>−0.6832</td>
<td>−0.3017***</td>
<td>−0.0704</td>
</tr>
<tr>
<td>const</td>
<td>4.9607***</td>
<td>0.9551**</td>
<td>−0.0330</td>
<td>−0.0422</td>
</tr>
</tbody>
</table>

System Statistics

- lags: 12
- obs: 359
- det(Ω): 2.1396
- llf: −2174.1274
- AIC: 13.2041
- SIC: 15.3242
- HQIC: 14.0471

Note: ***, ** and * denote significance at the 1%, 5% respectively 10% level. Ω is the variance-covariance matrix of the VAR’s forecast errors.
Figure 10: Autocorrelation Function of $Y_t$ with Confidence Intervals

Note: Bandpass-filtered moments computed from VAR described in Section 2. Bootstrapped standard-errors bands with Kilian (1998)'s small sample adjustment (1000 draws in first round, 2000 draws in second). Percentiles are shaded: 95% (light), 90% (middle), 68% (dark) and 38% (very dark). White line is median of bootstraps. Black line is point estimate of VAR.
B Identification of Persistence Shocks

This section describes the identification of the inflation persistence shocks. These are constructed as the single disturbance which maximizes the forecast error variance of inflation between the 2 to 20 year horizon and which is orthogonal to the previously identified shocks to technology and monetary policy. I extend the method of Uhlig (2004b) to accommodate this orthogonality constraint.

For this method it is convenient to express the identification in terms of an orthonormal matrix $\tilde{Q}$ and not in terms of the matrix of impact coefficients $Q$ defined in equation (3) above. These two are related via the Cholesky decomposition of the VAR’s forecast error variance, $\Sigma = E e_t' e_t$:

$$\Psi \equiv \text{chol} (\Sigma)$$

$$\tilde{Q} \equiv \Psi^{-1}Q$$

By construction we have $\tilde{Q}\tilde{Q}' = I$.

We seek the third column of $\tilde{Q}$, associated with the inflation persistence shocks, given the first two columns of $Q$ containing the impact coefficients of technology and monetary shocks. This third column of $\tilde{Q}$ solves the following variance maximization problem

$$\max_{\tilde{q}} h'_{\pi} \left( \sum_{k=24}^{240} C_k \Psi \tilde{q} \tilde{q}' \Psi' C_k' \right) h_{\pi}$$

$$= \tilde{q}' \left( \sum_{k=24}^{240} C_k' \Psi' h_{\pi} h_{\pi} \Psi C_k \right) \tilde{q} \quad \equiv S$$

---

40 The Cholesky decomposition is the unique triangular factorization of a positive definite matrix. $\Psi$ is lower triangular and we have $\Sigma = \Psi \Psi' = QQ'$. 

35
subject to

\[ \tilde{q}'\tilde{q} = 1 \]  
\[ (\Psi^{-1}Q_{12})' \tilde{q} = 0 \]

where \( C_k \) are the coefficients of the VAR’s vector moving average representation

\[ C(L) = \sum_{k=0}^{\infty} C_k L^k = A(L)^{-1} \]

\( Q_{12} \) contains the previously identified columns of \( Q \) and \( h_\pi \) selects inflation from the VAR defined in (2):

\[ h_\pi = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}' \]

[Uhlig (2004b)] solves the above problem without the orthogonality constraint (10). In this case the problem reduces to finding the largest eigenvector of the positive definite matrix \( S \) defined in (8) with \( \tilde{q} \) being its normalized eigenvector.

I extend his computations to handle the orthogonality constraint (10) as follows. Let \( B \) be an orthonormal basis for the nullspace of \( \Psi^{-1}Q_{12} \). Such a matrix is easily computed using algorithms based on the singular value decomposition\[^{41}\] of \( \Psi^{-1}Q_{12} \). The set of permissible vectors \( \tilde{q} \) is then

\[ \{ \tilde{q} : \tilde{q} = Bz \ \forall \ z \in \mathbb{R}^n \} \]

where \( n \) is the dimension of the nullspace (here: two, since there are two remaining columns in \( Q \)).

Reparametrized in terms of \( z \), the problem reduces to set \( z \) equal to the normalized

\[^{41}\text{See for instance the command } \texttt{null} \text{ in Matlab.}\]
eigenvector of of $B'SB$ associated with its largest eigenvalue, denoted $z^*$. All the necessary computation are part of standard libraries for linear algebra and pose no particular burden for the Bootstrap simulations.

The sign of $z^*$ (and thus the sign of the shock) is determined by making it raise inflation on impact. Let $Q_3$ denote the third column of $Q$. The sign of $z^*$ is then set such that the second element of $Q_3$ (the one associated with inflation) is positive. $Q_3$ is computed from

$$Q_3 = \Psi B z^*$$

**C No Long Run Effects of Monetary Policy**

This section describes the tests for zero long run effects of the Romer series on real output. This is an overidentifying restriction on the Romer coefficients in the VAR’s first equation for output growth.

Long-run restrictions are most conveniently combined with overidentifying restrictions using the analysis of Shapiro and Watson (1988). They showed how to cast long-run restrictions into coefficient restrictions on an instrumental variables regression. Without overidentifying restrictions, this is numerically equivalent to the matrix method of Blanchard and Quah (1989) used in the main text.

Without the overidentifying restriction, the long-run restriction can be implemented as follows: The structural equation for output growth is

$$a(L)\Delta y_t = W(L) \begin{bmatrix} \pi_t \\ i_t \end{bmatrix} + k(l)m_t + \sigma_z \varepsilon_t^z$$

(11)

---

42 Christiano, Eichenbaum, and Vigfusson (2003) do not use overidentifying restrictions, but they get also mileage out of the instrumental variables setup by recognizing nonstationary hours as a problem of weak instruments.
where $1/\sigma_z \begin{bmatrix} a(L) & -W(L) & -k(l) \end{bmatrix}$ corresponds to the first row of $Q^{-1}A(L)$ in the notation of equation (3).

Because of the correlation between the technology shock $\varepsilon_t^z$ and $i_t$ as well as $\pi_t$ and $m_t$, equation (11) cannot be estimated with OLS. But an instrumental variables regression works where $i_t$, $\pi_t$ and $m_t$ are instrumented for by their own lagged values. The restriction, that only $\varepsilon_t^z$ has a long-run impact on $y_t$ imposes $W(1) = 0$ and $k(1) = 0$ (this eliminates long-run effects of any other shocks than $\varepsilon_t^z$ operating through $i_t$, $\pi_t$ and $m_t$). The restricted IV regression is estimated by replacing $i_t$, $\pi_t$ and $m_t$ with their differences using $W(L) = (1 - L)\tilde{W}(L)$ and $k(l) = (1 - L)\tilde{k}(l)$, which holds for some $\tilde{W}(L)$ and some $\tilde{k}(l)$ whenever $W(1) = 0$. This is nicely illustrated by Shapiro and Watson (1988) and Francis, Owyang, and Theodorou (2003, Appendix A).

A zero effect of the Romer measure on permanent output means that $k(1) = 0$ without the need to impose it. I estimate (11) imposing only $W(1) = 0$ but not $k(1) = 0$. The Wald test for $k(1) = 0$ is insignificant with a p-value of 70%. Alternatively, technology shocks can be estimated by dropping $k(l) m_t$ from (11) as in a three-variable VAR. $m_t$ can then be added as an instrument. Hansen (1982)’s $J$ statistic for the estimation with efficient GMM cannot reject the overidentifying restriction. Using only $m_t$ as additional instrument, the $p$-value is 33%. Using $m_t$ and 12 of its lags, the $p$-value is 48%.

\footnote{\(\tilde{W}(L)\) has one lag less than $W(L)$ see for instance Hayashi (2000, p. 564).}
D Discussion of “Technology Shocks”

Permanent shocks to output are labeled here as “technology shocks”. Conditional on these shocks, the real rate is estimated to be pro-cyclical and a positive leading indicator of output. This is in line with predictions of standard RBC and New Keynesian models for technology shocks. (A common feature of these models is that they have a time-separable, iso-elastic utility of consumption.)

The kind of labor augmenting technology shocks specified in these models is the sole driver of permanent output if hours are stationary, otherwise they have to be identified from the permanent component of labor productivity (Gali 1999). Given the physical constraints of time, models sensibly assume hours to be stationary. Whether the post-war sample of U.S. hours data is better approximated by a stationary or a unit root process has generated an intense debate with good arguments on both sides.

What matters for the message of this paper, is whether the transmission mechanisms of simple RBC or New-Keynesian models predict the same comovements in output and interest rates for whatever shocks drive the stochastic trend in output. Non-technology sources of stochastic growth in output are for instance permanent changes in government

\[ y_t = a_t + \alpha(k_t - l_t) + l_t \]

where \( y_t \) is real output, \( a_t \) the technology shock, \( \alpha \) the income share of capital, \( k_t \) capital and \( l_t \) are hours (as throughout the paper, all quantities are per-capita). Stationarity of the capital-labor ratio is a key restriction of balanced growth (which is a maintained hypothesis). Stochastic trends in output must thus be coming either from technology and/or from hours.

See for instance Gali and Rabanal (2004), Francis and Ramey (2005a) and Christiano, Eichenbaum and Vigfusson (2003). Using quarterly hours data over my sample, unit root tests favor the stationary specification, mostly because the sample excludes the run-up in hours worked during the second half of the 1990’s. This was reported in the previous Working Paper version of this chapter.

Regardless of whether they are identified from labor productivity or output, my “technology shocks” are based on estimating a stochastic trend. Against this practice, a deeper critique has been levied by Chari, Kehoe, and McGrattan (2005) who are concerned with the large uncertainty and possibly misleading results arising from estimates of long-run shocks and responses from VARs with limited lag lengths. My paper implements its VAR with a limited lag length (8 lags) as well. It relies on obtaining good estimates from this specification.
spending or secular changes in workforce participation. These will now be discussed in more detail.

Generally, the following mechanisms will be at work: In standard models the real rate is proportional to expected consumption growth.\footnote{I have in mind models with standard preferences which are time-separable and iso-elastic. To a first order, they imply $r_t = \text{const} + \sigma E_t \Delta c_{t+1}$ where $r_t$ and $c_t$ are the logs of the real rate respectively consumption, and $\sigma$ is the relative risk aversion.} All it takes for a pro-cyclical and positively leading real-rate is that consumption is expected to be growing after a permanent increase in output. This rising profile will then be reflected in a higher interest rate – both in conjunction with the current increase in output and in anticipation of persistently higher levels of output in the future.\footnote{An obvious example would be permanent income effects from an increase in output. But note the subtle language above: It is important that consumption is expected to be growing after the impact, on impact disposable income may fall and thus can consumption. So the expected consumption growth may occur from an initially lower level compared to before the impact. See Aiyagari, Christiano, and Eichenbaum (1992) for the case of a permanent increase in government spending, which reduces disposable income. In response to preference shocks to demand, Baxter and King (1991) also find a pro-cyclical, positively leading real interest rate.} This applies both to the standard RBC as well as simple New-Keynesian models. Of course, different utility functions, or frictions in the decisions to consume/invest/work can change this prediction. My point is that the data is actually consistent with the simple model’s predictions of a pro-cyclical and positively leading real rate in response to permanent shocks to output.

Permanent shocks in government spending and their effects on output and interest rates are analyzed within a neoclassical model.\footnote{The effects work out as follows: The permanent increase in spending leads to a drop in consumer’s permanent income and thus in consumption. But the spending increase is also initially buffered by lower investment and the thus increased marginal productivity of capital leads to higher interest rates and a slowly increasing consumption profile. Together with higher work effort, output rises, too.} The same result can be found for the simple New-Keynesian model, see for example Gali (2003).\footnote{Gali (2003) specifies only a temporary shocks in government spending, but it is straightforwardly extended to a unit root process.}

Francis and Ramey (2005b) argue that the potentially permanent movements in hours per-capita are related to some secular changes in workforce participation: Increased school...
years, longevity of retirees (but unchanged retirement age) as well as increased government employment. Their suggestion is to prune the conventional per-capita units from these effects. Instead of dividing quantities by the entire civilian population of age 16 and older, this would account only for the population available for production in the private sector. Implicitly, this does not only redefine hours per-capita but also output per-capita. For a business-cycle model, such secular workforce effects are indeed best viewed as exogenous shifts in effective labor supply.

My estimated correlations pertain to conventionally measured per-capita output. To introduce some notation, $Y_t$, $C_t$, $K_t$ and $N_t$ are conventionally measured per-capita output, consumption, capital and hours. $\bar{N}_t = N_t / X_t$ is Francis and Ramey’s measure for hours, where $X_t$ reflects the exogenous and permanent workforce changes.\(^{52}\) In a neo-classical production function we have $Y_t = F(K_t, A_t, X_t, \bar{N}_t)$ where $A_t$ are conventional technology shocks. But also the workforce changes, $X_t$, influence conventional per-capita output very much like labor-augmenting technology shocks. Together with $A_t$, they make up for a permanent component in $Y_t$.

For the standard RBC model, King, Plosser, and Rebelo (1988b, 1988a, “KPR”) trace out how the effects of permanent technology shocks induce a concurrent rise in real rate and output (forecasting future high levels of output as well).\(^{53}\) Business cycle models typically abstract from population growth so that their quantities are best understood as matching per-capita aggregates. The question is now which per-capita units to use? A direct mapping of the present setting to the KPR model presumes that the representative household has preferences over $C_t$ and $\bar{N}_t$, not $N_t$. In response to an increase in his workforce participation

\(^{52}\)To be precise: $X_t$ reflects a difference in population measures. It is the ratios of Francis and Ramey (2005b, p. 10)’s “population available to carry out productive activity in the private sector” or “available workforce” over the conventional measure of the civilian population over age 16.

\(^{53}\)Partly, this results also from the choice of preferences and parameters of KPR. A permanent technology shocks initially reduces capital stock (in technology adjusted units) and would thus reduce the incentive to work because of a lower real wage. For standard preferences, this is however outweighed by positive incentives arising from the high incentives to invest and thus raise future productive capacity and wealth.
$X_t$, he benefits from the additional production without an increased disutility of labor.\footnote{In the spirit of \textit{Francis and Ramey} (2005b), this could be palatable when $X_t$ involves a change between occupations with different productivity at least in terms of private sector output (e.g. time spent in school or – no offense – government employment).} But also a model where the representative agent’s quantity decisions are measured in the new per-capita units of \textit{Francis and Ramey} would yield similar results. To a first order, deflating all quantities by $X_t$ corresponds to the transformations used by \textit{King, Plosser, and Rebelo} (1988a) to obtain a stationary economy in the presence of a stochastic trend.\footnote{This holds when the accuracy of the solution is limited to the first order, such that certainty equivalence can be posited. The analysis then mirrors closely the case of a deterministic trend, see \textit{King, Plosser, and Rebelo} (1988a) for details.} Its prediction for the correlation between real rate and (untransformed) output are the same as above see \textit{King, Plosser, and Rebelo} (1988a).
References


———. 2005b, October. “Measures of Per Capita Hours and their Implications for the Technology-Hours Debate.” Working Paper 11694, NBER.


