Lecture 5B: Default-Liquidity Spiral in Corporate Bond Market

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Brief Literature Review

- Growing literature on rollover risk:
  - Diminishing debt capacity: Acharya, Gale, and Yorulmzer (2009)
  - Dynamic debt runs: He and Xiong (2012)

- Structural credit risk models focus on fundamental default risk:
  - Exogenous default threshold: Merton (1973), Longstaff and Schwartz (1995);

- Empirical evidence on important liquidity effects in credit spreads:
  - Interpreted as a liquidity-premium effect.
  - Our model: an increase in liquidity premium also leads to higher default premium.
Empirical Literature

- Collin-Dufresne, Goldstein, and Martin (2001)
  - The change of credit spread should depend on changes of 1) default risk and 2) liquidity risk
  - Kitchen-sink approach to proxy for these two risk (macroeconomic and firm specific)
  - Residuals are largely unexplained (66%); commonality among these residuals, as the first-principal component explains 59%
  - An aggregate factor driving liquidity in the bond market, which is beyond typical liquidity measures (On-and-off-the-run spread, swap spreads, and the frequency of quotes)

- Longstaff, Mithal, and Neis (2005)
  - Use CDS data to extract the default component (liquidity-default decomposition)
  - Estimate an structural model by assuming independence between liquidity and default

- Chen, Lesmond, and Wei (2007)
  - More illiquid bonds earn higher yield spreads
  - Robust to issuers’ fixed effect (within estimator)
Model (1)

We build on Leland and Toft (1996) with an additional feature:
- Illiquid secondary bond markets.

A firm repays maturing bonds by issuing new bonds at market prices.
- The rollover gain/loss is absorbed by equity holders;
- The firm defaults when equity value drops to zero.

The unlevered firm value follows a log-normal process under the $Q$-measure:

$$\frac{dV_t}{V_t} = (r - \delta) \, dt + \sigma dZ_t.$$

- Riskfree rate $r$, payout rate $\delta$.

In bankruptcy creditors recover $\alpha$ fraction of the firm value.
Model (2): Debt Structure

- The firm commits to a stationary debt structure \((C, P, m)\):
  - aggregate face value \(P\) and annual coupon payment \(C\);
  - each bond has maturity \(m\);
  - debt expirations are uniformly spread across time, i.e., over \((t, t + dt)\), \(\frac{1}{m} dt\) fraction of the bonds matures.
- The firm issues new bonds with the same face value, coupon rate and maturity to replace maturing bonds.
- Over \((t, t + dt)\), the net cash flow to equity holders is (with \(\pi\) as marginal tax rate)

\[
NC_t = \delta V_t - (1 - \pi) C + \frac{1}{m} [\overline{d} (V_t, m) - P].
\]

- \(\overline{d} (V_t, m)\): market value of per unit newly issued bond;
- When the bond price drops, equity holders face rollover losses.
- Will show the loss is greater for short-term debt.
Model (3): Endogenous Default

- The firm defaults when $V_t$ drops to an endogenous threshold $V_B$.
  - At $V_B$, equity value $E(V_B) = 0$, i.e., the firm cannot raise any equity financing;
  - Optimality of $V_B$: smooth pasting $E'(V_B) = 0$.

- Intrinsic conflict of interest between debt and equity holders:
  - When the bond price falls (for either fundamental or liquidity reasons), equity holders bear the rollover loss while the maturing debt holders get paid in full.
  - Equity holders face a tradeoff: rollover loss vs option value of keeping the firm alive.
The secondary markets of corporate bonds are highly illiquid.

- Large bid-ask spreads and price impact.
- Edwards, Harris, and Piwowar (2007): bid/ask spread on corporate bonds ranges from 4 to 75 bps.
- Bao, Pan, and Wang (2009): trading cost (bid/ask spread & price impact) ranges from 74 to 221 bps; and the cost is higher for long-term bonds.

- When a bond holder sells a bond, he only recovers a fraction \((1 - k)\) of the value.
  - \(k\) represents the liquidity discount (trading cost, info problem,...)

- Each bond investor is subject to Poisson liquidity shocks with intensity \(\zeta\), a la Amihud and Mendelson (1986).
  - Upon the arrival of a liquidity shock, he has to sell his bond holdings.

- We assume no cost for trading equity and issuing new bonds.
Solving the Equilibrium

- For a given $V_B$, PDE for the debt value $d(V_t, \tau; V_B)$:

$$\left( r + \bar{\xi}k \right) d(V_t, \tau) = c - \frac{\partial d(V_t, \tau)}{\partial \tau} + (r - \delta) V_t \frac{\partial d(V_t, \tau)}{\partial V} + \frac{1}{2} \sigma^2 V_t^2 \frac{\partial^2 d(V_t, \tau)}{\partial V^2}.$$  

- At the bankruptcy, $d(V_B, \tau; V_B) = \frac{\alpha V_B}{m}$, for all $\tau \in [0, m]$.
- At maturity, $d(V_t, 0; V_B) = p$, for all $V_t > V_B$.

- ODE for equity value $E(V)$: (denote $\frac{1}{m} [d(V_t, m) - P] = d(V_t, m) - p$)

$$rE = (r - \delta) V_tE_V + \frac{1}{2} \sigma^2 V_t^2 E_{VV} + \delta V_t - (1 - \pi) C + d(V_t, m) - p.$$  

with boundary condition $E(V_B) = 0$:

- Closed-form solution for $E(V)$ using Laplace transform.

- Smooth pasting $E'(V_B) = 0$: closed-form solution for $V_B$. 
Laplace Transform (1)

- ODE:
  \[ rE = (r - \delta) VE + \frac{1}{2} \sigma^2 V^2 E_{VV} + d(V, m) + \delta V - [(1 - \pi) C + p]. \]

- Define \( v \equiv \ln \left( \frac{V}{V_B} \right) \), so
  \[ rE = \left( r - \delta - \frac{1}{2} \sigma^2 \right) E_v + \frac{1}{2} \sigma^2 E_{vv} + d(v, m) + \delta V_B e^v - [(1 - \pi) C + p], \]
  with the boundary conditions \( E(0) = 0 \) and \( E_v(0) = 1 \).

- Laplace transformation of \( E(v) \) as
  \[ F(s) \equiv L[E(v)] = \int_0^\infty e^{-sv} E(v) \, dv. \]
  implying:
  \[ rF(s) = \left( r - \delta - \frac{1}{2} \sigma^2 \right) L[E_v] + \frac{1}{2} \sigma^2 L[E_{vv}] + L[d(v, m)] + \frac{\delta V_B}{s - 1} - \frac{(1 - \pi) C + p}{s}. \]

- But \( L[E_v] = sF(s) - E(0) = sF(s) \) and \( L[E_{vv}] = s^2 F(s) - sE(0) - E_v(0) = s^2 F(s) - l \).
Laplace Transform (2)

- Hence we have

\[
\left[ r - \left( r - \delta - \frac{1}{2} \sigma^2 \right) s - \frac{1}{2} \sigma^2 s^2 \right] F(s) = L \left[ d(v, m) \right] - \frac{1}{2} \sigma^2 I + \frac{\delta V_B}{s-1} - \frac{(1-\pi)C + p}{s}.
\]

- Recall solutions to fundamental equation \( \eta > 0 \) and \( -\gamma < 0 \). Then,

\[
\frac{1}{2} \sigma^2 F(s) = -\frac{1}{(s-\eta)(s+\gamma)} \left\{ L \left[ d(v, m) \right] + \frac{\delta V_B}{s-1} - \frac{(1-\pi)C + p}{s} - \frac{1}{2} \sigma^2 I \right\}
\]

\[
= -\frac{1}{s-\eta} - \frac{1}{s+\gamma} \left\{ L \left[ d(v, m) \right] + \frac{\delta V_B}{s-1} - \frac{(1-\pi)C + p}{s} - \frac{1}{2} \sigma^2 I \right\}.
\]

- Recall that \( d(v, m) \) is given by

\[
d(V_t, m; V_B) = \frac{c}{r + \bar{\xi}k} + e^{-(r+\bar{\xi}k)\tau} \left[ p - \frac{c}{r + \bar{\xi}k} \right] (1 - F(\tau)) + \left[ \frac{\alpha V_B}{m} - \frac{c}{r + \bar{\xi}k} \right] G(\tau)
\]

we get

\[
\frac{1}{2} \sigma^2 F(s) = f_1(s) + f_2(s) + \ldots.
\]

- Laplace inversion to get \( \hat{f}_j(s) \), so we get \( \hat{F} \) which is just equity
Key Channels of Liquidity Effects

- Consider an unanticipated liquidity shock which increases $\zeta$ or $k$.
  - e.g., increased redemption risk, margin risk, or market illiquidity.
Baseline Model Parameters for Illustration

- Risk-free rate: $r = 8\%$.
- Tax rate: $\pi = 27\%$.
- Asset volatility $\sigma = 23\%$; payout rate $\delta = 2\%$.
- Trading cost $k = 1\%$; Intensity of liquidity shocks $\xi = 1$.
  - Consistent with Bao, Pan, and Wang (2009) who focus on a relatively liquid sample.
- Liquidation recovery rate: $\alpha = 0.5$.
- Debt maturities $m = 1$; total principal $P = 61.68$; total coupons $C = 6.39$.
- Current asset value: $V_t = 100$. 
Market Liquidity and Endogenous Default

Two channels of liquidity effects: liquidity premium and endogenous default risk.

- Panel A: Rollover Loss
- Panel B: Bankruptcy Boundary
- Panel C: Bond Spread
- Panel D: Composition of Bond Spread

Figure:
**Amplification by Short-term Debt**

- Shorter maturity forces equity holders to quickly realize rollover loss.
  - Rollover loss per unit of time: \( \frac{\bar{d}(V_t, m) - P}{m} \).
  - More severe conflict b/w debt- and equity-holders.
- Short-term maturity makes an individual bond safer, but a firm with more short-term debt is riskier.

![Graphs showing rollover loss, bankruptcy boundary, and bond spread](image)
- Bond market illiquidity reduces the firm’s initial leverage choice.
Implications: Predicting Defaults

- Our model predicts market liquidity as a new factor for predicting bond defaults, in addition to
  - Distance to default: leverage, asset volatility
  - Firms’ liquidity holdings: cash, credit lines
- The existing structural credit risk models have mixed successes:
  - Leland (2004): Leland model does a good job in capturing average default probabilities of bonds with different ratings.
  - Bharath and Shumway (2008): distance-to-default variable constructed from Merton model is not a sufficient statistic for default probability.
  - Davydenko (2007): distance to default cannot capture the cross section of bond spreads;
- Collin-Dufresne, Goldstein, and Martin (2001): standard variables cannot explain the changes of credit spreads.
- Das, Duffie, Kapadia, and Saita (2007): distance-to-default variables cannot fully capture default correlation observed in the data.
Implications: Decomposing Credit Spreads

- Both academics and policy makers have recognized the important effect of market liquidity on credit spreads, but tend to treat it as independent from default risk.
- Several studies, e.g., Longstaff, Mithal, and Neis (2005), Beber, Brandt, and Kavajecz (2008), and Schwarz (2009), decompose credit spreads to assess contributions of liquidity premium and default risk:

$$\text{CreditSpread}_{i,t} = \alpha + \beta \cdot \text{CDS } _i \text{ Spread}_{i,t} + \delta \cdot \text{LIQ}_{i,t} + \epsilon_{i,t}$$

- Default risk explains a majority part of the cross-sectional variation, although the liquidity effect is also significant.
- But these two effects are correlated through endogenous default.
  - How to classify the correlated part?
  - In the empirical analysis, the more precise measure of default risk (via traded prices) could have favored the default risk effect.
Several recent studies examine the impact of TAF on LIBOR-OIS spread.
  

They tend to control for default risk using certain credit spread, such as CDS spread or LIBOR-REPO spread.
  
  - Example: Taylor and Williams (2009)
  
    \[(\text{LIBOR} - \text{OIS})_t = a \cdot (\text{LIBOR} - \text{REPO})_t + b \cdot \text{TAF}_t + \epsilon_t\]

  - The control variables can also absorb liquidity effects and thus leading to an under-estimation.
Implications: Maturity Risk

- Our model implies that firms’ debt maturity structure is an important determinant of credit risk.
- Evidence: non-financial firms with more maturing long-term debt during the recent credit crisis period had to cut down more investment and had greater credit spread increases.
- Evidence on credit ratings had ignored maturity risk.
  - Gopalan, Song, and Yerramilli (2009).
Extension

▶ A more elaborate secondary market:
  ▶ Multiple types of bond investors with different frequencies of liquidity shocks;
  ▶ Multiple classes of long-term and short-term bonds with short-term debt being more liquid.

▶ Endogenous market segmentation in spirit of Amihud and Mendelson (1986):
  ▶ investors with higher liquidity needs self-select to short-term bonds;
  ▶ liquidity effect spill over across different segments through investors’ required bond returns.

▶ Endogenous debt maturity structure:
  ▶ The firm trades off short-term debt’s lower liquidity premium and the resulting higher default risk.
Conclusion

- A model of liquidity effects on credit spreads.
  - Two channels: liquidity premium and endogenous default.
  - The latter channel operates through firms' rollover risk.

- Several results:
  - Liquidity shocks increase credit spreads not only through higher liquidity premia, but also higher default probabilities.
  - Flight to quality: Bonds with weaker fundamentals are more exposed to liquidity shocks.
  - Shorter debt maturity exacerbates rollover risk and thus effects of liquidity deterioration on endogenous default.

- Implications:
  - 1) Liquidity can predict defaults; 2) caution against treating liquidity and default premia as independent; 3) maturity risk