Lecture 3A: Asset Pricing Models with Financial Intermediaries

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Financial intermediaries are huge and playing important roles in asset pricing and real economy

- Separation between ownership and control is ubiquitous
- Financial crisis reveals first-order importance of studying financial intermediaries

Especially for those sophisticated assets which NIPA households will no way be marginal investors!

Recent events: high spreads in debt markets Aug 2007 to Oct 2008

- In contrast, S&P or Dow Jones near all time high in Aug 2008
- In general, debt markets are more sophisticated

Role of distressed intermediary capital in the crises of market with complex asset classes (e.g. MBS)

- Narrative: intermediaries get in trouble, drive up required risk premium in intermediated asset
Two periods, $t = 0$ and $1$, interest rate $r$

Three risk neutral agent groups: households, bankers/monitors, entrepreneurs

Entrepreneurs with endowment $A_i$ index $i \in [0, 1]$, and $K_f = \int A_i dG (A_i)$

Households have large date 0 endowment

Entrepreneurs and bankers have limited endowment

There is an exogenous savings technology with gross return $r$
A Twist on (Three) Project Types

- Projects: invest $I$ at date 0, get $R$ with probability $p$ or 0 with probability $1 - p$
- Three projects available
  - Good project $G$, $p = p_H > p_L$
  - Bad project $B$, $p = p_L$ with entrepreneur’s private benefit of $B$
  - OK project $b$, $p = p_L$ with entrepreneur’s private benefit of $b < B$
- Parameter conditions $p_H R - rl > 0 > p_L R - rl + B$
- Project choice is unobservable to investors, so need incentive provision
Non-intermediated or Direct Finance (1)

- **Contract** \((T_0, R_f)\): \(T_0\) is households contribution at \(t = 0\); \(R_f \leq R\) is what entrepreneur gets at \(t = 1\) if success
  - So, households get \(R - R_f\) if success
- **Investment** \(T_0 + A \geq I\); IR constraint \(p_H (R - R_f) - rT_0 \geq 0\)
- **IC constraint** \(p_H R_f \geq p_L R_f + B\) implies

\[
R_f \geq \frac{B}{\Delta p}
\]

where \(\Delta p \equiv p_H - p_L\). Entrepreneur has to get something for incentive provision

- Projects get funded if the entrepreneur’s net worth is sufficiently high
  - Say the entrepreneur net worth \(A\) is low \(\Rightarrow\) high \(T_0 = I - A\)
  - For households to break even (IR of households), their repayment is high \(\Rightarrow\) entrepreneur’s share \(R_f = R - \frac{rT_0}{p_H} = R - \frac{r(I-A)}{p_H}\) is low \(\Rightarrow\) violating IC
Non-intermediated or Direct Finance (2)

- The cutoff $\bar{A}$ that satisfies IC with 100% pledging of endowment is
  \[ R - \frac{r (I - \bar{A})}{\rho_H} = \frac{B}{\Delta p} \Rightarrow \bar{A} = I - \frac{\rho_H}{r} \left( R - \frac{B}{\Delta p} \right) \]

- For entrepreneurs with $A > \bar{A}$, borrow and invest; otherwise cannot invest
- For $A > \bar{A}$ it is w.l.o.g to assume IC binding $R_f = \frac{B}{\Delta p}$; just contributes $\bar{A}$ to the firm (and saves $A - \bar{A}$)

- Intuition
  - $R - \frac{B}{\Delta p}$ is the pledgeable income given success
  - Today’s expected value of pledgeable income is $\frac{\rho_H}{r} \left( R - \frac{B}{\Delta p} \right)$, i.e., the maximum that investors are willing to contribute
  - Entrepreneur needs to fill the gap between this and investment $I$

- For $A < \bar{A}$, how about borrowing from banks and banks borrow from households?
  - So-called intermediated financing
Intermediated Financing

- Monitoring eliminates $B$ (bad) project, but entrepreneur can still choose $b$ project with smaller private benefit
- If not monitor (shirk), banker enjoys $C > 0$ as private benefit
  - As in Diamond (1984), monitoring activity is unobservable. We need to monitor the monitor
  - The paper models $C$ as cost...here $C$ is reduction of private benefit, simpler
- Say monitoring is not that costly

\[ \frac{C \Delta p}{p_H} < B - b \]

- Crucial model ingredient: Banker’s endowment $K_m$
  - It can be considered as (aggregate) banking/intermediary capital
- Bank capital market is competitive
  - $\beta \geq r$ will be determined by a standard supply-demand argument (see later)
Financing Contract with Intermediation

- The contracting involving informed capital, provided by banks, is similar.
- We need to specify $R_f$ (goes to firm) and $R_m$ (goes to banker), and households get $R - R_f - R_m$.
- Ex ante, the firm contributes $l_f$, banker $l_m$, and households $l_h$ so that
  \[ l = l_f + l_m + l_h \Rightarrow l_f \leq l - l_m - l_h \]
- Again, w.l.o.g, IC for entrepreneur $R_f = \frac{b}{\Delta p}$ binds in optimal contract.
- IC for banker
  \[ p_H R_m \geq p_L R_m + C \Rightarrow R_m \geq \frac{C}{\Delta p} \]
- In optimal contracting, $R_m = \frac{C}{\Delta p}$ binds as well.
Ex ante Contribution & Demand for Informed Capital

- Households break-even, contributing
  \[ I_h = \frac{p_H (R - R_f - R_m)}{r} \]

- For bankers, they contributes \( I_m \) today and in expectation gets
  \[ p_H R_m = \frac{p_H C}{\Delta p} \] tomorrow

- Bankers are earning an equilibrium return of \( \beta \), defined by
  \[ \beta I_m = \frac{p_H C}{\Delta p} \Rightarrow I_m (\beta) = \frac{p_H C}{\beta \cdot \Delta p} \]

  - This gives demand for informed capital given price \( \beta \)

- So, the entrepreneur contributes
  \[ I_f = I - \frac{p_H (R - R_f - R_m)}{r} - I_m \]
Two Interpretation of Optimal Contracting (1)

- Optimal contract just specifies ultimate sharing rules
- Plenty of flexibility in its implementation, which ties to application directly
- First interpretation (more direct)

<table>
<thead>
<tr>
<th>Asset</th>
<th>Debt (Household)</th>
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<tbody>
<tr>
<td>VC Financing</td>
<td>Firm Equity</td>
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Two Interpretation of Optimal Contracting (2)

- Second interpretation with bank loans

<table>
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<tr>
<th>Firm</th>
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<tr>
<td>Asset</td>
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Threshold Entrepreneurs Net Worth with Banks

- Previously we show that $A < \bar{A} = I - \frac{p_H}{r} \left( R - \frac{B}{\Delta p} \right)$ then no investment

- Now investment requires

$$A \geq I - \frac{p_H}{r} \left( R - R_f - R_m \right) - I_m$$

$$= I - \frac{p_H}{r} \left( R - \frac{b + c}{\Delta p} \right) - \frac{p_H}{\beta} \frac{C}{\Delta p}$$

$$= I - \frac{p_H}{r} \left( R - \frac{b}{\Delta p} \right) + \frac{p_H}{r} \frac{C}{\Delta p} \left( 1 - \frac{r}{\beta} \right) \equiv A$$

Replace $B$ in $\bar{A}$ by $b$, firm agency

Intermediary monitoring agency

- $\frac{p_H}{r} \left( R - \frac{b}{\Delta p} \right)$ is project’s discounted pledgeable income given perfect monitoring

  - A higher $b \Rightarrow$ a higher $A \Rightarrow$ less entrepreneurs get financed

- The more interesting part is the second term "intermediary agency"
Cutoff Net Worth in Intermediated Financing

- The new cut-off entrepreneur net worth

\[ A(\beta) = I - \frac{p_H}{r} \left( R - \frac{b}{\Delta p} \right) + \frac{p_H}{r} \frac{C}{\Delta p} \left( 1 - \frac{r}{\beta} \right) \]

Replace \( B \) in \( \bar{A} \) by \( b \), firm agency

Intermediary monitoring agency

- Greater bank agency problem (higher monitoring cost \( C \)) \( \Rightarrow \) less projects get funded (higher \( A \))

- How about the equilibrium return \( \beta^* \)? Intuitively, it is tied to the aggregate intermediary capital supply \( K_m \)
  
  - Introduce distribution on \( A \), pdf \( g(\cdot) \)
  
  - Given \( \beta \), entrepreneurs with \( A \in [\underline{A}(\beta), \bar{A}] \) gets informed bank capital, with an aggregate demand of

\[ D(\beta) = \int_{\underline{A}(\beta)}^{\bar{A}} g(A) \, dA \cdot I_m(\beta) = \int_{\underline{A}(\beta)}^{\bar{A}} g(A) \, dA \cdot \frac{p_H C}{\beta \cdot \Delta p} \]

- The equilibrium \( \beta^* \geq r \) is given by

\[ D(\beta^*) \leq K_m \text{ with equality only if } \beta^* > r \]
When Bank Capital is Abundant

In the diagram, the supply of intermediary capital, $K_m$, intersects with the demand curve, $I_m(\beta)$, at $r = \beta^*$. This point represents a high supply of bank capital, $K_m$.
When Bank Capital is Scarce

\[ I_m(\beta) \]

Intermediary Capital Demand

Intermediary Capital Supply, a low \( K_m \)

\[ K_m \]

\[ r \quad \beta^* \quad \text{Bank Rent}/\beta \]
The Role of Intermediary/Bank Net Worth

- $\beta \geq r$ is the equilibrium return earned by intermediary/bank
  - $1 - \frac{r}{\beta} \geq 0$ captures bank’s rent
- Higher banker’s rent $\beta \Rightarrow$ less projects get funded (higher $A$)
- The aggregate intermediary capital $K_m$ determines $\beta$
- In general equilibrium, the lower the supply $K_m$, the greater the rent, the higher the $\beta$
  - Well, you get more rents if your endowment is more scarce
  - Intensive margin: each firm attracts less initial bank capital $\frac{pH}{C} \frac{1}{\beta \cdot \Delta p}$
  - Extensive margin: the cut-off firm $A(\beta)$ goes up
- The paper further studies what if the bank capital $K_m$ fluctuates....
  - $A(\beta(K_m))$ and hence real activity fluctuates
- He-Krishnamurthy and Brunnermeier-Sannikov: an endogenous dynamics of $K_m$
There are more and more empirical papers in the topic of intermediary asset pricing.

Our annual review paper has a model section and empirical literature review section.

Goal: build a simple model as the framework to
- Understand the essence and connections between these results
- Clarify what the empirical results are able and unable to claim

In our view, the essence of intermediary asset pricing is that "intermediary is not a veil"
Simple CARA-Normal Model

- Draw the essence from He-Krishnamurthy (2012, Restud)
- Two period model, risky asset with exogenous supply $\theta$, dividend payout $\widetilde{D} \sim \mathcal{N}(\mu, \sigma^2)$
- CARA (exponential utility) agents, households with constant risk tolerance $\rho_H$ and managers with constant risk tolerance $\rho_M$
  - $u_i(W_i) = -e^{-\frac{1}{\rho_i} W_i}$, mean-variance preference
- Risk-free rate is exogenous 0; we derive time-0 asset price $p$
- Frictionless benchmark. Given $P$
  - Agent $i$’s demand of risky asset
    \[ x_i = \rho_i \frac{\mu - p}{\sigma^2} \]
- Equilibrium $x_H + x_M = \theta$ so
  \[ p = \underbrace{\mu}_{\text{expected return}} - \underbrace{\frac{\theta \sigma^2}{\rho_H + \rho_M}}_{\text{risk premium}} \]
Market Segmentation and Agency Friction

- Say households cannot directly get access to invest in risky asset
- As in reality, a manager can set up an intermediary, which essentially offers households’ access to risky asset
- But, subject to certain model hazard problem
  - Work or shirk; shirking hurts fund return by $\Delta$ but gives private benefit of $b$
- Affine contract between $H$ and $M$: fee $K$ and share $\phi$
  - No wealth effect in CARA preference, so ignore the fee $K$
- Say fund deliver $R_F$; manager gets $\phi R_F$ and households get $(1 - \phi) R_F$
- We will see that the agency problem imposes a minimum threshold for $\phi$
  - Skin-in-the-game requirement
Managers’ Problem and IC Constraint

- Manager is making decisions on 1) intermediary’s position $x_F$ and 2) shirking or not

$$\max_{x_F, s \in \{0, 1\}} \phi [x_F (\mu - p) - s\Delta] - \frac{x_F^2 \phi^2 \sigma^2}{2 \rho_M} + s b$$

- Working if and only if IC holds

$$\phi\Delta \geq b \Rightarrow \phi \geq \frac{b}{\Delta}$$

- Portfolio decision. Recall $x_M = \rho_M \frac{\mu - p}{\sigma^2}$, manager’s desired risk exposure. Then

$$x_F = \frac{x_M}{\phi}$$

- Households risk holding

$$x_H = (1 - \phi) x_F = \frac{1 - \phi}{\phi} x_M = \frac{1 - \phi}{\phi} \rho_M \frac{\mu - p}{\sigma^2}$$

- A lower bound on $\phi$ translates to an upper bound on $x_H$
IC Constraint and Equity Constraint

- Recall IC constraint $\phi \geq \frac{b}{\Delta}$
- Let $m \equiv \frac{\Delta}{b} - 1$. Then the sharing rule $\phi$, which is the share to manager, satisfies
  \[
  \phi \geq \frac{1}{1 + m}
  \]
- Equity implementation. This says that if manager contributes 1 dollar of inside equity, at most households are willing to contribute $m$ outside equity
- This model does not have leverage (or debt) constraint
  - Introducing regulatory capital type of constraint will add some additional cost
- But, equity constraint is more primitive than debt constraint
  - Leverage constraint binds only if equity constraint binds—otherwise just issue more equity at a fair price
Equilibrium Asset Prices

- Depending on parameters, equity constraint might not bind
- The equilibrium asset price is given by

\[
p = \begin{cases} 
\mu - \frac{\theta \sigma^2}{\rho_M + \rho_H} & \text{if } m > \frac{\rho_H}{\rho_M} \text{ so equity constraint slack} \\
\mu - \frac{\theta \sigma^2}{(1+m)\rho_M} & \text{if } m \leq \frac{\rho_H}{\rho_M} \text{ so equity constraint binds}
\end{cases}
\]

- When agency friction is small, \( m = \frac{\Delta}{b} - 1 \) is large, there is a large intermediation capacity, equity constraint is slack
- The equilibrium asset price is
  - sensitive to agency parameter \( m \) only if equity constraint binds
  - sensitive to \( \rho_M, \theta, \) and \( \sigma^2 \) in both regions, but more so when equity constraint binds
- Side note: \( \rho = \text{wealth} \times \frac{1}{\text{relative risk-aversion}} \). Capital loss drives down \( \rho_M \), more likely to bind
Pricing Kernel and Stochastic Discount Factor (SDF)

- In asset pricing, SDF is the word you often hear.
- No arbitrage $\iff$ exists an SDF that prices EVERYTHING
  - The SDF proposed by theory often is too simple (say two-factor)
- With structural model, SDF (pricing kernel) says that agent who can adjust his/her portfolio is happy
- Marginal investors are those unconstrained; whose FOC helps us to identify pricing kernel...
- High level take-away:
  - Manager’s pricing kernel always holds whether or not constraint binds
  - Households pricing kernel will not work when constraint binds
  - To show "intermediary is not a veil," someone need to show households pricing kernel fails to work
Equilibrium Risk-Return Relation and SDF Test

- The standard factor-based pricing is in terms of $\lambda$-$\beta$ language

$$\mu_i - r = \beta_i \cdot \lambda$$

- In CAPM, the factor is $R_M$. So $\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$ and $\lambda$ is the expected return of the asset with unit beta

- In standard setting,
  - The factor can be thought of the marginal investor’s wealth return $R_W$
  - The quantity of risk $\beta_i$ is again $\frac{Cov(R_i, R_W)}{Var(R_W)}$
  - The price of risk $\lambda$ equals relative risk aversion times wealth return volatility
Introducing Second Asset

- To show SDF test in our model, consider the second asset, say free of agency problem
  - Set $\theta_1 = \theta_2 = 1$
- CARA-Normal framework, iid, and equity constraint slack ($m_2 = \infty$)

$$p_2 = \mu_2 - \frac{\sigma_2^2}{\rho_M + \rho_H}$$

- And consider the situation $m_1 \leq \rho_H / \rho_M$ so constraint binds for asset 1
- Households terminal wealth

$$\frac{m}{1 + m} \tilde{D}_1 + \frac{\rho_H}{\rho_M + \rho_H} \tilde{D}_2$$

- Managers terminal wealth

$$\frac{1}{1 + m} \tilde{D}_1 + \frac{\rho_M}{\rho_M + \rho_H} \tilde{D}_2$$
Cross-Sectional Asset Pricing Test

- Factor based asset pricing holds for manager’s SDF, for both assets:

\[ p_1 - \mu_1 = \frac{\sigma_1^2}{(1 + m) \rho_M} = \frac{1}{\rho_M} \left( \frac{\sigma_1^2}{(1 + m)^2} + \frac{\rho_M^2 \sigma_2^2}{(\rho_M + \rho_H)^2} \right) \]

\[ \beta_1 = \frac{\text{Cov}(R_i, R_{W,M})}{\text{Var}(R_{W,M})} \]

\[ \text{ARA of } M \]

\[ \text{vol of } M's \text{ wealth} \]

\[ p_2 - \mu_2 = \frac{\sigma_2^2}{\rho_M + \rho_H} = \frac{1}{\rho_M} \left( \frac{\sigma_1^2}{(1 + m)^2} + \frac{\rho_M^2 \sigma_2^2}{(\rho_M + \rho_H)^2} \right) \]

\[ \beta_1 = \frac{\text{Cov}(R_i, R_{W,M})}{\text{Var}(R_{W,M})} \]

\[ \text{ARA of } M \]

\[ \text{vol of } M's \text{ wealth} \]

\[ \text{price of risk } \lambda \]

- One can show asset 2 holds for household but not asset 1
Empirical Literature

- Two camps
- Study some shocks, and show certain assets are affected more (than others)
  - CIP violation (Du, Tepper, Verdelhan, 2017)
  - CDS pricing after Tsunami shock (Siriwardane, 2016)
- SDF-type test
  - Gabaix, Krishnamurthy, and Vigneron (2007)
  - He, Kelly, and Manela (2017)
CIP Violation

- Du, Tepper, Verdelhan (2017) show CIP violation after crisis
- Higher demand of dollar, $\frac{S_t}{F_t} \left( 1 + i_t^{Euro} \right) > \left( 1 + i_t^{USD} \right)$

![Graph showing CIP violation over time]
Gabaix, Krishnamurthy, and Vigneron (2007)

- Study MBS market before crisis
- At that time, the major risk for institutional investors who specialize in MBS is prepayment risk
  - These investors’ pricing kernel is tightly linked to $CBAR_t - R_t$
    - $CBAR_t$: average coupon outstanding in the mortgage market; $R_t$: 10-year treasury
    - Time-varying price of risk $\lambda_t$
- Cross-sectionally, MBS that have higher $\beta$ to this risk earns higher return
  - A positive risk premium for prepayment risk
  - When $CBAR_t - R_t$ is high, households prepay $\Rightarrow$ loss of MBS investors
- Controlling interest rate fluctuation, households consumption $\uparrow$ when $CBAR_t - R_t$ $\uparrow$
  - Standard model will imply a negative risk premium
Gabaix, Krishnamurthy, and Vigneron (2007)
Construct an intermediary pricing kernel based on equity capital ratio of Primary Dealers

- Primary Dealers: counterparties of the Federal Reserve Bank of New York ("NY Fed" henceforth) in its implementation of monetary policy
- We choose them because they are large and active in many asset markets

\[
\eta_t = \frac{\sum_i \text{Market Equity}_{i,t}}{\sum_i \text{Market Equity}_{i,t} + \text{Book Debt}_{i,t}}
\]

In contrast to GKV (2007), we carry out traditional asset pricing test for a wide range of asset classes

- Equities, US government and corporate bonds, foreign sovereign bonds, options, credit default swaps (CDS), commodities, and foreign exchange (FX)

Theory suggests that the price of risk \( \lambda \) should be similar across these asset markets, a surprising result that holds roughly in the data
Evolution of Intermediary Capital Risk
Empirical Methodology

- Two factor structure, market return $R^M$ and intermediary capital risk $\eta^\Delta$

- For each asset class $k$
  - Time-series regression to obtain $\beta_i$
    \[
    R_{t+1}^i - r^f_t = \alpha^i + \beta^i_{\eta} \eta^\Delta_{t+1} + \beta^i_M (R^M_{t+1} - r^f_t) + \epsilon^i_{t+1}
    \]
  - Cross-sectional regression to obtain $\lambda^k_{\eta}$
    \[
    \hat{E} \left[ R_{t+1}^i - r^f_t \right] = \gamma^k + \lambda^k_{\eta} \hat{\beta}_{\eta}^i + \lambda^k_M \hat{\beta}_M^i + \nu^i_k
    \]

- They says the price of risk does not depend on asset class
  \[
  \lambda^k_{\eta} = \lambda_{\eta}
  \]

- These asset classes might be segmented, but if intermediaries like primary dealers are active on all of them, they will move around capital until their FOC holds
Price of Intermediary Capital Risk
Is Intermediary a Veil?

- Note, so far the evidence does not rule out that households SDF works as well
  - Standard consumption based framework goes a long-way in explaining equity market
  - There are some attempts to explain dynamics of index options based on standard consumption based framework, but no cross-section
  - Doubt these models work for other derivatives markets say CDS

- He, Kelly, Manela (2017) present two pieces of evidence along this line
  - First, other widely-used factors that work in equity market (FF five-factor, Pastor-Stambaugh liquidity factor) fail
  - Second, the equity capital ratio of non-Primary-Dealers works for equity but fails on others
    - Non-primary dealers tend to be smaller, standalone broker-dealers for the equity market but with little activity in derivatives markets
    - Their pricing kernel may reflect SDF of their clients (households) in equity market