Indeterminacy and Sunspots in Macroeconomics

Friday September 8th: Lecture 10
Gerzensee, September 2017
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Topics for Lecture 10

• Tying together the pieces
• Using the belief function to select an equilibrium
• Using dynamic and steady state indeterminacy to explain data
Reading

• Farmer, ReStuds “The Lucas Critique, Policy Invariance and Multiple Equilibria”

• Farmer and Nicolò “Keynesian Economics without the Phillips Curve”

• Farmer Prosperity for All: Chapter 9
The Asset Markets and the Labor Market

In this final lecture I will tie together two themes.

People die. New people are born.

These facts imply that the asset markets are inefficient.

The labor market is not an auction. There are not enough prices to allocate search effort.

This fact implies that the labor market is inefficient.
Dynamic Indeterminacy

Overlapping generations models are characterized by multiple equilibria

These models always display dynamic indeterminacy and sunspots

This feature explains sticky prices

Death is a fact of life!
Dynamic Indeterminacy

Dynamic indeterminacy explains

1) Why asset markets are excessively volatile

2) Why monetary shocks have real effects

3) Why prices are sticky
Steady State Indeterminacy

The auction model of the labor market cannot explain unemployment.

The classical search model can explain unemployment but it cannot explain how unemployment changes over time.

The Keynesian search model can explain both of these features.
Steady State Indeterminacy

Steady state indeterminancy explains

1) Why unemployment is persistent

2) Why the asset markets are closely correlated with unemployment
Resolving Indeterminacy with the Belief Function

Here, I will begin with the Farmer-Woodford model of dynamic indeterminacy and I will ask: How do people behave in a world where there are multiple possible equilibrium actions?

I will show how to resolve the indeterminacy of equilibrium with a belief function
Resolving Indeterminacy with the Belief Function

This is drawn from the paper, Farmer, ReStuds “The Lucas Critique, Policy Invariance and Multiple Equilibria”
Dynamic Indeterminacy and Sticky Prices

On Wednesday, I showed how to construct a simply monetary model where an equilibrium is a solution to the equation

\[ m_t = E_t \left[ \frac{m_{t+1} - g}{m_t} \right] \]

where

\[ m_t \equiv \frac{M_t}{p_t} \]

is real money balances.
Dynamic Indeterminacy and Sticky Prices

Here, I will show how this model has the potential to explain

1) Excess asset market volatility

2) Why prices appear to be ‘sticky’ in data
Excess Volatility

Note that any process of the form

\[ m_{t+1} = m_t^2 + g + \varepsilon_{t+1} \]

Satisfies the equation

\[ m_t = E_t \left[ \frac{m_{t+1} - g}{m_t} \right] \]
Stochastic equilibria

\[ m_{t+1} = m_t^2 + g + \varepsilon_{t+1} \]

If \( \varepsilon_{t+1} \in [-\bar{\varepsilon}, \bar{\varepsilon}] \), the economy converges to an invariant measure.
Excess Volatility

This raises two questions

1) What is $\varepsilon_{t+1}$?

2) How is the equilibrium selected?
$\varepsilon_{t+1}$ is a story that influences opinion

It is a common view that asset markets are not rational. I agree with that common view. But it does not mean that there is money to be made.

Markets are informationally efficient. They are not Pareto efficient.

$\varepsilon_{t+1}$ is a story that circulates and influences opinion
Stories and Beliefs

In a model where equilibrium is unique, there are forces acting that return the price back to equilibrium.

If demand exceeds supply at the current price, shortages will cause some demanders to bid up the price.

If supply succeeds demand at the current price, the glut will cause some suppliers to lower the price.

What enforces an equilibrium in the sunspot model with indeterminacy?
The Belief Function

I will show that expectations are anchored by a belief function.

A belief function is a mapping from current and past observable variables to beliefs about future variables: An example is:

\[
\frac{p_t}{p_{t+1}^E} = m_{t-1}^2 + g + \varepsilon_t + \frac{\varepsilon_{t+1}}{m_t}
\]
The Belief Function

Note that the belief function is state contingent.

If the Wall Street Journal (WSJ) predicts that the economy is in good shape, that is reflected in a high value of $\varepsilon_t$ and a high labor supply $n_t$.

But the prediction of next year’s price level is contingent on what people believe the WSJ will say next year. This is reflected in $\varepsilon_{t+1}$. 
The Belief Function and the Demand for Money

Recall that

\[
\frac{M_t^D}{p_t} = n_t = E_t \left[ \frac{p_t}{p_{t+1}^E} \right]
\]

\[
\frac{p_t}{p_{t+1}^E} = m_{t-1}^2 + g + \epsilon_t + \frac{\epsilon_{t+1}}{m_t}
\]

Putting these pieces together

\[
\frac{M_t^D}{p_t} \equiv m_t^D = [m_{t-1}^2 + g + \epsilon_t]
\]

This is invariant to changes in \( p_t \)
The Belief Function and the Supply of Money

The supply of money is determined by the fiscal rule

\[ m_t^S \equiv \frac{M_t^S}{p_t} = m_{t-1}^S \frac{p_{t-1}}{p_t} + g \]

This is NOT invariant to changes in \( p_t \)
Pinning Down the Equilibrium Price Level

If the quantity of money demanded is equal to the quantity of money supplied

\[ m_t^D = m_{t-1}^2 + g + \varepsilon_t = m_{t-1}^S \frac{p_{t-1}}{p_t} + g = m_t^S \]

This the way the actual price is determined when people forecast using the belief function

\[ \frac{p_t}{p_{t+1}} = m_{t-1} + \frac{\varepsilon_t}{m_{t-1}} \]

\[ \frac{p_t}{p_{t+1}} = m_{t-1} + g + \varepsilon_t + \frac{\varepsilon_{t+1}}{m_t} \]
Is it Rational?

We have established that if, at date $t$, people forecast the future using the function

$$\frac{p_t}{p_t^E} = m_{t-1} + g + \varepsilon_t + \frac{\varepsilon_{t+1}}{m_t},$$

Then prices at date $t$ will be given by the expression

$$\frac{p_{t-1}}{p_t} = m_{t-1} + \frac{\varepsilon_t}{m_{t-1}}.$$
Is it Rational?

We know from the money supply rule that

\[
\frac{p_{t-1}}{p_t} = m_{t-1} + \frac{\varepsilon_t}{m_{t-1}},
\]

If this is a rational expectations equilibrium, it must be that

\[
m_{t-1} = m_{t-2}^2 + g + \varepsilon_{t-1}
\]
Is it Rational?

If we substitute (2) into (1)

\[
\frac{p_{t-1}}{p_t} = m_{t-2}^2 + g + \varepsilon_{t-1} + \frac{\varepsilon_t}{m_{t-1}},
\]

We have established that if people at date \( t \) forecast using the rule

\[
\frac{p_t}{p_{t+1}^E} = m_{t-1}^2 + g + \varepsilon_t + \frac{\varepsilon_{t+1}}{m_t}
\]

It is rational to use the same rule at date \( t - 1 \)
To summarize

If agents form beliefs using this rule at every date

\[
\frac{p_t}{p_{t+1}} = m_{t-1}^2 + g + \varepsilon_t + \frac{\varepsilon_{t+1}}{m_t}
\]

Equilibrium prices will duplicate these beliefs

\[
\frac{p_t}{p_{t+1}} = m_{t-1}^2 + g + \varepsilon_t + \frac{\varepsilon_{t+1}}{m_t}
\]
Sticky Prices

It is often asserted that prices are ‘sticky’: What does this mean?

The main evidence for sticky prices at the aggregate level comes from the observation that, in monetary VARs, monetary shocks have real effects on GDP and employment that only slowly feed into prices.
Sticky Prices

To explain the VAR evidence when the equilibrium is determinate, the price must move one-for-one with a monetary shock. Can you explain why?
Sticky Prices

If the equilibrium is determinate, the price level must jump to keep the economy on the stable manifold. In this case the stable manifold is a point

\[ \frac{M_t}{p_t} \]

\[ \frac{M_{t-1}}{p_{t-1}} \]
Sticky Prices

If the equilibrium is indeterminate, the price level can be determined one period in advance. In this case the stable manifold is a one-dimensional set.
Excess Volatility

If there is NO fundamental uncertainty, asset prices can still undergo substantial fluctuations.

These are generated by sunspots aka animal spirits
The Implications for Macroeconomics

I have explained how dynamic indeterminacy can explain sticky prices and excess asset market volatility in a toy example.

Next: I will turn to how dynamic indeterminacy can explain sticky prices and excess asset market volatility in a calibrated DSGE model.

I will also explain how steady-state indeterminacy can explain the persistence of the unemployment rate.
Recall that the NK model has three equations:

(1) \[ \rho y_t = \rho E_t[y_{t+1}] - (i_t - E_t[\pi_{t+1}]) - r + \epsilon_t^D \]

(2) \[ i_t = r + (1 + \lambda)\pi_t + \mu(y_t - \bar{y}_t) + \epsilon_t^P \]

(3) \[ \pi_t = E_t[\pi_{t+1}] + \varphi(y_t - \bar{y}_t) + \epsilon_t^S \]
The Farmer Monetary Model

Farmer 2013 develops an alternative three equation monetary model

(1)  \[ \rho y_t = \rho E_t[y_{t+1}] - (i_t - E_t[\pi_{t+1}]) - r + \epsilon_t^D \]

(2)  \[ i_t = r + (1 + \lambda)\pi_t + \mu(y_t - \bar{y}_t) + \epsilon_t^P \]

(3)  \[ y_t - y_{t-1} + \pi_t = E_t[y_{t+1} - y_t + \pi_{t+1}] + \epsilon_t^S \]
The Farmer Monetary Model

The first two equations are the same as in the NK model

The NK Phillips curve is replaced by the belief function

\[
(3) \quad y_t - y_{t-1} + \pi_t = E_t[y_{t+1} - y_t + \pi_{t+1}] + \varepsilon_t^s
\]
To make decisions about how much to spend this year, we need to forecast our future income over the infinite future

Milton Friedman called this concept, permanent income

Income is growing over time because of inflation and because of real GDP growth
The Farmer Monetary Model

In the FM model, I assume that people forecast the future by looking at the past

$$E_t \left[ \frac{Y_{t+1}P_{t+1}}{Y_tP_t} \right] = \frac{Y_tP_t}{Y_{t-1}P_{t-1}}$$

This is a simple parameterization of a belief function
The Farmer Monetary Model

If we let $x_t$ be the log of nominal GDP growth, the belief function states that

$$E_t[x_{t+1}] = x_t$$

Where

$$x_t = y_t - y_{t-1} + \pi_t$$
Structural Forms

If we define

\[ X_t = [y_t, \pi_t, i_t]^T \]

The structural form of the NK and the FM model can both be written in the form

\[ A_0 X_t = A_1 X_{t-1} + FE_t[X_{t+1}] + A_2 \varepsilon_t + A_3 \]
Reduced Forms

The reduced form of the models are equations of the form

\[ X_t = AX_{t-1} + B\varepsilon_t + C \]

Using the Engle-Granger representation theorem

\[ \Delta X_t = C + \Pi X_{t-1} + B\varepsilon_t \]

And the steady state is

\[ \bar{X} = -\Pi^{-1}C \]
The Steady State of the NK Model

For the NK model, the steady state solves:

\[ i - \pi = r \]

\[ i - \pi = r + \lambda(\pi - \bar{\pi}) + \mu(y - \bar{y}) \]

\[ y = \bar{y} \]

The unique steady state is

\[ \{y = \bar{y}, \pi = \bar{\pi}, i = \bar{\pi} + r\} \]
The Steady State of the FM Model

For the FM model, the steady state solves:

\[ i - \pi = r \]

\[ i - \pi = r + \lambda (\pi - \bar{\pi}) + \mu (y - \bar{y}) \]

\[ 0 = 0 \]

There are only two steady state equations to determine three unknowns.
The Reduced Form of the NK Model

The reduced form of the NK model is a VAR

\[ X_t = AX_{t-1} + C + B \varepsilon_t \]
The Reduced Form of the FM Model

The reduced form of the FM model is a VECM

\[ \Delta X_t = \alpha \beta^T X_{t-1} + C + B \varepsilon_t \]
The Taylor Principle in the NK Model

We have established that, for the NK model, the equilibrium is determinate if the Taylor principle holds

This requires

\[ |1 + \lambda| > 1 \]
The Taylor Principle in the FM Model

Farmer and Nicolò show that an equivalent representation of the Taylor principle in the FM model, for the special case when $\rho = 1$ (log preferences) is given by the expression

$$\left| \frac{\lambda}{\lambda - \mu} \right| > 1$$
The Taylor Principle in the FM Model

For more general preferences, when $\rho \neq 0$, there is no simple analytic expression for dynamic determinacy.

But: for estimated values, the Taylor Principle fails whenever $\rho$ is slightly greater than one.
The Taylor Principle in the FM Model

This figure plots two roots of the structural form for a calibrated version of the FM model as functions of the risk aversion parameter $\rho$.

Dynamic determinacy requires both roots to be greater than 1.

Figure 1
The Taylor Principle in the FM Model

Figure 2: Data used to estimate the model
The Taylor Principle in the FM Model

Farmer and Nicolò “Keynesian Economics without the Phillips Curve” estimate the NK and the FM model and compare their ability to explain data

Dynamic indeterminacy allows for a richer propagation mechanism

Steady state indeterminacy implies a VECM rather than a VAR
The Taylor Principle in the FM Model

Bayesian econometrics allows the comparison of two models

We assume that both models are equally likely

We compute the likelihood of each model and combine it with the prior to generate a posterior

We compare the posteriors of the two models
The Taylor Principle in the FM Model

We compare the models separately for data before and after 1980 and we find that the data overwhelmingly favor the FM model for both sub-periods.
The Taylor Principle in the FM Model

Table 2: Model comparison

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<th>FM model</th>
<th>NK model</th>
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<tbody>
<tr>
<td>Pre-Volcker (54Q3-79Q2)</td>
<td>Log data density</td>
<td>1023.24</td>
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<tr>
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<td>Posterior Model Prob (%)</td>
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<tr>
<td>Post-Volcker (83Q1-07Q4)</td>
<td>Log data density</td>
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<td>Posterior Model Prob (%)</td>
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Bayesian Model Comparison
The Taylor Principle in the FM Model

Why do we find these results?

1) The data are persistent and a model with dynamic indeterminacy has a richer propagation mechanism

2) The data are non-stationary and cointegrated: the NK model cannot account for that feature
Conclusion

Indeterminacy, both dynamic and steady state, is a way of understanding how psychology influences economics.

Models where psychology matters, are also models where policy matters.

In Prosperity for All, I argue that governments should stabilize the stock market.