Marriage, Labor Supply, and Home Production

Jean-Marc Robin

Sciences Po and UCL

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INTRODUCTION

Goussé, Jacquemet, and Robin (forth.)
Question

How much of gender differences in labor supply can be explained by differences in

- wages
- preferences for consumption and leisure
- home production skills
- family values
What we do

- We take as given the changes in the distribution of individual wages, education and family values in the UK, 1991-2008.
- We construct a search-matching model of marriage and intrahousehold allocation of resources.
- We fit it to the observed changes in
  - the labor supply and work-in-household of husband and wife, and
  - the distributions of individual characteristics by marital status (married/cohabiting and single).
- Finally we run counterfactuals.
Collective models (Chiappori, Mazzocco,…). By modeling the marriage market we endogenize the sharing rule.

Macro literature on marriage (Greenwood, Guner, Knowles, Fernandez,…). We put more emphasis on modeling the changes in the cross sectional distributions of types.

Search-matching (Shimer-Smith and macro papers) instead of stable matching (Choo-Siow, Galichon-Salanié, Chiappori-Iyigun-Weiss,…)

Social norms (Akerlof-Kranton, Bertrand-Kamenica-Pan)
Roadmap

1. Model
2. Identification and estimation procedure
3. Results
THE MODEL

Extends Shimer and Smith (2000)
Private consumption and leisure

- Individuals differ in a vector of characteristics $i$ for males and $j$ for females (wage, education, family values, etc.).
- Individual $i$ draws utility $U_i(c, e, q)$ from consumption ($c$), leisure ($e \leq 1 - d$; $d$ is nonmarket time) and a public good ($q$).
- Indirect utility:

$$\psi_i(R, q) = \max_{c > 0, 1 - d \geq e > 0} U_i(c, e, q) \text{ s.t. } c + w_i e \leq R$$

for a given total private expenditure $R$ and public good $q$. 
Singles: \( q = F_i^0(d) \), where \( d \) is home-production time use

Couples: \( q = F_{ijz}^1(d_m, d_f) \equiv z \cdot F_{ij}^1(d_m, d_f) \)

- \( d_m, d_f \) are domestic time inputs
- \( z \) is a bliss shock – changes with proba \( \delta \), iid
- Hence, two types of complementarities: in inputs \( d_m, d_f \), and in exogenous types \( i, j \)
Saving time and money for home production

- **Singles:**
  \[ c + w_i e = w_i (1 - d) \equiv R \]

- **Couples:**
  \[ c_m + w_i e_m = w_i (1 - d_m) - t_m \equiv R_m \]
  \[ c_f + w_j e_f = w_j (1 - d_f) - t_f \equiv R_f \]

- **Transfers** \( t_m, t_f \) finance redistribution and a lump sum contribution:
  \[ t_m + t_f = C_{ij} \]

\( C_{ij} \) can be thought of as children’s consumption.
Marriage contracts

- A marriage contract (given match characteristics $i, j, z$) specifies flow utilities $u_m, u_f$ and promised values.

- No commitment. Marriage contracts must be renegotiated when environment changes (bliss shock). Implies promise keeping constraint.

Define continuation values $V^1_m(i, j, z')$ and $V^1_f(i, j, z')$ upon realization of the next match shock $z'$ and given $i, j$.

Define $V^0_i$ as the value of being single.

Present value of marriage to $i$ married with $j$ for a given marriage contract:

$$rW_m = u_m + \delta \left[ \int \max \{ V^0_i, V^1_m(i, j, z') \} \, dG(z') - W_m \right]$$
Nash bargaining

For given continuation values, spouses solve

$$\max_{d_m, d_f, t_m, t_f} [W_m - V_i^0]^\beta [W_f - V_j^0]^{1-\beta}$$

for

$$u_m = \psi_i(w_i(1 - d_m) - t_m, q), \quad u_f = \psi_j(w_j(1 - d_f) - t_f, q), \quad q = F_{ijz}^1(d_m, d_f)$$

subject to feasibility

$$C_{ij} = t_m + t_f$$

and participation constraints

$$W_m - V_i^0 \geq 0, \quad W_f - V_j^0 \geq 0$$
Limited commitment

Without commitment, continuation values must satisfy the promise keeping constraint

\[ W_m = V_m^1(i, j, z), \quad W_f = V_f^1(i, j, z) \]

i.e.

\[
(r + \delta) \left[ V_m^1(i, j, z) - V_i^0 \right] = u_m - rV_i^0 + \delta \int (V_m^1(i, j, z') - V_i^0)^+ \, dG(z')
\]

where \( x^+ = \max\{x, 0\} \)
EQUILIBRIUM
Parametric specification - Preferences

- Quasi-linear indirect utility:
  \[ \psi_i(R, q) = q \frac{R - A_i(w_i)}{B_i(w_i)} \]

  - \( A_i, B_i \) are functions of wage of other characteristics such as education.
  - \( A_i \) is minimal expenditure
  - \( B_i \) is an individual-specific price index

- Implies that we rule out corner solutions such as \( e = 1 - d \)

- By Roy’s lemma,
  \[ e = - \frac{\partial \psi_i(R, q)/\partial w_i}{\partial \psi_i(R, q)/\partial R} = A_i' + B_i' \frac{R - A_i}{B_i} \]
Stone-Geary home production:

\[
\begin{align*}
[couples] & \quad F_{ij}^{1}(d_m, d_f) = Z_{ij} (d_m - D_{i}^{1})^{K_{i}^{1}} (d_f - D_{j}^{1})^{K_{j}^{1}} \\
singles & \quad F_{i}^{0}(d) = (d - D_{i}^{0})^{K_{i}^{0}}
\end{align*}
\]

TFP $Z_{ij}$ is public good quality and incorporates complementarities wrt to exogenous characteristics.

Forces complementarity between time uses $d_m, d_f$. Now in reality we are not far from Leontieff.
Recursivity - Home production

- FOC of Nash bargaining problem + special form of indirect utility makes it easy to solve for home production time uses:

\[ w_i d_m^1 = w_i D_m^1 + K_m^1 X_{ij} \]
\[ w_j d_f^1 = w_j D_f^1 + K_f^1 X_{ij} \]

Leontieff if \( K_m^1 = K_f^1 = 0 \)

- Net total income:

\[ X_{ij} = R_m - A_i + R_f - A_j \]
\[ = w_i (1 - d_m^1) - A_i + w_j (1 - d_f^1) - A_j - C_{ij} \]
\[ = \frac{w_i (1 - D_m^1) + w_j (1 - D_f^1) - C_{ij} - A_i - A_j}{1 + K_m^1 + K_f^1} \]

- Recursivity: independence wrt transfers and values (cf Magnac-Lambert)
Transferability - Match surplus

There exists a match surplus that solves

\[ S_{ij}(z) = zF^1_{ij}X_{ij} - B_i rV^0_i - B_j rV^0_j + \frac{\delta}{r + \delta} \int \max\{S_{ij}(z'), 0\} \, dG(z') \]

where \( zF^1_{ij}X_{ij} = B_i u_m + B_j u_f \) is the sum of individual utility flows.

- Match formed if surplus positive
- Individual surpluses:

\[
(r + \delta)B_i[V^1_m(z) - V^0_i] = \beta S_{ij}(z) \\
(r + \delta)B_j[V^1_f(z) - V^0_j] = (1 - \beta) S_{ij}(z)
\]
Transferability - Transfers

- Transfers:
  \[ R_m - A_i = w_i(1 - d_m) - t_m - A_i = \beta_{ij}(z) X_{ij} \]

- Sharing rule:
  \[
  \beta_{ij}(z) = \beta \left( 1 - \beta \right) B_i r V_i^0 - \beta B_j r V_j^0 \]
  \[
  z F_{ij}^1 X_{ij}
  \]

Depends on bargaining parameter \( \beta \) and outside options
Outside option: The value of being/remaining single

- The values for singles solve

\[ B_i r V_i^0 = \max_{0<d<1} \left\{ F_i^0(d) [w_i(1 - d) + \mu - A_i] \right\} + \int \int \lambda n_f(j) \frac{\beta S_{ij}(z)^+}{r + \delta} \, dG(z) \, dj \]

- \( \lambda \): proba of meeting; \( n_m(i), n_f(j) \): distribution of types for singles
- Individuals have to predict measures \( n_m(i), n_f(j) \), which are endogenous.
Steady-state matching equilibrium

- **Marriage probability:**
  \[
  \alpha(i, j) \equiv \alpha_{ij} = \Pr \{ S_{ij}(z) > 0 \mid i, j \}
  \]

- **Steady state:**
  \[
  \delta [1 - \alpha(i, j)] m(i, j) = \lambda n_m(i) n_f(j) \alpha(i, j)
  \]

- **Accounting restrictions:**
  \[
  \int m(i, j) \, dj = \ell_m(i) - n_m(i), \quad \int m(i, j) \, di = \ell_f(j) - n_f(j),
  \]

- These 3 equations for 3 unknowns, that determine \( n_m(i) \) and \( n_f(j) \) given \( \ell_m(i), \ell_f(j) \) and \( \alpha(i, j) \).
Global equilibrium

- Equations for values and surplus + steady state
- Complicated fixed point. Existence of an equilibrium hard to show (see Shimer, Smith, ECMA, 2000). Non-uniqueness in general (Burdett, Coles, QJE, 1997).
IDENTIFICATION and ESTIMATION
Empirical specification

- **Preferences**: \[ A = a_0 + a_1 w + \frac{1}{2} a_2 w^2, \quad B = w^b, \text{ yielding} \]
  \[
  w_i e_m^1 = a_{1m} w_i + a_{2m} w_i^2 + b_m \beta_{ij}(z) X_{ij} \\
  w_j e_f^1 = a_{1f} w_j + a_{2f} w_j^2 + b_f [1 - \beta_{ij}(z)] X_{ij}
  \]

- **Public good quality**: \( Z_{ij} \) nonparametric (high-order polynomial incl. interactions).

- **Distribution of match-specific heterogeneity** is log normal:
  \[ G(z) = \frac{1}{z} \Phi\left(\frac{\ln z}{\sigma}\right) \]
Identification - Rates $\lambda, \delta$ and marriage probabilities $\alpha$

- Take a period of time (one year, three years, ...)
- Rates $\lambda$ and $\delta$ determining meetings and divorce risk and the conditional matching probability $\alpha_{ij}$ are identified from marriage and divorce flows.

$$MF(i, j) = \lambda n_m(i) n_f(j) \alpha_{ij}$$
$$DF(i, j) = m(i, j) \delta (1 - \alpha_{ij})$$

- Solve for $\alpha_{ij}$:

$$\alpha_{ij} = 1 - \frac{1}{\lambda n_m(i) n_f(j)} \frac{MF(i, j)}{DF(i, j)} = \frac{1}{\delta} \frac{DF(i, j)}{m(i, j)}$$

- In practice we estimate marriage probas from steady state restriction (curse of dimensionality)

$$\alpha_{ij} = \frac{\delta m(i, j)}{\delta m(i, j) + \lambda n_m(i) n_f(j)}$$
The parameters of preferences and home production are identified in much the same way as in standard labor supply model, the other spouse’s earnings acting as a source of non-earned income allowing to identify income effect parameters.

Eliminate $\beta_{ij}(z)$ from labor supply equations.

\[ w_i e_{m}^1 + \frac{b_m}{b_f} w_j e_{f}^1 = a_{1m} w_i + a_{2m} w_i^2 + \frac{b_m}{b_f} (a_{1f} w_j + a_{2f} w_j^2) + b_m X_{ij}. \]

$w_i e_{m}^1, w_j e_{f}^1$ depend on bliss shock, not RHS.

Other preference and home production parameters easily follow.
The bargaining parameter $\beta$ and the variance of match-specific shocks are identified from the first and second-order moments of individual leisure demands.
The public good quality parameter $Z_{ij}$ is identified from the structural link between public good quality and matching probability.

We show that $Z_{ij} \leftrightarrow \alpha_{ij}$ is invertible.

Only this latter step requires estimating values and using deep model restrictions.
RESULTS
British Household Panel Survey (BHPS) 1991-2008

- Singles and heterosexual couples aged 22-50 (about 1200 couples, 400 male singles and 400 female singles in every year)
- Data on CPI-deflated gross pay per month, number of hours normally worked per week (including paid and unpaid overtime hours), number of housework hours per week
- Short-term fluctuations of individual wages and hours are smoothed out by marital spell using moving averages (including zero hours).
- We drop out individuals with zero hours worked over long time periods.
- Heterogeneity: education, family values
### Family Values Index - PCA

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<td>Pre-school child suffers if mother works</td>
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<td>Family suffers if mother works full-time</td>
<td>-0.25</td>
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<td>Woman and family happier if she works</td>
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<td>Husband and wife should both contribute</td>
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<td>Full time job makes woman independent</td>
<td>0.12</td>
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<td>Husband should earn, wife stay at home</td>
<td>-0.21</td>
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<td>Children need father as much as mother</td>
<td>-0.05</td>
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<td>Employers should help with childcare</td>
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<td>Single parents are as good as couples</td>
<td>0.17</td>
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<td>Adult children should care for parents</td>
<td>-0.07</td>
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<td>Divorce better than unhappy marriage</td>
<td>0.12</td>
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<td>Attendance at religious services</td>
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<td>Cohabiting is always wrong</td>
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scale: 1 = agree, 5 = disagree
Estimation

- We split the whole period 91-08 into 6 three-year periods.
- For each subperiod we estimate by kernel density estimation the stocks \( m(i, j), n_m(i), n_f(j) \) and also marriage and divorce flows by education.
- We estimate marriage rates and probas \( \lambda, \delta, \alpha_{ij} \)
- We estimate preference and home production parameters, and \( \beta \) and \( \sigma \) (std of bliss shock) by nonlinear LS
- We recover \( Z_{ij} \) from \( \alpha_{ij} \)
Marriage probabilities

- We estimate low meeting rates and high marriage probabilities (around 0.5). So dating here is a pretty serious affair.
- Very stable marriage probabilities
By education

Symmetric with a strong diagonal.

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Symmetric, but only conservative care.

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Q4: conservative, Q1: progressive
By wage quartile

Not symmetric. Females care about male wage. Males don’t care.

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<td>Q4</td>
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<td>0.31</td>
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<td>0.30</td>
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<tr>
<td>Q1</td>
<td>0.30</td>
<td>0.31</td>
<td>0.27</td>
<td></td>
<td></td>
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</tbody>
</table>
By female wage ratio

Bertrand-Kamenica-Pan: “Women should not earn more than men.”
Preference parameters

\[ w_i e_m^1 = a_{1m} w_i + a_{2m} w_i^2 + b_m \beta_{ij}(z) X_{ij}, \quad c_m^1 = a_{0m} - \frac{1}{2} a_{2m} w_i^2 + (1 - b_m)[1 - \beta_{ij}(z)] X_{ij} \]

Education matters for private trade-off between consumption and leisure, not family values.

| \( a_{0f}[Ed = L] \) | -21.459 (7.802) | \( a_{2f} \) | -0.0031 (0.0007) |
| \( a_{0m}[Ed = L] \) | -26.634 (8.058) | \( a_{2m} \) | -0.0008 (0.0005) |
| \( a_{0f}[Ed = H] \) | -12.742 (5.007) | \( b_f[Ed = L] \) | 0.0303 (0.0119) |
| \( a_{0m}[Ed = H] \) | -13.579 (5.503) | \( b_f[Ed = H] \) | 0.0721 (0.0248) |
| \( a_{0f}[FVI] \) | 0.7455 (0.7355) | \( b_m[Ed = L] \) | 0.0345 (0.0122) |
| \( a_{0m}[FVI] \) | 0.6168 (0.6101) | \( b_m[Ed = H] \) | 0.0940 (0.0340) |
| \( a_{1f}[Ed = L] \) | 0.4403 (0.0175) | \( b_F[FVI] \) | -0.0023 (0.0020) |
| \( a_{1m}[Ed = L] \) | 0.3939 (0.0197) | \( b_m[FVI] \) | -0.0000 (0.0021) |
| \( a_{1f}[Ed = H] \) | 0.4350 (0.0215) |
| \( a_{1m}[Ed = H] \) | 0.3812 (0.0258) |
| \( a_{1f}[FVI] \) | 0.0184 (0.0046) |
| \( a_{1m}[FVI] \) | -0.0001 (0.0050) |
Home production parameters

\[ w_i d_m^1 = w_i D_m^1 + K_m^1 X_{ij} \]

Little effect of education, strong effect of family values.

<table>
<thead>
<tr>
<th></th>
<th>Singles</th>
<th>Couples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( D_f^0 [Ed = L] ) 0.0524 (0.0177)</td>
<td>( D_f^1 [Ed = L] ) 0.0701 (0.0105)</td>
</tr>
<tr>
<td></td>
<td>( D_m^0 [Ed = L] ) 0.0467 (0.0188)</td>
<td>( D_m^1 [Ed = L] ) 0.0660 (0.0095)</td>
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<tr>
<td></td>
<td>( D_f^0 [Ed = H] ) 0.0481 (0.0142)</td>
<td>( D_f^1 [Ed = H] ) 0.0564 (0.0094)</td>
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<tr>
<td></td>
<td>( D_m^0 [Ed = H] ) 0.0363 (0.0142)</td>
<td>( D_m^1 [Ed = H] ) 0.0708 (0.0085)</td>
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<tr>
<td></td>
<td>( D_f^0 [FVI] ) 0.0096 (0.0041)</td>
<td>( D_f^1 [FVI] ) 0.0159 (0.0029)</td>
</tr>
<tr>
<td></td>
<td>( D_m^0 [FVI] ) 0.0031 (0.0039)</td>
<td>( D_m^1 [FVI] ) -0.0073 (0.0025)</td>
</tr>
<tr>
<td></td>
<td>( K_f^0 ) 0.0177 (0.0070)</td>
<td>( K_f^1 ) 0.0183 (0.0038)</td>
</tr>
<tr>
<td></td>
<td>( K_m^0 ) 0.0002 (0.0034)</td>
<td>( K_m^1 ) 0.0056 (0.0026)</td>
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</tbody>
</table>
Public good complementarities $Z_{ij}$

\[ F_{ij}^1(d_m, d_f) = Z_{ij} (d_m - D_m^1)^{K_m^1} (d_f - D_f^1)^{K_f^1} \]

Essentially education

<table>
<thead>
<tr>
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<tr>
<td>Nonparametric estimation of $Z_{ij}$</td>
<td>0.72</td>
<td>0.83</td>
<td>0.86</td>
<td>0.87</td>
<td>0.82</td>
<td>0.78</td>
<td>0.81</td>
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<tr>
<td>Quadratic projection of ln $Z_{ij}$</td>
<td>0.68</td>
<td>0.77</td>
<td>0.80</td>
<td>0.80</td>
<td>0.75</td>
<td>0.72</td>
<td>0.75</td>
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<tr>
<td>Quadratic projection of ln $Z_{ij}$ without interactions</td>
<td>0.38</td>
<td>0.51</td>
<td>0.60</td>
<td>0.62</td>
<td>0.57</td>
<td>0.53</td>
<td>0.54</td>
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<td>Quadratic projection of ln $Z_{ij}$ with no interactions but...</td>
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<tr>
<td>$w_m * w_f$</td>
<td>0.43</td>
<td>0.54</td>
<td>0.63</td>
<td>0.65</td>
<td>0.59</td>
<td>0.56</td>
<td>0.57</td>
</tr>
<tr>
<td>$FVI_m * FVI_f$</td>
<td>0.37</td>
<td>0.51</td>
<td>0.61</td>
<td>0.63</td>
<td>0.57</td>
<td>0.54</td>
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<tr>
<td>$Ed_m * Ed_f$</td>
<td>0.66</td>
<td>0.72</td>
<td>0.76</td>
<td>0.75</td>
<td>0.69</td>
<td>0.67</td>
<td>0.71</td>
</tr>
<tr>
<td>$w_m * Ed_f$</td>
<td>0.43</td>
<td>0.55</td>
<td>0.64</td>
<td>0.66</td>
<td>0.61</td>
<td>0.57</td>
<td>0.58</td>
</tr>
<tr>
<td>$w_f * Ed_m$</td>
<td>0.53</td>
<td>0.61</td>
<td>0.68</td>
<td>0.67</td>
<td>0.63</td>
<td>0.58</td>
<td>0.62</td>
</tr>
<tr>
<td>$w_m * FVI_f$</td>
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</tr>
<tr>
<td>$FVI_m * Ed_f$</td>
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<td>0.52</td>
<td>0.60</td>
<td>0.63</td>
<td>0.59</td>
<td>0.55</td>
<td>0.54</td>
</tr>
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</table>
Fit of marriage probability by female wage ratio

![Graph showing the relationship between wife share of aggregate wage and average match probability. The graph compares actual and predicted values.]
Fit of mean wage

Dots and crosses: predictions, lines: actual
Dashed: single, solid: married, red: male, blue: female
Fit of mean FVI
Fit of education levels

Share of high school dropouts

Share of college educated
Fit of time uses

Market hours

Non market hours

- Married Men, Actual
- Married Men, Predicted
- Married Women, Actual
- Married Women, Predicted
- Single Men, Actual
- Single Men, Predicted
- Single Women, Actual
- Single Women, Predicted
Fit of wage and earnings ratios

Traditionalist couples

Wages

Earnings

Actual
Predicted

Fraction of couples

Wife share of aggregate wage

Wife share of total labor earnings
Fit of wage and earnings ratios

Progressive couples

**Wages**

- Actual
- Predicted

**Earnings**

- Actual
- Predicted

Fraction of couples

Wife share of aggregate wage

Wife share of total labor earnings
Male’s share of net total income $\beta_{ij}(z)$

- Aggregate share over time is flat around 0.55
By education

Compensating differentials

![Graphs showing changes in compensating differentials by education level between 1995 and 2005. The graphs compare the labor supply and home production of spouses with different levels of education.](image-url)
By FVI

Nothing

By female FVI

By male FVI
By female wage ratio, \( \frac{w_f}{w_m + w_f} \)

High earner gets more
COUNTERFACTUALS
## Number of marriages

<table>
<thead>
<tr>
<th></th>
<th>Actual 2000-02</th>
<th>Baseline sim.</th>
<th>(1) No wage gap</th>
<th>(2) Same preferences</th>
<th>(3) Same home-prod technology</th>
<th>(4) All progressive $FVI = 1$</th>
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<tbody>
<tr>
<td># single men</td>
<td>1416</td>
<td>1468</td>
<td>1475 0.5%</td>
<td>1258 -14.3%</td>
<td>1418 -3.4%</td>
<td>1586 8.0%</td>
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<tr>
<td># single women</td>
<td>1452</td>
<td>1509</td>
<td>1498 -0.7%</td>
<td>1305 -13.5%</td>
<td>1457 -3.4%</td>
<td>1633 8.2%</td>
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<tr>
<td># couples</td>
<td>3802</td>
<td>3745</td>
<td>3747 0.1%</td>
<td>3943 5.3%</td>
<td>3786 1.1%</td>
<td>3627 -3.2%</td>
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</table>
## Labor supply

<table>
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<th>Actual 2000-02</th>
<th>Baseline sim.</th>
<th>(1) No wage gap</th>
<th>(2) Same preferences</th>
<th>(3) Same home-prod technology</th>
<th>(4) All progressive FVI = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Married Men</td>
<td>42.99</td>
<td>42.49</td>
<td>42.37 -0.3%</td>
<td>41.77 -1.7%</td>
<td>42.55 0.2%</td>
<td>43.24 1.8%</td>
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<tr>
<td>Single Men</td>
<td>37.48</td>
<td>36.83</td>
<td>36.71 -0.3%</td>
<td>36.87 0.1%</td>
<td>36.78 -0.1%</td>
<td>36.49 -0.9%</td>
</tr>
<tr>
<td>Married Women</td>
<td>25.86</td>
<td>26.63</td>
<td>29.27 9.9%</td>
<td>28.66 7.6%</td>
<td>35.77 34.3%</td>
<td>34.53 29.6%</td>
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<tr>
<td>Single Women</td>
<td>30.07</td>
<td>29.56</td>
<td>32.29 9.3%</td>
<td>30.00 1.5%</td>
<td>34.46 16.6%</td>
<td>30.64 3.7%</td>
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## Home production

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<th>(1) No wage gap</th>
<th>(2) Same preferences</th>
<th>(3) Same home-prod technology</th>
<th>(4) All progressive $FVI = 1$</th>
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<tbody>
<tr>
<td>Married Men</td>
<td>5.13</td>
<td>5.33</td>
<td>5.37 0.8%</td>
<td>5.56 4.3%</td>
<td>5.35 0.5%</td>
<td>6.26 17.5%</td>
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<tr>
<td>Single Men</td>
<td>5.00</td>
<td>5.04</td>
<td>5.04 -0.1%</td>
<td>5.06 0.3%</td>
<td>5.04 0.1%</td>
<td>4.54 -9.8%</td>
</tr>
<tr>
<td>Married Women</td>
<td>14.99</td>
<td>15.52</td>
<td>14.35 -7.5%</td>
<td>16.45 6.0%</td>
<td>6.10 -60.7%</td>
<td>11.45 -26.2%</td>
</tr>
<tr>
<td>Single Women</td>
<td>10.00</td>
<td>10.01</td>
<td>9.30 -7.1%</td>
<td>10.63 6.2%</td>
<td>4.90 -51.0%</td>
<td>9.23 -7.8%</td>
</tr>
</tbody>
</table>

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Marriage, Labor Supply, and Home Production
COMPUTATIONAL DETAILS

This section shortly describes the numerical tools used in estimation. The best reference here is Trefethen (2013).
Chebyshev nodes

We discretize continuous functions on a compact domain using Chebyshev grids. For example, let \([\underline{x}, \overline{x}]\) denote the support of male wages, we construct a grid of \(n + 1\) points as

\[
    x_k = \frac{\overline{x} + \underline{x}}{2} + \frac{\overline{x} - \underline{x}}{2} \cos \frac{k\pi}{n}, \quad k = 0, \ldots, n.
\]
Clenshaw-Curtis quadrature I

Many equations involve integrals. Given Chebyshev grids, it is natural to use Clenshaw-Curtis quadrature to approximate these integrals:

\[
\int_{x}^{\bar{x}} f(x) \, dx \simeq \frac{\bar{x} - x}{2} \sum_{k=0}^{n} \omega_k f(x_j),
\]

where the weights \( \omega_j \) can be easily computed using Fast Fourier Transform (FFT).

Note that, although Gaussian quadrature provides exact evaluations of integrals for higher order polynomials than CC, in practice CC works as well as Gaussian. On the other hand, quadrature weights are much more difficult to calculate for Gaussian quadrature. See Trefethen (2008).
The following MATLAB code can be used to implement CC quadrature
Waldvogel (2006):

```matlab
function [nodes,wcc] = cc(n)
    nodes = cos(pi*(0:n)/n);
    N=[1:2:n-1]'; l=length(N); m=n-l;
    v0=[2./N./(N-2); 1/N(end); zeros(m,1)];
    v2=-v0(1:end-1)-v0(end:-1:2);
    g0=-ones(n,1); g0(1+l)=g0(1+l)+n; g0(1+m)=g0(1+m)+n;
    g=g0/(n^2-1+mod(n,2)); wcc=real(ifft(v2+g));
    wcc=[wcc;wcc(1)];
```
We need to solve functional fixed point equations. The standard algorithm to calculate the fixed point $u(x) = T[u](x)$ is to iterate $u_{p+1}(x) = Tu_p(x)$ on a grid. If the fixed point operator $T$ involves integrals, we simply iterate the finite dimensional operator $\hat{T}$ obtained by replacing the integrals by their approximations at grid points.
Example 1: equilibrium.

Using the previous approximations, an equation like

$$u(x) = T[u](x) = \frac{\ell(x)}{1 + \rho \int_x^x u(y) \alpha(x, y) \, dy}$$

becomes

$$\mathbf{u} = [u(x_k)]_{k=0,\ldots,n} = \hat{T}(\mathbf{u}) = \left[ \frac{\ell(x_k)}{1 + \rho \sum_{\ell=0}^n \omega_\ell u(x_\ell) \alpha(x_k, x_\ell)} \right]_{k=0,\ldots,n}.$$

It was sometimes necessary to “shrink” steps by using iterations of the form $u_{p+1} = u_p + \theta(Tu_p - u_p)$ with $\theta \in (0, 1]$. A stepsize $\theta < 1$ may help if $T$ is not everywhere strictly contracting.
Example 2: singles’ values.

$(B_i r V_i^0, B_j r V_j^0)$ solve the inhomogeneous Fredholm system

\[
B_i r V_i^0 = B_i u_i^0 + \frac{\lambda}{\delta} \beta \int (B_i r V_i^0 + B_j r V_j^0) \theta_{ij} n_f(j) \, dj,
\]

\[
B_j r V_j^0 = B_j u_j^0 + \frac{\lambda}{\delta} (1 - \beta) \int (B_i r V_i^0 + B_j r V_j^0) \theta_{ij} n_m(i) \, di.
\]
Integral equations IV

This is a linear system that can be solved for after discretizing the state space, or value function iteration. Suppose that \( i \) and \( j \) are discrete variables with weights \( \omega_i, \omega_j \). Define the matrices

\[
\Theta_m = -\frac{\lambda \beta}{\delta} \left[ \theta_{ij} n_f(j) \omega_j \right]_{i \times j}, \quad \Theta_f = -\frac{\lambda(1 - \beta)}{\delta} \left[ \theta_{ij} n_m(i) \omega_i \right]_{i \times j}^T,
\]

and

\[
\Delta_m = 1 - \frac{\lambda \beta}{\delta} \text{diag}\left( \sum_j \theta_{ij} n_f(j) \omega_j \right), \quad \Delta_f = 1 - \frac{\lambda(1 - \beta)}{\delta} \text{diag}\left( \sum_i \theta_{ij} n_m(i) \omega_i \right)
\]

Then

\[
\begin{bmatrix}
B_i r V_i^0 \\
B_j r V_j^0
\end{bmatrix} = \begin{bmatrix}
\Delta_m & \Theta_m \\
\Theta_f & \Delta_f
\end{bmatrix}^{-1} \begin{bmatrix}
B_i u_i^0 \\
B_j u_j^0
\end{bmatrix}.
\]
Interpolation

We can interpolate functions very easily between points \( y_0 = f(x_0), \ldots, y_n = f(x_n) \) using Discrete Cosine Transform (DCT):

\[
f(x) = \sum_{k=0}^{n} Y_k \cdot T_k(x),
\]

where \( Y_k \) are the OLS estimates of the regression of \( y = (y_0, \ldots, y_n) \) on Chebyshev polynomials

\[
T_k(x) = \cos \left( k \arccos \left( \frac{x - \overline{x}}{\overline{x} - \overline{x} \over 2} \right) \right),
\]

but are more effectively calculated using FFT.
A MATLAB code for DCT is, with \( y = (y_0, ..., y_n) \):

\[
Y = y([1:n+1 n:-1:2],:);
Y = \text{real}(\text{fft}(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) \cos(\text{acos}((2*x-(xmin+xmax))/(xmax-xmin))
*(0:n))*Y(1:n+1);
\]

A bidimensional version is

\[
Y = y([1:n+1 n:-1:2],:);
Y = \text{real}(\text{fft}(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
Y = Y(:,[1:n+1 n:-1:2]);
Y = \text{real}(\text{fft}(Y'/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)'];
f=@(x,y) \cos(\text{acos}((2*x-(xmin+xmax))/(xmax-xmin))\*(0:n))...\*Y(1:n+1,1:n+1)...\*\cos((0:n)'*\text{acos}((2*y'-(ymin+ymax))/(ymax-ymin))));
\]
The fact that the grid \((x_0, \ldots, x_n)\) is not uniform and is denser towards the edges of the support interval allows to minimize the interpolation error and thus avoids the standard problem of strong oscillations at the edges of the interpolation interval (Runge’s phenomenon).
Another advantage of DCT is that, having calculated \( Y_0, \ldots, Y_n \), then polynomial projections of \( y = (y_0, \ldots, y_n) \) of any order \( p \leq n \) are obtained by stopping the summation in (1) at \( k = p \). Finally, it is easy to approximate the derivative \( f' \) or the primitive \( \int f \) simply by differentiating or integrating Chebyshev polynomials using

\[
\cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)},
\]

and

\[
\int \cos(k \arccos x) \, dx = \begin{cases} 
  x & \text{if } k = 0, \\
  \frac{x^2}{2} \cos(k+1)x - \frac{\cos(k-1)x}{2(k-1)} & \text{if } k \geq 2.
\end{cases}
\]
In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary. Moreover, our experience is that the approximation:

\[
\int_{x}^{x} 1\{ t \leq x \} f(x) \, dx \approx \sum_{k=0}^{n} w_k 1\{ t \leq x_k \} f(x_k)
\]

gave similar results as integrating the interpolated function.
In the application we had to calculate the density of variables such as the wage ratio \( y_{ij} = \frac{w_j}{w_i+w_j} \) across couples \((i,j)\). A kernel density estimator of the distribution of \( M \) observations \( y_{ij} \) is

\[
\hat{f}(y) = \frac{1}{M} \sum_{(i,j)} K_h(y - y_{ij}),
\]

for a kernel \( K_h \) with width \( h \).
When we use the model to predict this density, we use our estimate of the match distribution \( m(w_m, w_f) \) (omitting socio-demographic characteristics for simplicity, which we average over), which we have tabulated it on the Chebyshev grid (using the same notation for the data and the grid points):

\[
\begin{align*}
  w_i &= \frac{\overline{w}_m + w_m}{2} + \frac{\overline{w}_m - w_m}{2} \cos \frac{i\pi}{n}, \quad i = 0, \ldots, n, \\
  w_j &= \frac{\overline{w}_f + w_f}{2} + \frac{\overline{w}_f - w_f}{2} \cos \frac{j\pi}{n}, \quad j = 0, \ldots, n.
\end{align*}
\]

Then we calculate

\[
\hat{f}(y) = \frac{\overline{w}_m - w_m}{2} \frac{\overline{w}_f - w_f}{2} \sum_{i,j=0}^{n} K_h \left( y - \frac{w_j}{w_i + w_j} \right) m(w_i, w_j).
\]

We do the same for earnings ratios.
To calculate the conditional mean matching probability given wage ratio, we use a Nadaraya-Watson estimator on data:

$$\hat{E} \left[ \alpha_{ij} \left| \frac{w_j}{w_i + w_j} = y \right. \right] = \sum_{(i,j)} \alpha_{ij} K_h \left( y - \frac{w_j}{w_i + w_j} \right) \frac{\sum_{(i,j)} K_h \left( y - \frac{w_j}{w_i + w_j} \right) m(w_i, w_j)}{\sum_{(i,j)} K_h \left( y - \frac{w_j}{w_i + w_j} \right) m(w_i, w_j)}.$$

The prediction is

$$\hat{E} \left[ \alpha(w_i, w_j) \left| \frac{w_j}{w_i + w_j} = y \right. \right] = \sum_{(i,j)} \alpha(w_i, w_j) K_h \left( y - \frac{w_j}{w_i + w_j} \right) m(w_i, w_j) \frac{\sum_{(i,j)} K_h \left( y - \frac{w_j}{w_i + w_j} \right) m(w_i, w_j)}{\sum_{(i,j)} K_h \left( y - \frac{w_j}{w_i + w_j} \right) m(w_i, w_j)},$$

where $\alpha(w_i, w_j)$ is the predicted matching probability evaluated on the Chebyshev grid.
CONCLUSION
What we do

- We develop a search-matching model of marriage formation and intrahousehold allocation of time and earnings.
- Matching along several dimensions: wages, family values and education
- We show identification and develop an estimation procedure
  - Lots of savory technical details have been omitted: Chebychev grids and polynomials, Clenshaw-Curtis quadrature, DCT interpolation (for simulation), kernel smoothing
- Fit is good over time despite time-constant parameters
- Counterfactual analyses show that family values and home production specialization are key to understand the gender division of labor.
Future work

- Wage shocks
- Labour market non participation
- Age
- Etc.
References


