Labour Market Dynamics and Heterogeneous Agents

Sorting

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Introduction

- The previous model is extended to allow for two-sided heterogeneity.
- There is still very little work with two-sided worker-firm heterogeneity. Yet there is a lot of interest in understanding the evolution of match quality in recessions and booms.
Questions

- What is the role of worker and job heterogeneity in explaining the macrodynamics of (un)employment?
- How does the business cycle affect sorting, i.e. the joint distribution of workers and jobs?
Contribution

- We develop an equilibrium random on-the-job search model of the Labor market, with ex-ante heterogeneous workers and firms, and aggregate productivity shocks. We prove that the model has a recursive structure where
  - knowledge of the current aggregate shock (and the stochastic process) is a sufficient statistic for decisions regarding which worker-firm matches to form or dissolve, and who change jobs
  - the decision of which types of vacancies to create depends on the current distribution of worker-types among the unemployed and the current distribution of worker-types across job-types

- We illustrate the quantitative implications of the model by fitting to US aggregate labor market data from 1951-2012.

- The model has rich implications for the cyclical dynamics of the distribution of skills of the unemployed, the distribution of types of vacancies posted, and sorting between heterogeneous workers and firms.
1. THE MODEL
Time, agents and aggregate shocks

- Time is discrete and indexed by $t$.
- There is a continuum of workers indexed by type $x \in [0, 1]$, with distribution $\ell(x)$.
- There is a continuum of potential jobs indexed by $y \in [0, 1]$.
- The aggregate state of the economy is $z_t$. 
Distributions of workers and jobs at end of $t - 1$

- $h_t(x, y)$ is the distribution of worker-firm matches at the beginning of period $t$ (prior to realization of $z_t$)
- $u_t(x)$ is the distribution of unemployed workers at the beginning of period $t$ (prior to realization of $z_t$):

$$u_t(x) = \ell(x) - \int h_t(x, y) \, dy$$
Timing

- At the beginning of period $t$, $z_t$ is updated to $z'$ from $z_{t-1} = z$ according to a Markov transition probability $\pi(z, z')$.
- Following the realization of $z_t$ the timing is assumed to be:
  1. Separations occur.
  2. Workers search for a job and firms post vacancies.
  3. Meetings occur.
Following the realization of $z_t$ job separations occur.
Job separations

- Let $P_t(x, y)$ denote the present value an $(x, y)$ match given the aggregate state of the economy at $t$.
- Let $B_t(x)$ be the value of unemployment to a type-$x$ worker.
- Assuming no fixed investment in job posts, matches are endogenously destroyed iff $P_t(x, y) < B_t(x)$.
- If $P_t(x, y) \geq B_t(x)$, exogenous job destruction occurs with probability $\delta$.
- The layoff rate is thus

$$
\mathbf{1}\{P_t(x, y) < B_t(x)\} + \delta \times \mathbf{1}\{P_t(x, y) \geq B_t(x)\}
$$

endogenous

exogenous
Distributions at $t+$ after job separations

- The distribution of worker-firm matches that survive the destruction shocks is

$$h_{t+}(x, y) = (1 - \delta) 1\{P_t(x, y) \geq B_t(x)\} h_t(x, y)$$

- The distribution of unemployed workers after any job separation is

$$u_{t+}(x) = \ell(x) - \int h_{t+}(x, y) \, dy$$

$$= u_t(x) + \int \left[ 1\{P_t(x, y) < B_t(x)\} + \delta 1\{P_t(x, y) \geq B_t(x)\} \right] h_t(x, y) \, dy$$
Following the realization of $z_t$ and job separations, workers search for a job.
Aggregate search effort

- Workers search both when unemployed and employed.
- Together these workers produce aggregate search effort

\[ L_t = \int u_t^+(x) \, dx + s \int \int h_t^+(x, y) \, dx \, dy \]

where \( s \) is the relative effectiveness of search effort by the employed.
Following the realization of $z_t$ and job separations firms post vacancies.
Vacancy creation

- The cost of posting \( v \) vacancies is an increasing, convex function \( c(v) \).
- Firms of type \( y \) choose to post \( v_t(y) \) vacancies so as to equate the marginal cost of a recruiting to the marginal return

\[
c'[v_t(y)] = q_t J_t(y)
\]

where \( J_t(y) \) denotes the value of a vacancy and \( q_t \) the probability of a contact per vacancy (derived later).
- The aggregate number of vacancies solves

\[
V_t \equiv \int v_t(y) \, dy = \int (c')^{-1} (q_t J_t(y)) \, dy
\]
Then workers and vacancies meet.
Meeting rates

- The total measure of meetings between workers and firms at time $t$ is given by
  
  $$ M_t = M(L_t, V_t) $$

- The probability an unemployed worker contacts a vacancy is $\lambda_t = M_t/L_t$.

- The probability an employed worker contacts a vacancy is $s\lambda_t$.

- The probability per unit of recruiting intensity $v_t(y)$, that a firm contacts a searching worker is $q_t = M_t/V_t$. 
VALUES
The value of unemployment

- The planning horizon for workers and firms is infinite.
- The present value of unemployment is the expected discounted sum of future earnings conditional on being employed in period $t$ and given $z_t$ and distributions $h_t$.
- In period $t$, home production is $b(x, z_t)$.
- In period $t + 1$,
  - unemployed workers expect to receive offers with probability $\lambda_t$.
  - Firms make take it or leave it offers to unemployed workers.
The value of unemployment

- Hence, whether or not unemployed workers receive an offer, the continuation value is their reservation value $B_{t+1}(x)$.
- Workers (and firms) are risk neutral and discount the future at rate $r$.

$$B_t(x) = b(x, z_t) + \frac{1}{1 + r} \mathbb{E}_t \left[ (1 - \lambda_{t+1}) B_{t+1}(x) + \lambda_{t+1} \int B_{t+1}(x) \frac{V_{t+1}(y)}{V_{t+1}} dy \right]$$

$$= b(x, z_t) + \frac{1}{1 + r} \mathbb{E}_t B_{t+1}(x)$$
The value of unemployment

Therefore $B_t(x) = B(x, z_t)$ with

$$B(x, z) = b(x, z) + \frac{1}{1 + r} \int B(x, z') \pi(z, z') \, dz'$$

This is a simple linear equation.
The value of a match

- The present value of a match \((x, y)\) at \(t\), \(P_t(x, y)\), is the expected discounted sum of worker and employer future earnings.
- In period \(t\), the output of a match \((x, y)\) is \(p(x, y, z_t)\).
- In period \(t + 1\),
  - The employee meets a firm of type \(y'\) with probability \(s\lambda_{t+1}v_{t+1}(y')/V_{t+1}\).
  - Firms engage in Bertrand competition.
    - The worker moves to firm \(y'\) if \(P_{t+1}(x, y') > P_{t+1}(x, y)\) and s/he pockets \(P_{t+1}(x, y)\).
    - The worker stays if \(P_{t+1}(x, y') \leq P_{t+1}(x, y)\) and the match continues with value \(P_{t+1}(x, y)\).
The value of a match

- Hence the continuation values is either unemployment $B_{t+1}(x)$ or the current match value $P_{t+1}(x, y)$ whether the worker moves or stays.

\[
P_t(x, y) = p(x, y, z_t) + \frac{1}{1 + r} \mathbb{E}_t \left[ (1 - \delta) \mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} P_{t+1}(x, y) \right.
\]

\[
\left. + \mathbf{1}\{P_{t+1}(x, y) < B_{t+1}(x)\} + \delta \mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} \right] B_{t+1}(x).
\]

- The continuation value does not depend on distribution $h_{t+1}(x, y)$. 
The surplus of a match

- Define match surplus as \( S_t(x, y) = P_t(x, y) - B_t(x, y) \).
- There is a solution \( S_t(x, y) = S(x, y, z_t) \) such that

\[
S(x, y, z) = s(x, y, z) + \frac{1 - \delta}{1 + r} \int S(x, y, z')^+ \pi(z, z') \, dz'
\]

where \( s(x, y, z) = p(x, y, z) - b(x, z) \) and we denote \( x^+ = \max\{x, 0\} \).
Expected firm profit on a new match

Given that a the firm meets a searching worker, the expected firm profit depends on whether the contacted worker is employed or unemployed:

\[ J_t(y) = \int \frac{u_{t+}(x)}{L_t} [P_t(x, y) - B_t(x)]^+ \, dx \]

\[ + \iint \frac{sh_{t+}(x, y')}{L_t} [P_t(x, y) - P_t(x, y')]^+ \, dx \, dy' \]

\[ = \int \frac{u_{t+}(x)}{L_t} S_t(x, y)^+ \, dx \]

\[ + \iint \frac{sh_{t+}(x, y')}{L_t} [S_t(x, y) - S_t(x, y')]^+ \, dx \, dy' \]
Law of motion for updating worker distributions

- At the end of the period we have the distribution of jobs

\[
\begin{align*}
    h_{t+1}(x, y) &= h_t(x, y) \left[ 1 - \int s\lambda_t \frac{v_t(y')}{V_t} \mathbf{1}\{S_t(x, y') > S_t(x, y)\} \, dy' \right] \\
    &+ \int h_t(x, y') s\lambda_t \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x, y) > S_t(x, y')\} \, dy'
\end{align*}
\]

exit because of poaching

entry by poaching

\[
+ u_{t+1}(x)\lambda_t \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x, y) \geq 0\}
\]

entry from unemployment

- And unemployment

\[
\begin{align*}
    u_{t+1}(x) &= u_t(x) \left[ 1 - \int \lambda_t \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x, y) \geq 0\} \, dy \right]
\end{align*}
\]
Computation of the stochastic search equilibrium

1. Once and for all, solve for the fixed point in $S(x, y, z)$ independently of the actual realization of aggregate productivity shocks.

2. Then recursive: Given an initial distribution of workers across jobs, $h_0(x, y)$, and a realized sequence of aggregate productivity shocks $\{z_0, z_1, \ldots, z_T\}$ we can solve for the sequence of distributions of unemployed worker types, worker-firm matches, and vacancies $\{v_t(y), h_{t+1}(x, y)\}_{t=0}^T$. 
2. ESTIMATION
A parametric specification

- Meeting function
  \[ M_t = M(L_t, V_t) = \min\{\alpha \sqrt{L_t V_t}, L_t, V_t\}, \quad \alpha > 0 \]

- Vacancy costs
  \[ c(v) = \frac{c_0}{1+c_1} v^{1+c_1}, \quad c_0 > 0, \quad c_1 > 0 \]

- Value added
  \[ p(x, y, z) = z \left( p_1 + p_2 x + p_3 y + p_4 x^2 + p_5 y^2 + p_6 x y \right) \]

- Home production
  \[ b(x) = 0.7 \times p(x, y^*(x, 1), 1) \quad y^*(x, 1) = \arg \max_y S(x, y, 1) \]

- Worker type distribution
  \[ x \sim \text{Beta}(\beta_1, \beta_2) \]

- Aggregate shocks
  \[ \ln z_t = \rho \ln z_{t-1} + \sigma \sqrt{1 - \rho^2} \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \]
Estimation

- We HP filter the log transformed data (1951-2012).
- We calculate means, volatilities (standard deviations) and correlations.
- We estimate the model parameters by method of simulated moments.
- The model is solved at a weekly frequency and the simulated data is then aggregated (exactly as the BLS data) to form quarterly moments.
Identification

- \( \alpha, s, \) and \( \delta \) (mobility) are identified from transition rates between unemployment and employment, between jobs, and from employment to unemployment.
- \( \sigma \) and \( \rho \) (process for \( z \)) are identified from aggregate output (GDP).
- \( c \) (vacancy cost) is identified from vacancies.
- \( \beta \) (worker heterogeneity) is identified from unemployment duration patterns (number of workers unemployed 5, 15 and 27 or more weeks).
- \( p \) (match value added) is identified from the cross-sectional dispersion in value added per job across firms (from Bloom et al., 2014).
Method of moments

- Let $\hat{m} = (\hat{m}_1, ..., \hat{m}_N)$ denote the $N \times 1$ vector of data moments that we want to fit.

- Let $m(\theta) = (m_1(\theta), ..., m_N(\theta))$, for $\theta$ a vector of $K$ parameters, be the corresponding theoretical moments.

- We estimate $\theta$ by

$$
\hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{N} \omega_i \left( \frac{\hat{m}_i - m_i(\theta)}{\hat{m}_i} \right)^2
$$

where $\omega_i$ are fixed weights that we choose to force a better fit of some particular moments (or not).

- More usual:

$$
\hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{N} \omega_i \left( \hat{m}_i - m_i(\theta) \right)^2
$$

where $\omega_i$ is the inverse of an estimate of $\text{Var}(\hat{m}_i)$. 
Standard errors

- Under standard regularity conditions,
  \[
  \left[ \sum_{i=1}^{N} \frac{\omega_i}{\hat{m}_i^2} \frac{\partial m_i(\theta_0)}{\partial \theta} \frac{\partial m_i(\theta_0)}{\partial \theta^\top} \right] \left( \hat{\theta} - \theta_0 \right) = \sum_{i=1}^{N} \frac{\omega_i}{\hat{m}_i^2} (\hat{m}_i - m_i(\theta)) \frac{\partial m_i(\theta_0)}{\partial \theta} + o_P(1).
  \]

- Assuming that, for a large sample size \( T \),
  \[
  \hat{m} \sim \mathcal{N} \left( m(\theta_0), \hat{\Sigma} \right),
  \]
  with \( \hat{\Sigma} = o_P(1/T) \), then
  \[
  \hat{\theta} \sim \mathcal{N}(\theta_0, \hat{J}^{-1} \hat{I} \hat{J}^{-1}),
  \]
  for
  \[
  \hat{J} = \sum_{i=1}^{N} \frac{\omega_i}{\hat{m}_i^2} \frac{\partial m_i(\hat{\theta})}{\partial \theta} \frac{\partial m_i(\hat{\theta})}{\partial \theta^\top} = \hat{M}^\top \hat{\Omega} \hat{M}, \quad \hat{\Omega} = \text{diag} \left( \frac{\omega_1}{\hat{m}_1^2}, \ldots, \frac{\omega_L}{\hat{m}_L^2} \right)
  \]
  and
  \[
  \hat{I} = \hat{M}^\top \hat{\Omega} \hat{\Sigma} \hat{\Omega} \hat{M}.
  \]
The variance-covariance matrix of the vector of moments

- The vector of moments consists in sample averages, standard deviations and correlations of some vector \( y_t = (y_{1t}, \ldots, y_{Lt}) \) of variables.

- Let

  \[
  f_{1i}(y_t, m) = y_{it} - \mu_i, \quad f_{2ij}(y_t, m) = (y_{it} - \mu_i)(y_{jt} - \mu_j) - \rho_{ij}\sigma_i\sigma_j
  \]

where \( m = [\mu; \sigma; \rho] \) is the vector of parameters, for \( \mu = (\mu_1, \ldots, \mu_L) \) the vector of means, \( \sigma = (\sigma_1, \ldots, \sigma_L) \) the vector of standard deviations, and \( \rho = (\rho_{ij})_{i>j} \) the vector of (non trivial) correlations.

- The vector of moments \( \hat{m} \) is obtained as the solution to

  \[
  \hat{E}f_{1i}(y_t, \hat{m}) = \frac{1}{T} \sum_{t=1}^{T} f_{1i}(y_t, \hat{m}) = 0, \quad \hat{E}f_{2ij}(y_t, \hat{m}) = 0, \quad \forall i, j.
  \]
The variance-covariance matrix of the vector of moments

- Let \( f_1 = (f_{11}, \ldots, f_{1L}) \) and \( f_2 = (f_{2ij})_{i \geq j} \). Let also \( f = [f_1; f_2] \).
- First, we calculate the Jacobian of the transformation \( m \mapsto \widehat{Ef}(y_t, m) \), that is \( \hat{D} = \frac{\partial \widehat{Ef}(y_t, m)}{\partial m} \bigg|_{m=\hat{m}} \).
- We have: \( \frac{\partial \widehat{Ef}_1(y_t, m)}{\partial \mu_k} \bigg|_{m=\hat{m}} = -1_{i=k} \), where \( 1_{i=k} = 1 \) if \( i = k \) and 0 otherwise,

\[
\frac{\partial \widehat{Ef}_1(y_t, m)}{\partial \rho_{kl}} \bigg|_{m=\hat{m}} = \frac{\partial \widehat{Ef}_2(y_t, m)}{\partial \mu_k} \bigg|_{m=\hat{m}} = 0,
\]

and

\[
\frac{\partial \widehat{Ef}_2(y_t, m)}{\partial \sigma_k} \bigg|_{m=\hat{m}} = -\rho_{ij}[1_{i=k}\sigma_j + 1_{j=k}\sigma_i]
\]

\[
\frac{\partial \widehat{Ef}_2(y_t, m)}{\partial \rho_{kl}} \bigg|_{m=\hat{m}} = -1_{ij=kl}\sigma_i\sigma_j.
\]
The variance-covariance matrix of the vector of moments

- Second, we need to estimate the variance of $f(y_t, m)$. Given the autocorrelated nature of $y_t$, we use the Newey-West estimator:

$$
\hat{S} = \sum_{p=-q}^{q} \frac{q-|p|}{q} \sum_{t=1+p}^{T-p} f(y_t, \hat{m}) f(y_{t-p}, \hat{m})^\top,
$$

where $q$ is of the order of $T^{-1/3}$. Start low and increase progressively until $\hat{S}$ stabilizes.

- Finally, we can estimate the asymptotic variance of $\hat{m}$ as

$$
\hat{\Sigma} = \left( \hat{D}^\top \hat{S}^{-1} \hat{D} \right)^{-1}.
$$
Implementation details

- In practice, we use a lag order of 8 in the Newey-West covariance estimator. The fixed weights used in estimation $\omega$, are equal to 100 for $E[U]$ and $sd[VA]$; 10 for $E[J2J]$, $E[sd\text{ labor prod}]$, $sd[U]$, $sd[V/U]$, $sd[sd\text{ labor prod}]$, $sd[V]$, $autocorr[VA]$, $corr[sd\text{ labor prod}, VA]$; and 1 for all other moments.

- These weights were selected to ensure the model replicated the variability and persistence of output, the level and variability of the unemployment rate and variability of vacancy creation; moments that have been of primary interest in the related literature.
Implementation details

- The moments are not necessarily smooth functions of the parameters due to simulation noise.
- To estimate the derivative of each moment with respect to each parameter $\partial m(\hat{\theta})/\partial \theta$, we simulate the model for 101 equally spaced values for each parameter $\theta_k \in [0.5\hat{\theta}_k, 1.5\hat{\theta}_k]$ holding all other parameters at their estimated values, and saving the vector of moments for each evaluation.
- We then fit a polynomial of degree 9 for each moment as a function of each parameter.
- The derivative of this polynomial evaluated at $\hat{\theta}$ is our estimate of $\partial m(\hat{\theta})/\partial \theta$. 
Implementation details

- Missing data for the job-to-job and productivity series means that the cells of the covariance matrix of the data are calculated using only the maximal available data for each cell.
- The covariance matrix $\hat{S}$ is not guaranteed to be positive semi-definite (although it will be asymptotically).
- To ensure invertibility, we multiply the diagonal of $\hat{S}$ by $1 + \lambda$ where $\lambda$ is chosen as small as possible such that all eigenvalues of $\hat{S}$ are positive.
MODEL FIT
Moments

Right amplification of aggregate shocks.

<table>
<thead>
<tr>
<th>Fitted Moments</th>
<th>Data</th>
<th>Model</th>
<th>Fitted Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd[$\text{GDP}$]</td>
<td>0.033</td>
<td>0.034</td>
<td>sd[$\text{UE}$]</td>
<td>0.127</td>
<td>0.127</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td></td>
<td>(0.011)</td>
<td></td>
</tr>
<tr>
<td>sd[$\text{U}$]</td>
<td>0.191</td>
<td>0.203</td>
<td>sd[$\text{EU}$]</td>
<td>0.100</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td></td>
<td></td>
<td>(0.011)</td>
<td></td>
</tr>
<tr>
<td>sd[$\text{U}^{5p}$]</td>
<td>0.281</td>
<td>0.315</td>
<td>sd[$\text{EE}$]</td>
<td>0.095</td>
<td>0.112</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td></td>
<td></td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>sd[$\text{U}^{15p}$]</td>
<td>0.395</td>
<td>0.413</td>
<td>sd[$\text{V/UE}$]</td>
<td>0.381</td>
<td>0.306</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td></td>
<td></td>
<td>(0.029)</td>
<td></td>
</tr>
<tr>
<td>sd[$\text{U}^{27p}$]</td>
<td>0.478</td>
<td>0.439</td>
<td>sd[$\text{V}$]</td>
<td>0.206</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td></td>
<td></td>
<td>(0.015)</td>
<td></td>
</tr>
</tbody>
</table>

sd[$\text{sd labor prod}$] | 0.039 | 0.038 |
|                          | (0.005) |       |

Note: Newey-West standard errors in brackets.
Moments

Right signs for correlations

<table>
<thead>
<tr>
<th>Fitted Moments</th>
<th>Data</th>
<th>Model</th>
<th>Fitted Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>autocorr[$GDP$]</td>
<td>0.932</td>
<td>0.991</td>
<td>corr[$UE, GDP$]</td>
<td>0.878</td>
<td>0.978</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td></td>
<td></td>
<td>(0.122)</td>
<td></td>
</tr>
<tr>
<td>corr[$U, GDP$]</td>
<td>−0.860</td>
<td>−0.983</td>
<td>corr[$EU, GDP$]</td>
<td>−0.716</td>
<td>−0.910</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td></td>
<td></td>
<td>(0.133)</td>
<td></td>
</tr>
<tr>
<td>corr[$V, GDP$]</td>
<td>0.721</td>
<td>0.996</td>
<td>corr[$UE, EE$]</td>
<td>0.695</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td>(0.149)</td>
<td></td>
<td></td>
<td>(0.108)</td>
<td></td>
</tr>
<tr>
<td>corr[$V, U$]</td>
<td>−0.846</td>
<td>−0.975</td>
<td>corr[sd labor prod, GDP]</td>
<td>−0.366</td>
<td>−0.365</td>
</tr>
<tr>
<td></td>
<td>(0.119)</td>
<td></td>
<td></td>
<td>(0.260)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Newey-West standard errors in brackets.
Unemployment prediction given filtered $z_t$

- We first filter out $z_t$ so as to exactly fit GDP (depends on $h_{t+}$).
- Then we predict the other variables ($h_{t+1}$ in particular).
Vacancies and mobility prediction given filtered $z_t$
PARAMETER ESTIMATES
**Estimated parameters**

Parameters precisely estimated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching ( M = \min{\alpha\sqrt{LV}, L, V} )</td>
<td>0.497</td>
<td>0.083</td>
</tr>
<tr>
<td>Search intensity</td>
<td>0.027</td>
<td>0.007</td>
</tr>
<tr>
<td>Vacancy posting costs ( c[v(y)] = \frac{c_0}{1+c_1}v(y)^{1+c_1} )</td>
<td>0.028</td>
<td>0.014</td>
</tr>
<tr>
<td>Exogenous separation ( \delta )</td>
<td>0.013</td>
<td>0.001</td>
</tr>
<tr>
<td>Productivity shocks ( \sigma )</td>
<td>0.077</td>
<td>0.009</td>
</tr>
<tr>
<td>Gaussian copula ( \sigma, \rho )</td>
<td>0.999</td>
<td>0.001</td>
</tr>
<tr>
<td>Worker heterogeneity ( \beta_1 )</td>
<td>2.148</td>
<td>(0.192)</td>
</tr>
<tr>
<td>Beta ( \beta_1, \beta_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Search intensity</td>
<td>0.027</td>
<td>(0.192)</td>
</tr>
<tr>
<td>Value added ( p(x, y, z) = )</td>
<td>p_1: 0.003, p_2: 2.053, p_3: -0.140, p_4: 8.035, p_5: -1.907, p_6: 6.596</td>
<td></td>
</tr>
<tr>
<td>Exogenous separation ( \delta )</td>
<td>0.013</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Productivity shocks ( \sigma )</td>
<td>0.077</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Gaussian copula ( \sigma, \rho )</td>
<td>0.999</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Note: \( r \) is fixed at 0.05 annually.
Production function

Varies more across workers than firms
Worker ability distributions

Unemployed are mostly low ability workers.
Equilibrium vacancy creation $v(y)$

More vacancies are created in booms. No lateral shift.
Relative home-to-market productivity $b(x)/p(x, y, z)$

This is not a small surplus economy ($b/p \ll 1$)
Feasible matches
In booms, there is more mismatch. In recessions, shrinks toward optimal matches.
Distribution of matches

Once employed they move more quickly to better matches in booms than in recessions.

$z$ at the 1st decile

$z$ at 9th decile
Cyclicality of Unemployment and Transitions

Compare U, EU and UE rates by education in CPS with model prediction for different skill groups: $x \in [0, \bar{x}]$ for different $\bar{x} \leq 1$. 

Unemployment by education  |  UE rate by education  |  EU rate by education 

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**Unemployment by education**

**UE rate by education**

**EU rate by education**
Wages

- There is a simple way of maintaining the recursive structure of the model and of tracking wage distributions at the same time.
- Simply assume that wage contracts are state-contingent and employers commit to a fixed surplus sharing until the next poaching event:

\[ W_t(\sigma, x, y) = B_t(x) + \sigma S_t(x, y) \]

- The piece rate \( \sigma \) is
  - \( \sigma = 0 \) out of unemployment
  - \( \sigma = \frac{S_t(x, y')}{S_t(x, y)} \) as a result of the Bertrand competition between two firm \( y \) and \( y' \) such that \( S_t(x, y) > S_t(x, y') \)
Wages

- In any period, a contract $\sigma$ induces a wage $w_t(\sigma, x, y)$ that is such that

\[
W_t(\sigma, x, y) = B_t(x) + \sigma S_t(x, y)
= w_t(\sigma, x, y) + \frac{1}{1 + r} \mathbb{E}_t B_{t+1}(x)
+ \frac{1 - \delta}{1 + r} \mathbb{E}_t \left[ 1 \{ S_{t+1}(x, y) \geq 0 \} s \lambda_{t+1} \int I_{t+1}(\sigma, x, y, y') \frac{v_{t+1}(y')}{V_{t+1}} \, dy' \right],
\]

where $I_{t+1}(\sigma, x, y, y')$ is the second best of the three values: $S_{t+1}(x, y')$, $S_{t+1}(x, y)$, $\sigma S_{t+1}(x, y)$. That is,

\[
I_{t+1}(\sigma, x, y, y') = \begin{cases} 
S_{t+1}(x, y) & \text{if } S_{t+1}(x, y') > S_{t+1}(x, y), \\
S_{t+1}(x, y') & \text{if } \sigma S_{t+1}(x, y) < S_{t+1}(x, y') \leq S_{t+1}(x, y), \\
\sigma S_{t+1}(x, y) & \text{if } S_{t+1}(x, y') \leq \sigma S_{t+1}(x, y).
\end{cases}
\]
Wages

• Solving for wages, we obtain

\[ w_t(\sigma, x, y) = \sigma p(x, y, z_t) + (1 - \sigma) b(x, z_t) - \Delta \]

• \( \Delta \) is a discount for future renegotiation opportunities:

\[
\Delta = \frac{1 - \delta}{1 + r} \mathbb{E}_t \left[ \mathbb{1} \{ S_{t+1}(x, y) \geq 0 \} \cdot \lambda_{t+1} \int \Delta I_{t+1}(\sigma, x, y, y') \frac{v_{t+1}(y')}{V_{t+1}} \, dy' \right]
\]

where

\[
\Delta I_{t+1}(\sigma, x, y, y') = \begin{cases} 
(1 - \sigma) S_{t+1}(x, y) & \text{if j2j mobility} \\
S_{t+1}(x, y') - \sigma S_{t+1}(x, y) & \text{if counteroffer} \\
0 & \text{if status quo}
\end{cases}
\]
References