Wage posting

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Road map

1. Equilibrium wage dispersion
2. The Burdett-Mortensen model
3. Estimation
Monopsony wage

- Diamond (1971)
- Suppose that a set of identical firms seek to maximize profit flow \((p - w)\ell(w)\), where \(\ell(w)\) is the labour force that they expect if they offer a wage \(w\).
- Consider a symmetric equilibrium.
- A firm offering less than \(\phi\) in equilibrium attracts no worker and all firms offering more than \(\phi\) attract the same number of workers. Hence, firms’ best strategy is to offer the smallest acceptable wage, that is \(\phi\).
- Now, if \(F\) is a mass point at \(\phi\), then \(\phi = b\). Unemployed workers accept first offer.
Heterogeneous workers

- Albrecht and Axell (1984)

Suppose that workers differ in their opportunity costs of employment, \( b \), which are distributed according to some \( H \).

Heterogeneity in \( b \) induces heterogeneity in \( \phi \), say \( \phi_1 < \ldots, < \phi_N \).

Albrecht and Axell show that there exists an equilibrium such that different firms may offer different wages, equal to either one of the reservation wages.

A firm offering \( \phi_i \) hire any worker with a reservation wages greater or equal to \( \phi_i \).

In equilibrium all firms make same profit. It thus has to be that \( \ell(\phi_i) > \ell(\phi_{i-1}) \) to compensate for a lower marginal profit.
On-the-job search

- Burdett and Mortensen (1998): Equilibrium wage dispersion can be generated among \textit{ex ante} identical workers by allowing workers to search for a better job while employed.
- Same insight as Albrecht and Axell’s: heterogeneous wage offers can be supported by heterogeneous reservation wages.
- Stronger result. BM show that the unique Nash equilibrium is in mixed strategy form.
- The Burdett-Mortensen model predicts:
  1. Equilibrium wage dispersion between \textit{ex ante} identical workers across \textit{ex ante} identical firms.
  2. Larger firms pay higher wages
  3. Workers with more tenure tend to be better paid and less mobile than more junior workers (“wage ladder effect”)
The Burdett-Mortensen model
Assumptions

- $L$ identical workers and $N$ firms
- Matches are exogenously destroyed at rate $\delta$
- Job offer arrival rate to unemployed workers is $\lambda_0$
- Job offer arrival rate to employed workers is $\lambda_1$
- UI benefit is $b$
Steady state unemployment

- For unemployment rate $u$ to remain constant over time, flows into unemployment must balance outflows:

$$\lambda_0 u F(\phi) = \delta (1 - u)$$

- In equilibrium no firm can offer a wage lower than the reservation wage $\phi$, hence $F(\phi) = 0$.

- It thus follows that the equilibrium unemployment rate is $u = \frac{\delta}{\delta + \lambda_0}$. 
Let $F$ denote the wage offer distribution and let $G$ be the distribution of wages in a cross-section of employees.

In steady state, flows in and out of the stock $(1 - u)G(w)$ must balance each other out:

$$\lambda_0 u F(w) = [\delta + \lambda_1 (1 - F(w))] G(w)(1 - u)$$

Using $\lambda_0 u = \delta (1 - u)$,

$$G(w) = \frac{F(w)}{1 + \kappa_1 F(w)} \Leftrightarrow F(w) = \frac{(1 + \kappa_1) G(w)}{1 + \kappa_1 G(w)}$$

where $\kappa_1 = \lambda_1 / \delta$ is the number of alternative offers that an employee expects to receive before being laid off.
Steady state firm size

- \( \ell(w) \) is the steady-state size of firm \( w \):
  \[
  \ell(w) = \frac{L \cdot g(w)}{N \cdot f(w)} \left( = \frac{\text{\# workers in firms } w}{\text{\# firms } w} \right)
  = \frac{L}{N} \frac{1 + \kappa_1}{\left[1 + \kappa_1 \overline{F}(w)\right]^2}.
  \]

- \( \ell(w) \) is increasing.

- Equivalent flow equation:
  \[
  [\lambda_0 u + \lambda_1 G(w) (1 - u)] L f(w) = [\delta + \lambda_1 \overline{F}(w)] \ell(w) f(w) N
  \]
Nash equilibrium

- Firms seek to maximize profit flows

\[ \pi(p) = \max_{w \geq \phi} (p - w) \ell(w) \]

subject to the steady-state employment condition:

\[ \ell(w) = \frac{L}{N} \frac{1 + \kappa_1}{[1 + \kappa_1 F(w)]^2}. \]

- In equilibrium,
  - all firms make the same profit for all wages in the support of \( F \)
  - unemployed workers apply the reservation wage strategy: accept any wage offer \( w \) that is greater or equal than \( \phi \) such that

\[ \phi = b + (\lambda_1 - \lambda_0) \int_{\phi}^{\bar{w}} \frac{F(w) \, dw}{\rho + \delta + \lambda_1 F(w)}, \]

- employed workers accept any wage that is strictly greater than current wage.
No mass property

- There is no mass point in the equilibrium wage distribution.
- Otherwise, were there be a mass of firms offering the same wage $w$, then any one of these firms would be better off offering slightly more. The additional wage cost would be more than compensated by the additional labor force that this deviating firm would be able to attract.
- Technically: the function $\ell(w)$ solving the (now exact) flow condition

$$
\left[\lambda_0 u + \lambda_1 G(w^-) (1 - u)\right] Lf (w) = \left[\delta + \lambda_1 \overline{F}(w)\right] \ell(w)f(w) N
$$

$$
\Leftrightarrow \left[\delta + \lambda_1 G(w^-)\right] (1 - u) L = \left[\delta + \lambda_1 \overline{F}(w)\right] \ell(w) N
$$

jumps discontinuously at any point of discontinuity of $F$ or $G$.

- We write $G(w^-) = \lim_{x \uparrow w} G(x)$ instead of $G(w)$ because only firms posting a wage *strictly* less than $w$ lose their employees when they are contacted by a firm posting wage $w$. 

Equilibrium solution

- Equal profit condition:

$$\forall w \in \text{Supp}(F), (p - w) \ell(w) = (p - \underline{w}) \ell(\underline{w})$$

where \(\underline{w} = \inf \text{[Supp}(F)]\).

- As \(\ell(\underline{w}) = \frac{L}{\bar{N}} \frac{1+\kappa_{1}}{[1+\kappa_{1} \bar{F}(\underline{w})]^2} = \frac{L}{\bar{N}} \frac{1}{1+\kappa_{1}}\), the equal profit condition writes

$$\forall w \in \text{Supp}(F), (p - w) \frac{(1 + \kappa_{1})^{2}}{[1 + \kappa_{1} \bar{F}(w)]^2} (w) = (p - w)$$

$$\Leftrightarrow 1 + \kappa_{1} \bar{F}(w) = (1 + \kappa_{1}) \sqrt{\frac{p - w}{p - \underline{w}}}.$$ 

- The optimal minimum wage offer is \(\underline{w} = \phi\). The firm offering the lowest wage cannot raise its profit by offering a lower wage than \(\phi\).

- The wage offer density \(f\) is increasing.
Heterogeneous firms

- Mortensen (1990); Bontemps, Robin, and van den Berg (2000); Bontemps, Robin, and Van den Berg (1999)
- Now, suppose that firms differ in productivity \( p \) and let \( \Gamma \) denote the productivity distribution.
- Let \( w(p) = \arg \max_{w \geq \phi} (p - w) \ell(w) \) given \( F \).
- The wage offer correspondence \( w(p) \) is increasing.
  - For all \( p_1 > p_2 \), let \( w_1 \in w(p_1) \) and \( w_2 \in w(p_2) \), then,
    
    \[
    \begin{align*}
    (p_1 - w_1) \ell(w_1) & \geq (p_1 - w_2) \ell(w_2) > (p_2 - w_2) \ell(w_2) \geq (p_2 - w_1) \ell(w_1) \\
    = A_1 & = A_2 \quad = A_3 \quad = A_4
    \end{align*}
    \]

    \[ \Rightarrow A_1 - A_4 = (p_1 - p_2) \ell(w_1) \geq A_2 - A_3 = (p_1 - p_2) \ell(w_2) \]

    As \( \ell(w_1) \geq \ell(w_2) \) it follows that \( w_1 \geq w_2 \).
- In addition, one can show that if \( \Gamma \) is continuous, then \( w(p) \) must be reduced to a singleton and the function \( w(p) \) is almost surely strictly increasing.
Equilibrium solution

- FOC of the profit maximization problem \( \pi(p) = \max_{w \geq \phi} (p - w) \ell(w) \), for \\
  \( \ell(w) = \frac{L}{N} \frac{1 + \kappa_1}{[1 + \kappa_1 \overline{F}(w)]^2} \):

  \[-\ell(w) + (p - w) \ell'(w) = 0\]
  \[\Leftrightarrow -1 + \frac{2\kappa_1 (p - w)}{1 + \kappa_1 \overline{F}(w)} f(w) = 0\]

- Equilibrium: \( F(w(p)) = \Gamma(p) \). Hence, \( f(w)w'(p) = \gamma(p) \).

- The FOC can thus be transformed into the following first-order differential equation:

  \[-w'(p) + \frac{2\kappa_1 (p - w)}{1 + \kappa_1 \overline{\Gamma}(p)} \gamma(p) = 0\]

  with initial condition \( w(p) = \phi \).

- Solution: \( w(p) = p - \left[1 + \kappa_1 \overline{\Gamma}(p)\right]^2 \left(\int_p^p \frac{1}{[1 + \kappa_1 \overline{F}(x)]^2} dx + \frac{p - \phi}{(1 + \kappa_1)^2}\right)\)
Equilibrium solution (alternative)

- By the Envelope theorem,

\[
\pi'(p) = \ell(w(p)) = \frac{L}{N} \frac{1 + \kappa_1}{\left[1 + \kappa_1 \bar{F}(w(p))\right]^2} = \frac{L}{N} \frac{1 + \kappa_1}{\left[1 + \kappa_1 \bar{\Gamma}(p)\right]^2}
\]

- Hence,

\[
w(p) = p - \frac{\pi(p)}{\ell(w(p))} = p - \frac{N}{L(1 + \kappa_1)} \left[1 + \kappa_1 \bar{\Gamma}(p)\right]^2 \pi(p)
\]

with

\[
\pi(p) = \pi(p) + \int_p^p \pi'(p) dp, \quad \pi(p) = \frac{L}{N(1 + \kappa_1)}(p - \phi)
\]
Discrete distribution of productivity

- Theory: Mortensen (1990)
- Cumbersome!
Continuous distribution of productivity

- 1999 paper has both $b$ and $p$ heterogeneous but $\lambda_0 = \lambda_1$.
- Last restriction eliminated by Shephard (forth.) who uses the model to analyse the impact of UK tax credit reforms.
- 2000 paper only has $p$ heterogeneous. By some editorial hazard, it was published after the other one.
BRVdB (2000)

- Unemployed at first observation time ($x = 0$):
  - $t_{0b} =$ elapsed, $t_{0f} =$ residual unemployment duration
  - $d_{0b} = 1$ or 0 whether unemployment duration left-censored
  - $d_{0f} = 1$ or 0 whether unemployment duration right-censored
  - $w_0 =$ wage accepted by unemployed individuals
- Employed at first observation time ($x = 1$):
  - $t_{1b} =$ elapsed, $t_{1f} =$ residual employment duration
  - $d_{1b} = 1$ if job duration left-censored, otherwise $= 0$
  - $d_{1f} = 1$ if job duration right-censored, otherwise $= 0$
  - $w_1 =$ wage of employees at time of first interview
  - $v = 1$ if job-to-unemployment transition, $= 0$ if j-t-j transition
Likelihood

- **Worker unemployed** at the time of the first interview:

\[
\frac{\delta}{\delta + \lambda_0}
\]

(proba of being unemployed)

\[
\frac{\lambda_0^{2-d_{0b}-d_{0f}} \exp [-\lambda_0 (t_{0b} + t_{0f})]}{f(w_0)^{1-d_{0f}}}
\]

(density of \(t_{0b}, t_{0f}, d_{0b}, d_{0f}\))

(density of accepted wage)

- **Worker employed** at the time of the first interview:

\[
\frac{\lambda_0}{\delta + \lambda_0}
\]

(proba of being employed)

\[
\times \left( \frac{g(w_1)}{f(w_0)} \right)
\]

(density of current wage)

\[
\times \left( \frac{\delta}{\delta + \lambda_1 \overline{F}(w_1)} \right)^{2-d_{1b}} \exp \left[- \left( \frac{\delta}{\delta + \lambda_1 \overline{F}(w_1)} \right) (t_{1b} + t_{1f}) \right]
\]

(density of \(t_{1b}, t_{1f}, d_{1b}, d_{1f}\))

\[
\times \left[ \left( \frac{\delta}{\delta + \lambda_1 \overline{F}(w_1)} \right)^v \cdot \left( \frac{\lambda_1 \overline{F}(w_1)}{\delta + \lambda_1 \overline{F}(w_1)} \right)^{1-v} \right]^{1-d_{1f}}
\]

(proba of exit destination)
Multi-stage estimation

- Treats $F, G$ as a nuisance parameters.

First, estimate $G$ and $g$ using a non parametric estimator (empirical cdf for $G$ and a kernel density estimator for $g$, for example). For any $\kappa_1$, let

$$\hat{F}(w; \kappa_1) = \frac{1 - \hat{G}(w)}{1 + \kappa_1 \hat{G}(w)} \quad \text{and} \quad \hat{f}(w; \kappa_1) = \frac{1 + \kappa_1}{(1 + \kappa_1 \hat{G}(w))^2} \hat{g}(w).$$

Second, replace $\bar{F}$ and $f$ by these expressions in the expression for the likelihood and maximize it with respect to $\lambda_0, \kappa_1$ and $\delta$.

Third, use the FOC of the profit maximization programme to estimate a productivity value for each observed wage:

$$-1 + \frac{2\kappa_1 (p - w)}{1 + \kappa_1 \bar{F}(w)} f(w) = 0 \iff p - w = \frac{1 + \kappa_1 \bar{F}(w)}{2\kappa_1 f(w)}$$

$$\Rightarrow \hat{p}_i = w_i + \frac{1 + \hat{\kappa}_1 \hat{G}(w_i)}{2\hat{\kappa}_1 \hat{g}(w_i)}.$$
Results

- BM models fits turnover and wage distributions well.
- But estimated distribution of firm productivity implausible: exceedingly long right tail.
- Why?
  - High-productivity firms have a lot of market power in the BM model.
  - This tends to concentrate wages towards the upper part of the distribution.
  - In order to generate the very long, thin tails of observed wage distributions, productivity distributions with much longer and thinner tails are thus necessary.
  - Irreconcilable with actual productivity distributions.
Extensions of Burdett-Mortensen model

- Wage-tenure contracts: Stevens (2004); Burdett and Coles (2003, 2010)
- Piece-rate contracts: Barlevy (2008) adds productivity shocks and human capital accumulation in a simple way
- Endogenous search intensity: Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005)
- Wage-experience contracts: Burdett, Carrillo-Tudela, and Coles (2009)
- Discrimination: Bowlus and Eckstein (2002); Meghir, Narita, and Robin (2015)
- Aggregate shocks: Moscarini and Postel-Vinay (2013, 2016)
- Etc.
References I


References II


References III

Review of Economic Dynamics, 19, 135–160.

Shephard, A. (forth.): “Equilibrium Search and Tax Credit Reform,” International 
Economic Review.

Firms’ Strategies for Recruitment and Retention,” Review of Economic Studies, 
71, 535–551.