Advanced Topics in Monetary Economics II

Carl E. Walsh

UC Santa Cruz

August 18-22, 2014

© Carl E. Walsh, 2014.
Uncertainty

- **Standard analysis with additive errors leads to certainty equivalence (linear structural equations, quadratic objective function)**
  - Optimal policy depends on future forecasts but not the uncertainty surrounding those forecasts.
- **Suppose the true model of the economy is given by**
  \[
  y_{t+1} = A_1 y_t + A_2 y_{t/t} + Bi_t + u_{t+1},
  \]
  (1)
  where \( y_t \) is a vector of macroeconomic variables (the state vector), \( y_{t/t} \) is the optimal, current estimate of \( y_{t/t} \), and \( i_t \) is the policy maker’s control instrument.
- **\( u_{t+1} \)** represents a vector of additive, exogenous stochastic disturbances, assumed equal to \( Ce_{t+1} \) where the vector \( e \) is a set of mutually and serially uncorrelated disturbances with unit variances.
- \( A_1, A_2, \) and \( B \) are matrices of the model parameters.
Sources of model specification error

Suppose the policy maker’s estimates of $A_1$, $A_2$, and $B$ are denoted $\bar{A}_1$, $\bar{A}_2$, and $\bar{B}$, while $\bar{y}_{t/t}$ denotes the policy maker’s estimate of the current state $y_t$.

Then, letting $A = A_1 + A_2$ and $\bar{A} = (\bar{A}_1 + \bar{A}_2)$, we can write the policy maker’s perceived model in the form

$$y_{t+1} = \bar{A}\bar{y}_{t/t} + \bar{B}i_t + \bar{C} (e_{t+1} + w_{t+1})$$

(2)

where

$$w_{t+1} = \bar{C}^{-1} [(A - \bar{A}) \bar{y}_{t/t} + (B - \bar{B}) i_t + (C - \bar{C}) e_{t+1}]$$

$$+ \bar{C}^{-1} A_1 (y_t - y_{t/t}) + \bar{C}^{-1} \bar{A} (y_{t/t} - \bar{y}_{t/t}).$$

(3)
Sources of model specification error

\[ w_{t+1} = \bar{C}^{-1} \left[ (A - \bar{A}) \bar{y}_{t/t} + (B - \bar{B}) i_t + (C - \bar{C}) e_{t+1} \right] \]
\[ + \bar{C}^{-1} A_1 (y_t - y_{t/t}) + \bar{C}^{-1} \bar{A} (y_{t/t} - \bar{y}_{t/t}). \]

1. **Model mis-specification**: errors that arise if the policy maker’s estimate of the parameters of the model differs from their true values. This term also captures errors in modelling the structural impacts of exogenous disturbances.

2. **Imperfect information**: errors the policy maker incurs in estimating the current state of the economy.

3. **Asymmetric and/or inefficient forecasting**: informational asymmetries such as occur when the private sector has different information than the policy maker does.
Multiplicative uncertainty: Brainard (AER 1967)

- Model ($y$ is the output gap, $\pi$ is inflation):
  \[
  \pi_t = \beta E_t \pi_{t+1} + \kappa_t x_t + e_t
  \]

- Assume $\kappa_t$, is stochastic, $\kappa_t = \bar{\kappa} + \nu_t$, where $\nu_t$ is a white noise process.

- Under discretion, the policy maker takes $E_t y_{t+1}$ and $E_t \pi_{t+1}$ as given. The first order condition yields
  \[
  x_t = - \left( \frac{\bar{\kappa}}{\lambda + \sigma_v^2} \right) e_t
  \]

- Reaction with more caution.

- Does Brainard’s result generalize? No. Craine (JEDC 1979), Söderström (SJE 2002).
Robust control

- Standard approaches to decision problems are based on Bayesian expected utility maximization.
- Deals with uncertainty but not Knightian risk.
- Example: Ellsberg paradox.
  - Two urns each with 100 red and blue balls. Urn A has 50 red, 50 blue balls. Bet on extracting a color of your choice from an urn of your choice.
  - Which urn would you choose from?
- Robust control approach: Hansen and Sargent (JME 2003, JME 2012) and monetary policy applications in Giordani and Söderlind (JEDC 2004), Walsh (JMCB 2004), Dennis, Leitemo, and Söderström (JEDC 2009), Dennis (JEDC 2014).
Robust control: Hansen and Sargent

- Policy maker has reference model of form
  \[ y_{t+1} = \bar{A}y_t + \bar{B}i_t + \bar{C}e_{t+1} \]

- True model is
  \[ y_{t+1} = \bar{A}y_t + \bar{B}i_t + \bar{C}(e_{t+1} + w_{t+1}), \quad (4) \]
  
  - In the robust control literature, \( w_{t+1} \) represents unknown specification errors.
  - \( w_{t+1} \) is not simply an exogenous disturbance like \( e_{t+1} \) but may depend on the history of \( y_t \).

- The policy maker views \( \bar{A}\bar{y}_{t/t} + \bar{B}u_t + \bar{C}e_{t+1} \), i.e., the case \( w_{t+1} = 0 \), as a good “approximating model” to the true but unknown model.

- It is a good approximation in the sense that
  \[ \sum_{i=0}^{\infty} \beta^i w'_{t+i} w_{t+i} \leq \eta_0, \quad (5) \]
The intuition

- Strategic game involving the policy maker and an evil agent who attempts to make life hard for the policy maker.
- Leads to a min-max strategy by the policy maker, with the policy instrument chosen to minimize the worst-case outcome.
- Equilibrium is given by the solution to

\[ \min_w \max_u \mathbb{E}_t \left[ \sum_{t=0}^{\infty} \beta^t \{ -r(y_t, u_t) + \theta \beta w'_{t+1} w_{t+1} \} \right] \]

where \( y_t \) is the state, \( u_t \) is the policy maker’s control, \( r(y_t, u_t) \) is the quadratic loss, and \( \beta \) is the discount factor.
- As \( \theta \to \infty \), evil agent is more constrained. Standard case when \( \theta = \infty \).
Using a distorted model

- The policy maker replaces the model of the economy with a distorted model, one that incorporates the worst case process for $w_{t+1}$.
- The evil agent sets $w_{t+1}$ to maximize the policy maker's loss function.
  - The value of $w_{t+1}$ for which the worst-case outcome occurs can be expressed as a function of the state vector, $Ky_t$.
- Substituting $w_{t+1} = Ky_t$ into (4) yields the distorted model:
  \[ y_{t+1} = (\bar{A} + \bar{C}K)y_t + \bar{B}i_t + \bar{C}e_{t+1}. \] (6)
- The policy maker now treats this distorted model as the true model of the economy and minimizes loss subject to (6).
  - Once the policy maker has substituted in $Ky_t$ for $w_{t+1}$, the policy problem is reduced to a standard one – certainty equivalence holds.
Using a distorted model

- A Bayesian policy maker, faced with model uncertainty, assigns a probability to each possible model, where these probabilities reflect the policy maker’s assessment of the likelihood of each model.

- A policy maker concerned with robustness bases policy on a distorted model but then proceeds to act as if there were no longer any model uncertainty.

- An example: Does it pay to underestimate inflation persistence or over estimate it? Yes.
  - Coenen (JEDC 2007), Angeloni, Coenen, and Smets (SJPE 2003), and Walsh (AER 2003).
Preference for robustness

- The same policy that results from a policy maker employing the distorted model is also obtained when the policy maker believes the true model is given by

\[ y_{t+1} = \tilde{A}y_t + \tilde{B}i_t + \tilde{C}e_{t+1}, \]

and she maximizes an objective function that contains an additional adjustment for risk.

- Specifically, the policy maker’s preferences incorporate an additional sensitivity to risk. Similar risk sensitive preferences have been studied by Epstein and Zin (EMT 1989) and Weil (QJE 1990).

- Preferences are of the form

\[ V_t = U[c_t, V_{t+1}] \]

- Allows risk aversion and elasticity of intertemporal substitution to be separated.
Comparing the standard optimal rules and robust control

- Both approaches lead to same instrument or targeting rule.
- Consider simple example – new Keynesian model with $\psi_t$ the Langrangian multiplier on the NKPC.
- Standard approach – first order conditions:

  $$\pi_t + \psi_t - \psi_{t-1} = 0$$

  and

  $$\lambda x_t - \kappa \psi_t = 0.$$ 

- Combine to yield robustly optimal targeting rule:

  $$\pi_t = -\left( \frac{\lambda}{\kappa} \right) (x_t - x_{t-1}).$$
Comparing the two approaches
Equilibrium under the robustly optimal rule

- Together with the inflation adjustment equation, this yields

\[ \left[ 1 + \beta + \left( \frac{\kappa^2}{\lambda} \right) \right] x_t = \beta E_t x_{t+1} + x_{t-1} - \left( \frac{\kappa}{\lambda} \right) e_t, \]  

which can be jointly solved with the process for \( e_t \) given by

\[ e_t = \rho_e e_{t-1} + \epsilon_t. \]  

under rational expectations.
Robust control problem is

\[
\min_{x, \pi} \max_{w, e} E^r_{t} \sum_{i=0}^{\infty} \beta^i \left[ \left( \frac{1}{2} \right) \pi^2_{t+i} + \left( \frac{1}{2} \right) \lambda_{x} x^2_{t+i} \right.
\]

\[
- \left( \frac{1}{2} \right) \beta \theta w^2_{t+1+i}
\]

\[
+ \psi_{t+i} \left( \pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right)
\]

\[
+ \varphi_{t+i} \left( \rho_{e} e_{t+i} + \varepsilon_{t+i+1} + w_{t+i+1} - e_{t+i+1} \right).
\]
Robust control

- The policy maker’s first order conditions include

  \[ \pi + \psi_t - \psi_{t-1} = 0, \]  
  \[ \lambda x_t - \kappa \psi_t = 0, \]  

- The evil agent’s first order conditions include

  \[ -\varphi_t + \rho_e \varphi_t - \left( \frac{1}{\beta} \right) \varphi_{t-1} = 0, \]  
  and
  \[ -\theta w_{t+1} + \varphi_t = 0. \]
Robust control

Equilibrium

- Combining first two with the inflation adjustment equation yields

\[
1 + \beta + \left( \frac{\kappa^2}{\lambda} \right) x_t = \beta E_t^{rc} x_{t+1} + x_{t-1} - \left( \frac{\kappa}{\lambda} \right) e_t, \quad (13)
\]

which is identical to equation obtained in standard case except for the formation of expectations.

- Evil agent’s first order conditions imply that

\[
\varphi_{t-1} = \beta \rho_e \varphi_t - \beta \psi_t = \beta \rho_e \varphi_t - \beta \left( \frac{\lambda}{\kappa} \right) x_t.
\]

- Advancing this expression one period, taking expectations, solving the resulting expression forward implies that

\[
\omega_{t+1} = - \left( \frac{\beta \lambda}{\kappa \theta} \right) \sum_{i=0}^{\infty} (\beta \rho_e)^i E_t^{rc} x_{t+1+i}. \quad (14)
\]
Extensions: A simple rule for the evil agent

- Suppose the evil agent commits to a contingency rule that makes misspecification a function of exogenous state variables.
- Specifically, assume evil agent commits to $w_{t+1} = Ke_t$.
- Define $\tilde{\rho}_e = \rho_e + K$. Then, for any choice of $K$ such that $|\tilde{\rho}_e| < 1$, the policy maker’s problem becomes

$$
\min_i E_t^r \sum_{i=0}^{\infty} \beta^i \left\{ \left( \frac{1}{2} \right) \pi_{t+i}^2 + \left( \frac{1}{2} \right) \lambda x_{t+i}^2 - \left( \frac{1}{2} \right) \beta \theta (Ke_{t+i})^2 \\
+ \psi_{t+i} \left( \pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right) \\
+ \varphi_{t+i} \left( \tilde{\rho}_e e_{t+i} + \varepsilon_{t+i+1} - e_{t+i+1} \right) \right\}.
$$
Extensions: A simple rule for the evil agent

- This is a standard problem with the shock process replaced by the distorted process

\[ e_{t+1} = \tilde{\rho}_e e_t + \varepsilon_{t+1}. \quad (15) \]

- The policy maker (and the public) takes \( \tilde{\rho}_e \) (i.e., \( K \)) as given.

- The optimal targeting criterion is \( x_t = x_{t-1} - (\kappa / \lambda) \pi_t \), which is independent of \( \tilde{\rho}_e \) (and therefore \( K \)).

- This independence reflects the fact that the standard targeting criterion is designed to be robust with respect to exactly the type of model mis-specification of the disturbance process that is reflected in (15). (see Giannoni and Woodford 2003).
Using multiple models

- McCallum has long argued for evaluating simple rules in multiple models.
- Levin and Williams (JME 2003).
- Forward-looking models too easy to control – rules optimal in forward-looking models tend to do poorly in backward-looking models.
- Standard approach – use different models, but outcomes are evaluated using a fixed loss function.
  - What if loss function for model A is different from loss function for model B?
  - Theory says this will be the case.
- Example: greater nominal rigidity reduces output elasticity of inflation.
  - requires greater output gap variability to control inflation;
  - but theory says weight on gap fluctuations in loss function should fall.
Structural inflation inertia

- Important for relative performance of different targeting rules (Walsh AER 2003).
- Suppose
  \[
  \pi_t - \gamma \pi_{t-1} = \beta (E_t \pi_{t+1} - \gamma \pi_t) + \kappa x_t + e_t.
  \]
- Woodford (2003) shows that second order approximation to welfare is proportional to
  \[
  - \left( \frac{1}{2} \right) \sum_{i=0}^{\infty} [(\pi_t - \gamma \pi_{t-1})^2 + \lambda x_t^2].
  \]
- Define \( z_t \equiv \pi_t - \gamma \pi_{t-1} \). Then model becomes
  \[
  \min \left( \frac{1}{2} \right) \sum_{i=0}^{\infty} [(\pi_t - \gamma \pi_{t-1})^2 + \lambda x_t^2]
  \]
  subject to
  \[
  z_t = \beta E_t z_{t+1} + \kappa x_t + e_t.
  \]
- Loss independent of \( \gamma \)!
Several alternative assumptions possible about the relationship between monetary and fiscal policies.

- Fiscal policy assumed to adjust to ensure the government’s intertemporal budget is always in balance, while monetary policy is free to set the nominal money stock or the nominal rate of interest – described as a Ricardian regime (Sargent 1982), monetary dominance, or one with fiscal policy passive and monetary policy active (Leeper JME 1991).

- The fiscal authority sets its expenditure and taxes without regard to intertemporal budget balance. Seigniorage must adjust to ensure intertemporal budget constraint is satisfied. Case of fiscal dominance (or active fiscal policy) and passive monetary policy.
Intertemporal budget balance and seigniorage

- The intertemporal budget constraint implies that any government with a current outstanding debt must run, in present value terms, future surpluses.
- One way to generate a surplus is to increase revenues from seigniorage.
- Let \( s_t^f \equiv t_t - g_t \) be the primary fiscal surplus excluding seigniorage revenue.
- The government’s budget constraint can be written as

\[
\begin{align*}
    b_{t-1} &= R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}^f + R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i} \\
\end{align*}
\]  

(16)

- The current real liabilities of the government must be financed by, in present value terms, either a fiscal primary surplus \( R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}^f \) or by seigniorage.
Sargent and Wallace (FRB Minn QR 1981) – “unpleasant monetarist arithmetic” in a regime of fiscal dominance:

- If the present value of the fiscal primary surplus is reduced, the present value of seigniorage must rise to maintain intertemporal budget balance.
- Reducing inflation now can mean higher inflation in the future.
Suppose that the initial nominal stock of money is set exogenously by the monetary authority. Does this mean that the price level is determined solely by monetary policy, with no effect of fiscal policy? Fiscal policy can still affect the initial equilibrium price level, even when the initial nominal quantity of money is given and the government’s intertemporal budget constraint must be satisfied at all price levels. It does so if it affects the equilibrium real rate of interest.
Intertemporal budget balance and the nominal interest rate

Example

- Assume perfect foresight equilibrium.
- The government’s budget constraint must be satisfied and the real demand for money must equal the real supply of money.
- Assume money demand is

\[ \frac{M_t}{P_t} = f(R_{m,t}), \tag{17} \]

- Given \( R_m \), (17) implies a proportional relationship between the \( M \) and \( P \). If the initial money stock is \( M_0 \), then the initial price level is \( P_0 = M_0 / f(R_m) \).
The government’s budget constraint given by

\[ g_t + rb_{t-1} = t_t + (b_t - b_{t-1}) + m_t - \left( \frac{1}{1+\pi_t} \right) m_{t-1}. \]  

Consider a stationary equilibrium. The budget constraint becomes

\[ g + \left( \frac{1}{\beta} - 1 \right) b = t + \left( \frac{\pi_t}{1+\pi_t} \right) m = t + \left( \frac{\beta R_m - 1}{\beta R_m} \right) f(R_m), \]

Suppose the fiscal authority sets \( g, t, \) and \( b \). Then (19) determines the nominal interest rate \( R_m \).

Given the interest rate, \( P_0 \) is given by equation (17) as \( P_0 = M_0 / f(R_m) \), where \( M_0 \) is the initial money stock.
In subsequent periods, the price level is equal to \( P_t = P_0 (\beta R_m)^t \) where \( \beta R_m = (1 + \pi_t) \) is the gross inflation rate. The nominal stock of money in each future period is endogenously determined by \( M_t = P_t f(R_m) \).

Nominal interest rate must be such as to generate enough seigniorage to satisfy the government’s budget constraint.

In this example, even though the monetary authority has set \( M_0 \) exogenously, the initial price level is determined by the need for fiscal solvency.
A number of researchers (Woodford JME 1995, Cochrane NBER Macro Ann 1999, Sims ET 1994, unpub 2003, EER 2011) have examined models in which fiscal factors replace the money supply as the key determinant of the price level.

The government’s intertemporal budget constraint may not be satisfied for arbitrary price levels. Following Woodford (JME 1995), these regimes are described as non-Ricardian. The intertemporal budget constraint is satisfied only at the equilibrium price level, and the government’s nominal debt plays a critical role in determining the price level.
The fiscal theory of the price level

The basic idea

- If multiple equilibrium price levels are possible, does fiscal policy pin down a unique equilibrium?
- Restrict analysis to perfect-foresight equilibria for simplicity.
- Under standard assumptions, the household intertemporal budget constraint takes the form

\[
d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} (y_{t+i} - \tau_{t+i}) = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ c_{t+i} + \left( \frac{i_{t+i}}{1 + i_{t+i}} \right) m^d_{t+i} \right].
\] (20)

where \( \lambda_{t,t+i} = \prod_{j=1}^{i} \left( \frac{1}{1 + r_{t+j}} \right) \) with \( \lambda_{t,t} = 1. \)
The fiscal theory of the price level

The basic idea

- The budget constraint for the government sector, in real terms,

\[ g_t + d_t = \tau_t + \left( \frac{i_t}{1 + i_t} \right) m_t + \left( \frac{1}{1 + r_t} \right) d_{t+1}. \]

Recursively substituting for future values of \( d_{t+i} \), this budget constraint implies

\[ d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} [g_{t+i} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T \to \infty} \lambda_{t,t+T} d_T, \quad (21) \]

where \( \bar{s}_t = i_t m_t / (1 + i_t) \) is the government’s real seigniorage revenue.
The fiscal theory of the price level

The basic idea

Definition

Policy paths for \( (g_{t+i}, \tau_{t+i}, s_{t+i}, d_{t+i})_{i \geq 0} \) such that

\[
d_t + \sum_{i=0}^{\infty} \lambda_{t, t+i} [g_{t+i} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T \to \infty} \lambda_{t, t+T} d_T = 0
\]

for all price paths \( p_{t+i}, i \geq 0 \) are called Ricardian policies.

Definition

Policy paths for \( (g_{t+i}, \tau_{t+i}, \bar{s}_{t+i}, d_{t+i})_{i \geq 0} \) for which \( \lim_{T \to \infty} \lambda_{t, t+T} d_T \) may not equal zero for all price paths are called non-Ricardian.
The fiscal theory of the price level

The basic idea

- Now consider a perfect-foresight equilibrium; \( y_t = c_t + g_t \) and \( m_t^d = m_t \). Substituting in (20) and rearranging yields

\[
d_t + \sum_{i=0}^{\infty} \lambda_{t, t+i} \left[ g_{t+i} - \tau_{t+i} - \left( \frac{i_{t+i}}{1 + i_{t+i}} \right) m_{t+i} \right] = 0. \tag{22}
\]

- Thus, an implication of the representative household’s optimization problem is that equation (22) must hold in equilibrium.
The fiscal theory of the price level

Non-Ricardian policies

- Under a non-Ricardian policy, budget balance imposes an additional condition that must be satisfied in equilibrium.
- This requirement can be written as

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ \tau_{t+i} + \bar{s}_{t+i} - g_{t+i} \right].$$  \hspace{1cm} (23)

- At time $t$, the government’s outstanding nominal liabilities $D_t$ are predetermined by past policies.
- Given the present discounted value of the government’s future surpluses, the only endogenous variable is the current price level $P_t$. The price level must adjust to ensure equation (23) is satisfied.
The fiscal theory of the price level

Non-Ricardian policies

- Suppose the real demand for money is given by
  \[ \frac{M_t}{P_t} = f(1 + i_t). \]  
  \[ (24) \]

- Equations (23) and (24) must both be satisfied in equilibrium.
- Which two variables are determined jointed by these two equations depends on the assumptions that are made about fiscal and monetary policy.
Suppose the fiscal authority determines $g_{t+i}$ and $\tau_{t+i}$ for all $i \geq 0$, and the monetary authority pegs the nominal rate of interest $i_{t+i} = \bar{i}$ for all $i \geq 0$.

Seigniorage is equal to $\bar{i} f(1 + \bar{i}) / (1 + \bar{i})$ and so is fixed by monetary policy. With this specification of monetary and fiscal policy, the right side of

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ \tau_{t+i} + \bar{s}_{t+i} - g_{t+i} \right]$$

is given.
The fiscal theory of the price level

Non-Ricardian policies

- Since $D_t$ is predetermined at date $t$, this equation can be solved for the equilibrium price level $P_t^*$ given by

$$P_t^* = \frac{D_t}{\sum_{i=0}^{\infty} \lambda_{t,t+i} [\tau_{t+i} + \bar{s}_{t+i} - g_{t+i}]}.$$

- The current nominal money supply is then determined by

$$M_t = P_t^* f (1 + \bar{i}).$$

- One property of this equilibrium is that changes in fiscal policy ($g$ or $\tau$) directly alter the equilibrium price level, even though seigniorage is unaffected.
The fiscal theory of the price level

Non-Ricardian policies

- In standard infinite horizon, representative agent models, a tax cut (current and future government expenditures unchanged) has no effect on equilibrium (i.e., Ricardian equivalence holds) – the government cannot engineer a permanent tax cut unless government expenditures are also cut (in present value terms).

- If budget balance holds only when evaluated at the equilibrium price level, the government can plan a permanent tax cut. If it does, the price level must rise to ensure the new, lower value of discounted surpluses is again equal to the real value of government debt.
The fiscal theory of the price level

Empirical evidence

- Under the fiscal theory of the price level, intertemporal budget balance holds at the equilibrium value of the price level.
- Under traditional theories of the price level, it holds for all values of the price level.
- If we only observe equilibrium outcomes, it will be impossible empirically to distinguish between the two theories.
  - As Sims (ET 1994) puts it, “Determinacy of the price level under any policy depends on the public’s beliefs about what the policy authority would do under conditions that are never observed in equilibrium.”