Advanced Topics in Monetary Economics II

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August 18-22, 2014

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Analysis so far has been conducted within the context of a closed economy. 

Need to incorporate open economy aspects

- How does the transmission process of monetary policy differ in the open economy?
- How are the objectives of policy affected?
- How are conclusions about policy implementation affected?
The NK open economy model

- Clarida, Galí, Gertler (JME 2002): Simplified small open economy NK model.
  - The model consists of households who supply labor, purchase goods for consumption, and hold money and bonds, and firms who hire labor and produce and sell differentiated products in monopolistically competitive goods markets.
  - Consumption risk is shared internationally.
  - Each firm sets the price of the good it produces, but not all firms reset their price each period.
  - Households and firms behave optimally: households maximize the expected present value of utility and firms maximize profits.
Household utility

- Households consume a CES composite of home and foreign goods, defined as

\[ C_t = \left[ (1 - \gamma)^{\frac{1}{a}} \left( C_t^h \right)^{\frac{a-1}{a}} + \gamma^{\frac{1}{a}} \left( C_t^f \right)^{\frac{a-1}{a}} \right]^{\frac{a}{a-1}} \]

for \( a > 1 \).

- \( C^h \) (and \( C^f \)) are Dixit-Stiglitz aggregates of differentiated goods produced by domestic (and foreign) firms.

- Demand for good produced by firm \( j \):

\[ c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} C_t^h; \quad P_t^h = \left[ \int_0^1 p_{jt}^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} \]
Household utility

- Let $P^h_t$ ($P^f_t$) be the average price of domestically (foreign) produced consumption goods. The problem of minimizing the cost $P^h_t C^h_t + P^f_t C^f_t$ of achieving a given level of $C_t$ yields:

$$C^h_t = (1 - \gamma) \left( \frac{P^h_t}{P_t} \right)^{-a} C_t; \quad C^f_t = \gamma \left( \frac{P^f_t}{P_t} \right)^{-a} C_t$$

- The aggregate (CPI) price index is

$$P_t \equiv \left[ (1 - \gamma) \left( \frac{P^h_t}{P_t} \right)^{1-a} + \gamma \left( \frac{P^f_t}{P_t} \right)^{1-a} \right]^{\frac{1}{1-a}}.$$

- And

$$P^j_t C^j_t = (1 - \gamma) \left( \frac{P^j_t}{P_t} \right)^{1-a} P_t C_t, \ j = h, f$$
Household utility

- Household utility depends on its consumption of the composite good, money holdings, and on its labor supply:

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right].
\]

- Intertemporal optimization implies the standard Euler condition,

\[
C_{t-\sigma} = \beta E_t R_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma},
\]

- Optimal labor-leisure choice requires that the marginal rate of substitution between leisure and consumption equal the real wage. This condition takes the form

\[
\chi \frac{N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}.
\]
Law of one price

- Assuming law of one price,

\[ P_f^t = S_t P_t^* , \]

where \( P_t^* \) is the foreign currency price of foreign consumption goods and \( S_t \) is the nominal exchange rate (price of foreign currency in terms of domestic currency).

- Assumes complete exchange rate pass-through.

- The terms of trade is equal to the relative price of foreign and domestic goods:

\[ \Delta_t \equiv \frac{P_f^t}{P_h^t} . \] (1)

- The real exchange rate is defined as the relative price of foreign produced goods (in terms of domestic currency) relative to the home country's consumer price index:

\[ Q_t = \frac{P_f^t}{P_t} = \frac{S_t P_t^*}{P_t} \] (2)
International risk sharing

- Complete set of contingent claims imply, for some state $s$,

$$
\left( \frac{V_{t,t+1}(s)}{P_t} \right) C_t^{-\sigma} = \tilde{p}_{t+1}(s) \beta \left( \frac{1}{P_{t+1}(s)} \right) C_{t+1}(s)
$$

where $V(s)$ is price of claim to one unit of domestic currency in state $s$ and $\tilde{p}(s)$ is the probability of state $s$.

- For resident of foreign country, it must also hold that

$$
\left( \frac{V_{t,t+1}(s)}{S_t P_t^*} \right) (C_t^*)^{-\sigma} = \tilde{p}_{t+1}(s) \beta \left( \frac{1}{S_{t+1}(s) P_{t+1}^*(s)} \right) (C_{t+1}^*(s))^{-\sigma}
$$
International risk sharing

- Since
  \[ \frac{S_t P^*_t}{P_t} = Q_t \]

  we obtain
  \[ \left( \frac{C_{t+1}(s)}{C_t} \right)^{-\sigma} = \left( \frac{Q_t}{Q_{t+1}(s)} \right) \left( \frac{C^*_t(s)}{C^*_t} \right)^{-\sigma} \]

- Or
  \[ \frac{C^\sigma_t}{Q_t (C^*_t)^\sigma} = \frac{C^\sigma_{t+1}(s)}{Q_{t+1}(s) (C^*_t(s))^{\sigma}} = \frac{C^\sigma_{t+1}(s')}{Q_{t+1}(s') (C^*_t(s'))^{\sigma}} \]

- But this implies \( C^\sigma_t \) and \( Q_t (C^*_t)^\sigma \) must move proportionately, or
  \[ C_t = \nu Q_t^{1/\sigma} C^*_t \]

  for some \( \nu > 0 \).
International risk sharing

Assuming initial symmetric equilibrium with zero net asset positions and ex ante identical environments, set \( \nu = 1 \).

Taking logs of

\[
C_t = Q_t^{\frac{1}{\sigma}} C_t^*
\]

When log linearized,

\[
c_t = \left( \frac{1}{\sigma} \right) q_t + c_t^*
\]
International risk sharing

- Summing over all states,

\[ \sum \left( \frac{V_{t,t+1}(s)}{P_t} \right) C_t^{-\sigma} = \beta \sum \tilde{p}_{t+1}(s) \left( \frac{1}{P_{t+1}(s)} \right) C_{t+1}^{-\sigma} \]

\[ = \beta E_t \left( \frac{1}{P_{t+1}} \right) C_{t+1}^{-\sigma} \]

Or

\[ C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma} \]

where

\[ R_t \equiv \frac{1}{\sum V_{t,t+1}(s)}. \]
International risk sharing

- Domestic household can also hold foreign nominal bond, which implies

\[
\left(\frac{1}{P_t}\right) C_t^{-\sigma} = \beta E_t \left(\frac{1}{S_t}\right) R_t^* \left(\frac{S_{t+1}}{P_{t+1}}\right) C_{t+1}^{-\sigma}
\]

\[
c_t = E_t c_{t+1} - \frac{1}{\sigma} [r_t^* - E_t \pi_{t+1} + E_t (s_{t+1} - s_t)]
\]

- This implies UIP:

\[
i_t = i_t^* + E_t (s_{t+1} - s_t)
\]

- Or in real terms,

\[
r_t - E_t \pi_{t+1} = r_t^* - E_t \pi_{t+1}^* + (E_t q_{t+1} - q_t)
\]
Summarizing the log linear model

- Domestic prices:

  \[ p_t = (1 - \gamma) p_t^h + \gamma p_t^f = p_t^h + \gamma \delta_t \]

  so

  \[ \pi_t = \pi_t^h + \gamma (\delta_t - \delta_{t-1}) \]

- Euler condition:

  \[ c_t = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) \]

  \[ = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) \left[ i_t - E_t \pi_{t+1}^h - \gamma E_t (\delta_{t+1} - \delta_t) \right] \]
Summarizing households

- When log linearized,

\[ q_t = s_t + p_t^* - p_t = (1 - \gamma) \delta_t \]

\[ c^h_t = -a \left( p^h_t - p_t \right) + c_t; \quad c^f_t = -a \left( p^f_t - p_t \right) + c_t \]

\[ c^h_t - c^f_t = a \left( p^f_t - p^h_t \right) = a\delta_t \]

\[ c_t = (1 - \gamma) c^h_t + \gamma c^f_t = c^h_t - a\gamma\delta_t \]

\[ \eta n_t + \sigma c_t = w_t - p_t = w_t - p^h_t - \gamma\delta_t \]
Goods clearing:

\[ y_t = (1 - \gamma) c_t^h + \gamma c_t^{h*} = (1 - \gamma) (c_t + a\gamma\delta_t) + \gamma c_t^{h*} \]

Assume

\[ c_t^{h*} = y^* + a\delta_t \]

\[ \sigma (E_t c_{t+1}^* - c_t^*) = r_t^* - E_t \pi_{t+1}^* = \rho_t^* \]

Then

\[ y_t = (1 - \gamma) c_t^h + \gamma c_t^{h*} = (1 - \gamma) (c_t + a\gamma\delta_t) + \gamma (y_t^* + a\delta_t) \]

\[ = (1 - \gamma) c_t + (2 - \gamma) a\gamma\delta_t + \gamma y_t^* \]
The log linear Euler equation

So combining results,

\[ c_t = \left( \frac{1}{1-\gamma} \right) y_t - \left[ \frac{(2-\gamma)a\gamma}{1-\gamma} \right] \delta_t + \left( \frac{\gamma}{1-\gamma} \right) y_t^* \]

Using this in Euler condition,

\[ y_t = E_t y_{t+1} - \left( \frac{1+w}{\sigma} \right) \left( i_t - E_t \pi^h_{t+1} \right) + wE_t \left( y_{t+1}^* - y_t^* \right) \]

where

\[ w \equiv (2-\gamma)(a\sigma - 1)\gamma \]
Effects of world output

- A rise in world output has two effects on domestic output:
  - A contractionary effect due to expenditure switching as a rise in world output improves the terms of trade (a real appreciation as the price of foreign goods falls as their supply increases). This reduces demand and domestic production.
  - An expansionary effect due to the direct demand effect through higher exports.
- If \( w > 0 \) (\( w < 0 \)), contractionary (expansionary) effects dominate.
Open economy expectational IS curve

- IS curve becomes

\[ y_t = E_t y_{t+1} - \left( \frac{1}{\sigma \gamma} \right) \left( i_t - E_t \pi_{t+1}^h - \rho^* \right) \]

where \( \sigma \gamma \equiv (1 + w) / \sigma \).

- This is the small open economy equivalent to the closed economy expectational IS curve.

- There are two primary differences between the open and the closed economy versions of this relationship.
  
  - First, the elasticity of demand with respect to the real interest rate is no longer equal to the elasticity of intertemporal substitution, \( 1 / \sigma \). Instead, it equals \( 1 / \sigma \gamma = [1 - \gamma(1 - w)] / \sigma \) which depends on the “openness” of the economy.
  
  - The real interest rate \( \rho^* \) depends on developments in the real of the world.
Inflation in domestic goods prices

- To determine the rate of inflation of domestically produced goods, assume Calvo price adjustment:

\[ \pi^h_t = \beta E_t \pi^h_{t+1} + \kappa \left( w_t - p^h_t - z_t \right). \]

- The real consumption wage is \( w_t - p^c_t \), and this is related to the real product wage \( w_t - p^h_t \) by the terms of trade: \( p^c_t = p^h_t + \gamma \delta_t \). Since households equate the real consumption wage to the marginal rate of substitution between leisure and consumption,

\[ \eta n_t + \sigma c_t = w_t - p^c_t = w_t - p^h_t - \gamma \delta_t. \]

- Hence, real marginal cost is \( w_t - p^h_t - z_t = \eta n_t + \sigma c_t + \gamma \delta_t - z_t. \)
We can now eliminate consumption and the terms of trade to obtain an expression for real marginal cost solely in terms of domestic output and foreign variables.

Since $y_t = n_t + z_t$, $y_t = c_t + (\gamma w / \sigma) \delta_t$, and $y_t = y_t^* + (1 / \sigma \gamma) \delta_t$,

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \kappa (\eta + \sigma \gamma) (y_t - \tilde{y}_t),$$

where

$$\tilde{y}_t \equiv [(\sigma - \sigma \gamma) y_t^* - (1 + \eta) \epsilon_t] / (\eta + \sigma \gamma)$$
Parallels with the closed economy NK model

- Define the output gap as
  \[ x_t \equiv y_t - \tilde{y}_t. \]  

  (3)

- Then model becomes

  \[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma_\gamma} \right) \left( i_t - E_t \pi_t^h - \tilde{\rho}_t \right) \]  

  \[ \pi_t^h = \beta E_t \pi_{t+1}^h + \kappa (\eta + \sigma_\gamma) x_t. \]  

  (5)

  where

  \[ \tilde{\rho}_t = \rho^* + \sigma_\gamma E_t (\tilde{y}_{t+1} - \tilde{y}_t). \]  

  (6)
Policy objectives

- The SOE model, as Clarida, R., J. Galí, and M. Gertler (2001) have emphasized, is isomorphic to the closed economy new Keynesian model.
- The parallel is even stronger if the central bank’s objective can be represented as minimizing a quadratic form of the output gap and the inflation rate of domestically produced goods $\pi^h_t$.
- In this case, the central bank’s policy involves minimizing

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \left( \pi^h_{t+i} \right)^2 + \lambda x^2_{t+i} \right]$$

subject to the inflation adj. eq. and Euler eq.
- All the conclusions about policy reached there would apply without modification to the small open economy.
- The critical requirement is that the inflation rate appearing in the central bank’s objective function must be $\pi^h$ and not the inflation rate in consumer prices $\pi^c$. 
Policy implications

- Policy trade-offs only arise from inflation shocks as long as $GDP$ inflation and not $CPI$ inflation enters the objective function of the central bank.
  - If $CPI$ inflation matters, trade-offs are more complicated. An appreciation reduces firms’ marginal costs and reduces $GDP$ inflation.
  - In face of a positive shock to aggregate spending, the central bank must raise the nominal interest rate to stabilize the output gap. But this leads to an appreciation of the exchange rate and a decline in $CPI$ inflation.
Second order approximation to utility

- Losses relative to optimal policy for domestic representative household is

\[ W = - \left( \frac{1 - \alpha}{2} \right) \left( \frac{\varepsilon}{\lambda} \right) \sum_{t=0}^{\infty} \beta^t \left[ \pi_{h,t}^2 + \frac{\lambda(1 + \varphi)}{\varepsilon} x_t^2 \right] + t.i.p. \]

- Taking conditional expectations and letting $\beta \to 1$,

\[ V = - \left( \frac{1 - \alpha}{2} \right) \left( \frac{\varepsilon}{\lambda} \right) \left[ \text{var}(\pi_{h,t}) + \frac{\lambda(1 + \varphi)}{\varepsilon} \text{var}(x_t) \right] \]

- Stabilize output gap and domestic price inflation.
Optimal policy

- Distortions in the closed economy:
  - Steady-state markup and sticky prices
  - Use fiscal subsidy to deal with first, price stability to deal with second.

- Distortions in the open economy:
  - Steady-state markup, sticky prices, and possibility of affecting terms of trade to benefit domestic consumers affects incentives of central bank.
    - An monetary expansion that lowers domestic interest rates causes a depreciation of the currency and lowers prices of home production relative to foreign goods. This competitive devaluation increases demand for home production, and increases $C_t$ relative to $C_t^*$.
    - In special case of $\sigma = \eta = \gamma = 1$, can still use fiscal subsidy to deal with first, domestic price stability to deal with second.
Comparing policies

- Galí and Monacelli (RESTud 2005) compare optimal policy to a Taylor rule based on domestic price inflation (DITR), a Taylor rule based on CPI inflation (CITR), and an exchange rate peg. They calibrate domestic and world productivity shocks using Canadian and U.S. data respectively.
- In face of a domestic productivity shock, DITR leads to a rise in the CPI (there is a real depreciation due to the productivity shock) while under CITR, the domestic price level falls (which requires a negative output gap).
- Under a peg, responses are similar to under CITR but peg makes domestic and CPI price levels stationary (as they are under optimal policy).
Optimal policy involves greater terms of trade and nominal exchange rate volatility than DITR or CITR (but domestic inflation and output gap are keep equal to zero under the optimal policy).

Peg yields large welfare losses as it makes TOT too smooth. DITR does better than CITR (but that is presumably because under the case considered, it is optimal to stabilize the domestic price level).
Other distortions in the open economy

- Deviations from the law of one price (Monacelli JMCB 2005)
- Standard models assume complete pass through from foreign prices to domestic prices.
- Law of one price does not hold: empirical evidence strongly rejects complete pass through.
Law of one price

- Define $s_t$ as the nominal exchange rate (domestic currency price of one unit of foreign currency).
- The real exchange rate in log terms is

$$q_t = s_t + p_t^* - p_t$$

$$= \left( s_t + p_t^* - p_t^f \right) + p_t^f - p_t$$

$$= \psi_t + (1 - \gamma) \delta_t$$

where $\psi_t \equiv s_t + p_t^* - p_t^f$ is the deviation from the law of one price.
Imported goods prices

- Assume retailers import consumer goods and adjust the domestic currency price at which they sell the goods according to a Calvo mechanism.
- Inflation of the domestic price of imported goods will be equal to

\[ \pi_t^f = \beta E_t \pi^f_{t+1} + \kappa_f \psi_t. \]

- Notice that the law of one price gap \( \psi_t \) serves as the marginal cost variable.
- Importers pay \( p_t^* + s_t \) for foreign goods and sell them at \( p_t^f \), so retailer will want to raise price if \( \psi_t > 0 \) and lower them if \( \psi_t < 0 \).
Policy implications

- Deviations from law of one price act like cost shocks – forces policy trade-offs
- Standard problem:
  - with multiple sources of nominal rigidity, a policy of price stability cannot offset all the nominal distortions.
- Policies that achieve price stability let deviations from law of one price lead to output gap volatility.
  - Interest rate movements to insulate demand from $E_t \Delta \psi_{t+1}$ do not insulate inflation.
Preference differences and local currency pricing
Engel AER 2011

- Similar to Clarida, Galí, Gertler (JME 2002). Two countries, equal size (CGG allow populations to differ).

- Two generalizations:
  - Different preferences – home agents may put higher weight $v/2 > 1/2$ in utility on home produced goods, weight $1 - v/2$ on foreign produced goods.
  - Allow for violations of the law of one price (LOP) so goods may be sold at different prices in home and foreign countries.

- Continuum of households (normalized to 1) in each country whose utility is defined over consumption of goods and disutility of labor.

- Each country produces a continuum of goods, each by a monopolist. Goods are imperfect substitutes.

- Trade in a complete set of nominal denominated contingent claims.
Preference differences and local currency pricing

- Utility of home country representative household:

\[ U_t(h) = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \frac{C_{t+j}(h)^{1-\sigma}}{1-\sigma} \right] - \frac{N_{t+j}(h)^{1+\phi}}{1+\phi} \right\}, \quad \sigma > 0, \quad \phi \geq 0. \]

- \( C_t(h) \) is a consumption aggregate:

\[ C_t(h) = C_{Ht}(h)^{\nu/2} C_{Ft}(h)^{1-\nu/2}, \quad 0 \leq \nu \leq 2 \]

and for foreign household

\[ C^*_t(h) = C^*_{Ht}(h)^{1-\nu/2} C^*_{Ft}(h)^{\nu/2}, \quad 0 \leq \nu \leq 2. \]

where home bias for \( \nu > 1 \).

- Each \( C_{kt}(h) \) is CES aggregate of individual goods with elasticity of substitution between home and foreign varieties equal to \( \xi \) (same elasticity for home and foreign households).
Preference differences and local currency pricing: firms

- Production function of domestic firm $f$:

$$Y_t(f) = A_t N_t(f)$$

where $A_t$ is common productivity shock.

- Profits:

$$\Gamma_t(f) = P_{Ht}(f) C_{Ht}(f) + E_t P_{Ht}^*(f) C_{Ht}^*(f) - W_t N_t(f)$$

  - $P_{Ht}(f)$ is home-currency price of good sold in home country and $P_{Ht}^*(f)$ is foreign-currency price of good sold in foreign country.

- $E_t$ is exchange rate – home currency per unit of foreign currency (a rise is a home depreciation).

- Output of firm $f$ equals sales:

$$Y_t(f) = C_{Ht}(f) + C_{Ht}^*(f)$$

- Similar equations for foreign firms $(Y_t^*(f), A_t^*, \eta_t^*)$. 

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August 18-22, 2014
Preference differences and local currency pricing: prices

- CPI for domestic and foreign countries:

\[ P_t = k^{-1} P_{Ht}^{v/2} P_{Ft}^{1-v/2}; \quad P_t^* = k^{-1} P_{Ht}^{*1-v/2} P_{Ft}^{v/2} \]

where

\[ k = \left(1 - \frac{v}{2}\right)^{1-v/2} \left(\frac{v}{2}\right)^{v/2} \]

- Terms of trade:

\[ S_t = \frac{P_{Ft}}{P_{Ht}}; \quad S_t^* = \frac{P_{Ht}^*}{P_{Ft}^*} \]

- So

\[ \frac{P_{H,t}}{P_t} = k \frac{P_{H,t}}{P_{Ht}^{v/2} P_{Ft}^{1-v/2}} = k S_t^{v/2-1}; \quad \frac{P_{H,t}^*}{P_t^*} = k (S_t^*)^{v/2} \]

- Both \( P_{Ht} \) and \( P_{Ft} \) are CES aggregates of individual goods’ prices.
Preference differences and local currency pricing

- Real exchange rate:

\[ Q_t \equiv \frac{E_t P^*_t}{P_t} = \frac{E_t P^*_{Ht}^{1-v/2} P^*_{Ft}^{v/2}}{P^*_{Ht}^{v/2} P^*_{Ft}^{1-v/2}} = \frac{E_t P^*_{Ht}}{P^*_{Ht}} (S^*_t)^{-v/2} (S_t)^{(v/2)-1} \]

- Under PPP, \( E_t P^*_{H,t} = P_{H,t} \) and \( E_t P^*_{F,t} = P_{F,t} \), so

\[ S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{E_t P^*_{Ft}}{E_t P^*_{Ht}} = \frac{P^*_{Ft}}{P^*_{Ht}} = (S^*_t)^{-1} \]

- And

\[ \frac{E_t P^*_t}{P_t} = S_t^{v-1} = 1 \text{ if } v = 1 \text{ (no home bias)} \]
International risk sharing

- Standard result:
  \[ C_t = Q_t^{\frac{1}{\sigma}} C_t^* \]

- When PPP does not hold,
  \[
  \left( \frac{C_t}{C_t^*} \right)^\sigma = Q_t = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}} \left( S_t^* \right)^{-\nu/2} \left( S_t \right)^{(\nu/2)-1}
  \]

- Under PPP and identical preferences (i.e., \( \nu = 1 \)), \( Q_t = 1 \) (LOOP) so \( C_t = C_t^* \).
Market clearing

- Market clearing for domestic firms:

\[ Y_t = C_{Ht} + C_{Ht}^* = \frac{v}{2} \left( \frac{P_t}{P_{Ht}} \right) C_t + \left( 1 - \frac{v}{2} \right) \left( \frac{P_t^*}{P_{Ht}^*} \right) C_t^* \]

\[ = k^{-1} \left[ \frac{v}{2} S_t^{1-v/2} C_t + \left( 1 - \frac{v}{2} \right) (S_t^*)^{-v/2} C_t^* \right] \]

- And for foreign firms:

\[ Y_{t^*} = C_{Ft} + C_{Ft}^* = \left( 1 - \frac{v}{2} \right) \left( \frac{P_t}{P_{Ft}} \right) C_t + \frac{v}{2} \left( \frac{P_t^*}{P_{Ft}^*} \right) C_t^* \]

\[ = k^{-1} \left[ \left( 1 - \frac{v}{2} \right) S_t^{-v/2} C_t + \left( \frac{v}{2} \right) (S_t^*)^{1-v/2} C_t^* \right] \]
Employment

- Employment is

\[ N_t = \int_0^1 N_t(f) df = A_t^{-1} \int_0^1 Y_t(f) df = A_t^{-1} (C_{Ht} V_{Ht} + C_{Ht} V_{Ht}^*) \]

where \( V \) and \( V^* \) are price dispersion measures:

\[ V_{Ht} = \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df, \quad V_{Ht}^* = \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}^*} \right)^{-\xi} df \]

- With producer currency pricing, law of one price holds, and \( V_{H,t} = V_{H,t}^* \).

- With local currency pricing, dispersion of home produced goods prices sold at home and in foreign country can differ.

- So LCP implies two distortions.
Global social welfare

- Second-order welfare approximation around efficient steady state (from the perspective of global optimal) is

\[ E_t \sum_{j=0}^{\infty} \beta^j X_{t+j} \]

where

\[ X_t = \left( \frac{\sigma}{D} + \phi \right) (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\nu}{4} \left( \frac{2 - \nu}{D} \right) m_t^2 \]

\[ + \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_H}^2 + \left( \frac{2 - \nu}{2} \right) \sigma_{p_H^*}^2 + \frac{\nu}{2} \sigma_{p_F}^2 + \left( \frac{2 - \nu}{2} \right) \sigma_{p_F}^2 \right] \]

and \( \tilde{y}_t^R \) and \( \tilde{y}_t^W \) are relative and world income and terms such as \( \sigma_{p_H}^2 \) represent the cross sectional variance of prices (in this case, home good prices), and a variable \( \tilde{x} \) denotes a deviation around the efficient stochastic equilibrium and \( D = \sigma \nu (2 - \nu) + (\nu - 1)^2 \).
Global social welfare

- Currency misalignment is defined as average deviation of consumer prices in the foreign country from consumer prices in the domestic country:

\[ M_t = \left( \frac{E_t P^*_H, t}{P_{H,t}} \right)^{1/2} \left( \frac{E_t P^*_F, t}{P_{F,t}} \right)^{1/2} \]

- In terms of log linear deviations:

\[ m_t = \frac{1}{2} \left( e_t + p^*_{H,t} - p_{H,t} + e_t + p^*_{F,t} - p_{F,t} \right) \]

- The terms in the loss function associated with \( m \) and the price dispersion terms all arise because differences in relative prices generate welfare losses.
Global social welfare: Local currency pricing

- Even from the perspective of the domestic economy, there are two distortions because dispersion of relative prices at home and in foreign market differ and both result in an inefficient allocation of labor across firms.

- In terms of deviation from efficient levels:

\[ s_t = \left( \frac{2\sigma}{D} \right) \tilde{y}_t^R - \left( \frac{v - 1}{D} \right) m_t \]

so even if output gaps are zero, \( s \) deviates from efficient level if \( m \neq 0 \); reflects an inefficient relative price distortion.

- If preferences symmetric as in CGG (\( v = 1 \)), effect disappears.
Global social welfare: Producer currency pricing

- If law of one price holds, $\sigma_{p_H}^2 = \sigma_{p_{H*}}^2$ since $p_{Ht}(f) = p_{Ht}^*(f) + e_t$ so cross sectional variation is the same (and same holds for $p_F$ and $p_{F*}$). Also under LOOP, $m = 0$, so loss becomes

$$
\left( \frac{\sigma}{D} + \phi \right) \left( \tilde{y}_t^R \right)^2 + (\sigma + \phi) \left( \tilde{y}_t^W \right)^2 + \frac{\xi}{2} \left( \sigma_{p_H}^2 + \sigma_{p_{F*}}^2 \right).
$$
Traded and nontraded goods
Kirsanova, Leith and Wren-Lewis (EJ 2006) and Leith and Wren-Lewis (unpub 2006)

- Introduce traded and nontraded goods.
- Two sectors, relative price dispersion in each sector matters for domestic welfare.
- New relative price that matters:

\[ T_t = \frac{P_{N,t}}{P_{H,t}} \]

where \( P_N \) is price index for nontraded goods and \( P_H \) is price index for tradeable goods produced at home.

- Welfare can’t be reduced to a single output gap and a domestic price inflation measure.
Four different types of firms: firms producing (1) domestic good, (2) importing consumption goods, (3) importing investment goods, and (4) exporting goods.

Within each category, continuum of firms each producing a differentiated product, all with sticky prices.

Domestic goods firms use labor and capital to produce output which they sell to a retailer.

Retailer transforms domestic goods into a homogeneous final good sold to households.

Relative prices: (1) domestic goods and final good; (2) domestic goods and imported consumption goods; (3) consumption and investment goods; (4) final good and export goods.

Deviations from UIP.
The monetary union model of Benigno (JIE 2004)

- Two regions and a single monetary authority. Two fiscal authorities.
- Symmetric preferences.
- Each household is a consumer of all goods and a producer of a differentiated product.
- Terms of trade \( T = \frac{P_F}{P_H} \), where \( P_H \) is price index of goods produced in region \( H \).
The sticky-price equilibrium

- Inflation rates:

\[
\pi_t^H = \beta E_t \pi_{t+1}^H + k_C^H y_t^W + (1 - n) k_T^H (\hat{T}_t - \tilde{T}_t)
\]

\[
= \beta E_t \pi_{t+1}^H + k_C^H y_t^H + (1 - n) \left( k_T^H - k_C^H \right) (\hat{T}_t - \tilde{T}_t)
\]

\[
\pi_t^F = \beta E_t \pi_{t+1}^F + k_C^F y_t^W - nk_T^F (\hat{T}_t - \tilde{T}_t)
\]

\[
= \beta E_t \pi_{t+1}^F + k_C^F y_t^F - n \left( k_T^F - k_C^F \right) (\hat{T}_t - \tilde{T}_t)
\]

\[
\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H
\]

- Looks like the wage-price block when wages and prices are sticky.
Welfare

- Assume a fiscal subsidy to offset the distortions due to monopolistic competition.
- Benigno then shows that the second-order approximation to the welfare of the representative household is

\[ W_t = -\Omega E_0 \sum_{t=0}^{\infty} \beta^t L_t \]

\[ L_t = \Lambda \left( y_t^W \right)^2 + n(1-n)\Gamma \left( \hat{T}_t - \tilde{T}_t \right)^2 + \gamma \left( \pi_t^H \right)^2 \]

\[ + (1-\gamma) \left( \pi_t^F \right)^2 + t.i.p. + o \left( \parallel \xi \parallel^3 \right) \]

- Union-wide output gap, terms of trade gap, and inflation rates in each region matter.
  - Movements in \( \hat{T}_t \) reduce welfare if they deviate from \( \tilde{T}_t \). Relative prices in the two regions should move to reflect changes in \( \tilde{T} \).
- Inflation enters as weighted average of inflation squared in two regions.
Basic intuition similar to that with sticky wages and prices, or any two sector model with differing degrees of nominal rigidity.

With multiple nominal rigidities, the single instrument of monetary policy cannot eliminate all distortions.

Stabilizing a measure of inflation does not ensure that relative prices can adjust appropriately when prices in both countries are sticky.
Which inflation rate?

- The loss function involves

\[ \gamma \left( \pi_t^H \right)^2 + (1 - \gamma) \left( \pi_t^F \right)^2 \]

where

\[ \gamma = \frac{nd^H}{nd^H + (1 - n)d^F} \]

\[ d^H = \frac{\alpha^H}{(1 - \alpha^H)(1 - \beta \alpha^H)} \]

\[ d^F = \frac{\alpha^F}{(1 - \alpha^F)(1 - \beta \alpha^F)} \]
Which inflation rate?

- If prices are equally sticky in each region, $\alpha^H = \alpha^F$ and $\gamma = n$, then weight related to size.
  - In this case, if $\pi^R \equiv \pi^F - \pi^H$,
    \[ \gamma (\pi^H_t)^2 + (1 - \gamma) (\pi^F_t)^2 = (\pi^W_t)^2 + n(1 - n) (\pi^R_t)^2. \]

- If $\alpha^H = \alpha^F$, optimal to set $\pi^W = n\pi^H + (1 - n)\pi^F = 0$ since relative prices are out of the monetary authority’s control.
- $\pi^W = \pi^{HICP}$: harmonized index of consumer price inflation.
- If prices are flexible in one regions (say $F$), then $d^F = 0$ and $\gamma = 1$ – welfare only depends on $H$ inflation.