Advanced Topics in Monetary Economics II

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Optimal policy: Topics to cover

The LQ approach

1. Policy objectives
2. Optimal policy under discretion and commitment
3. The zero lower bound
Decomposing output

- Consider the following decomposition of output:

\[ Y_t = \left( \frac{Y_t}{Y^f_t} \right) \left( \frac{Y^f_t}{Y^\text{pot}_t} \right) \left( \frac{Y^\text{pot}_t}{Y^e_t} \right) Y^e_t \]

where \( Y^e_t \) is the efficient level of output and \( Y^f_t \) is the level of output under flexible prices and wages, \( Y^\text{pot}_t \) is potential output (output with constant markups and flexible prices).

- In terms of log deviations around the steady state,

\[ y_t - y^e_t = x_t + x^f_{\text{pot}} + x^\text{pot}_t. \]

- In baseline NK model, \( x^f_{\text{pot}} = 0 \) (markups constant) and \( x^\text{pot}_t \) is a constant (related to steady-state markups). So

\[ \sigma_{y - y^e}^2 = \sigma_x^2 \]

and minimizing output around the efficient level is the same as minimizing \( x_t \) (Blanchard-Galí ‘divine coincidence’ JMCB 2007).
Decomposing gaps

- In general,
  \[ \sigma^2_{y-y^e} = \sigma^2_x + 2\sigma_{xx} f_{pot} + t.i.p. \]
  and minimizing \( \sigma^2_x \) does not necessarily minimize \( \sigma^2_{y-y^e} \). Covariances can matter.

- Optimal policy may not involve minimizing \( y_t - y_t^e \) since resulting fluctuations in price and wage inflation may be too costly.
  - Theory of second best in the face of multiple distortions.
Decomposing gaps

- Which fluctuations in $y_t$ are due to fluctuations in $x_t$, which due to $x_t^{fpot}$, which to $x_t^{pot}$, which to $y_t^e$?
- Which fluctuations in $y_t$ are efficient? Which are inefficient?
- Chari, Kehoe, and McGrattan (AEJ Macro 2009) emphasized need to distinguish between efficient and inefficient sources of fluctuations.
  - May be hard to identify sources of fluctuations.
Gaps and wedges

- Efficient requires that
  \[ mrs_t = mpl_t \]

- Shocks that cause wedge between these two can be
  - Exogenous
    - Efficient: preference shifts \( \chi_t \)
    - Inefficient: shifts in desired markups to wages and prices \( \hat{\mu}_t^w, \hat{\mu}_t^p \)
  - Endogenous
    - Inefficient: nominal wage and price rigidities \( \varphi_t^w, \varphi_t^p \).
The case of flexible prices and wages

Flex-price/wage output

- With flexible prices and wages,
  \[ \omega_t = mpl_t - \hat{\mu}_t; \quad \omega_t = mrs_t + \hat{\mu}_w \]

- Suppose marginal rate of substitution is
  \[ mrs_t = \eta \hat{n}_t + \sigma \hat{c}_t + \hat{\chi}_t \]
  and marginal productivity is
  \[ mpl_t = \hat{y}_t - \hat{n}_t = \hat{z}_t. \]

- Then, the efficient labor equilibrium condition yields
  \[ \eta \hat{n}_t + \sigma \hat{c}_t + \hat{\chi}_t + \hat{\mu}_w = \omega_t = \hat{z}_t - \hat{\mu}_t. \]

- Now using the fact that \( \hat{y}_t = \hat{z}_t + \hat{n}_t \) and \( \hat{y}_t = \hat{c}_t \), the flexible-price equilibrium output \( \hat{y}_t^f \) can be expressed as
  \[ \hat{y}_t^f = \left( \frac{1 + \eta}{\eta + \sigma} \right) \hat{z}_t - \left( \frac{1}{\eta + \sigma} \right) (\hat{\mu}_t + \hat{\mu}_w + \hat{\chi}_t). \]
The wedges $\varphi_w^t$ and $\varphi_p^t$ are defined by

$$\omega_t = mpl_t - \hat{\mu}_t - \varphi_t^p$$

$$\omega_t = mrs_t + \hat{\mu}_w^t + \varphi_w^t$$

The efficiency wedge is

$$mrs_t - mpl_t = (\hat{\omega}_t - \hat{\mu}_w^t - \varphi_w^t) - (\hat{\omega}_t + \hat{\mu}_t + \varphi_t^p)$$

$$= - (\hat{\mu}_w^t + \hat{\mu}_t) - (\varphi_w^t + \varphi_t^p)$$
Measuring wedges

- The observed wedge:

\[
(\eta \hat{n}_t + \sigma \hat{c}_t) - (\hat{y}_t - \hat{n}_t) = - (\hat{\mu}_t + \hat{\mu}_w^t + \hat{\chi}_t) + (\phi_w^t + \phi_p^t)
\]

- LHS is observable: RHS isn’t.

- With log utility and \( \hat{y} = \hat{c} \),

\[
\hat{n}_t = - \left( \frac{1}{1 + \eta} \right) (\hat{\mu}_t + \hat{\mu}_w^t + \hat{\chi}_t - \phi_w^t - \phi_p^t)
\]

so employment gives a direct measure of the labor wedge but not its sources.
Efficiency gaps

- Galí, Gertler, and López-Salido (REStat 2007) define the “inefficiency gap” as the gap between the household’s marginal rate of substitution between leisure and consumption ($mrs_t$) and the marginal product of labor ($mpl_t$).

- They divide this gap into the wedge between the real wage and the marginal rate of substitution, which they label the wage markup, and the wedge between the real wage and the marginal product of labor (the price markup).

$$mrs_t - mpl_t = (mrs_t - \omega_t) + (\omega_t - mpl_t)$$

- Based on United States data, they conclude the wage markup accounts for most of the time series variation in the inefficiency gap.

- Consistent with the importance of nominal wage rigidity.
Empirical estimates of the wedges: Galí, Gertler, and Lopez-Salido (REStat 2007)

- Assume $\sigma = \eta = 1$, so $lw_t = (\hat{n}_t + \hat{c}_t) - (\hat{y}_t - \hat{n}_t)$.

Figure: Galí, Gertler, and Lopez-Salido’s price gap and wage gap.
Empirical estimates of the wedges: Galí, et. al. (REStat 2007)

Figure: Galí, Gertler, and Lopez-Salido’s efficiency, wage, and price gaps and US unemployment rate (right axis).

Karabarbounis (RED 2013): looos at OECD countries – it’s the wage.
Decomposing gaps using DSGE models

- Is it a good shock or a bad shock? (Chari, Kehoe, and McGrattan 2009)
- Suppose there are shocks to the disutility of labor and to the wage markup:
  \[
  mrst = \frac{\chi_t N_t^{\eta}}{C_t^{-\sigma}} = \frac{(W_t / P_t)}{\mu_t^w} \Rightarrow \frac{\mu_t^w \chi_t N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}
  \]

- Linearized:
  \[
  \eta \hat{n}_t + \sigma \hat{c}_t - (\hat{w}_t - \hat{p}_t) = - (\hat{\chi}_t + \hat{\mu}_t^w)
  \]
- What matters is \( \mu_t^w + \chi_t \) – one is a bad shock (\( \mu_t^w \)) and one is a good shock (\( \chi_t \)).
- Can we identify the two shocks?
Explaining the labor wedge

- Galí, Smets and Wouter (NBER Macro Annual 2011): using unemployment as an observable (see exercise).
- Sala, Söderström, and Trigari (Riskbank WP 2010): alternatives with markups and preferences as source of persistence.
- Justiniano, Primiceri, and Tambalotti (AEJ Macro 2013): measurement error and persistence:
  - If wage series \((\hat{w}_t^m - \hat{p}_t)\) measures \((\hat{w}_t - \hat{p}_t)\) with error, then
    \[\eta \hat{n}_t + \sigma \hat{c}_t = (\hat{w}_t^m - \hat{p}_t) - (e_t^m + \hat{\chi}_t + \hat{\mu}_t^w)\]
  - Use multiple (2) series \(\hat{w}_{1,t}^m\) and \(\hat{w}_{2,t}^m\) to reduce measurement error.
  - Assume \(\chi_t\) is AR(1) and \(\mu_t^w\) is i.i.d.
Sala, Söderström, and Trigari (2010)

Figure 10: Decomposing the fundamental wedge into its inefficient and efficient components

(a) Inefficient, AR(1) wage markup shock
(b) Efficient, AR(1) wage markup shock
(c) Inefficient, AR(1) labor disutility shock
(d) Efficient, AR(1) labor disutility shock

This figure shows the estimated paths for the fundamental wedge and its inefficient and efficient components in the model with measurement errors under different assumptions about the labor market shocks. Parameters are set to their posterior median values.
Summary

- Gaps/wedges can arise from efficient shocks and inefficient shocks.
- Implications for policy are different, so identifying nature of shocks is important.
- Serious identification issues:
  - Is the labor wedge due to preference shocks, wage markup shocks, or measurement error?
- Karabarbounis (RED 2013) does Galí, Gertler, Lopez-Salido exercise for several OECD countries.
  - Concludes $\omega - mpl \approx 0$, but $mrs - \omega \neq 0$. 
Optimal policy – welfare and policy objectives

- Given the specification of the economic environment, what are the appropriate objectives of the central bank?
- Standard to assume central bank is concerned with minimizing a quadratic loss function that depended on output and inflation – plausible, but ultimately *ad hoc*. Common in the Barro-Gordon tradition.
- Woodford (2003) has provided the most detailed analysis of the link between a welfare criteria derived as a log-linear approximation to the utility of the representative agent and the type of quadratic loss functions so common in the literature.
Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi^2_{t+i} + \lambda (x_{t+i} - x^*)^2 \right]. \quad (1)$$

$x_t$ is the gap between output and the output level that would arise under flexible prices, and $x^*$ is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the steady-state level of output.
Policy weights

- Theory says something about the weights in the loss function:

\[
E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right],
\]

where

\[
\Omega = \tilde{Y} U_c \tilde{\kappa} \theta, \quad \text{and} \quad \lambda = \frac{\tilde{\kappa} (\sigma + \eta)}{\theta} = \frac{\kappa}{\theta}.
\]

- More general case,

\[
\Omega = \tilde{Y} U_c \tilde{\kappa} (1 + \psi \theta) \theta, \quad \text{and} \quad \lambda = \tilde{\kappa} \frac{(\sigma + \psi)}{(1 + \psi \theta) \theta}, \quad \psi = \frac{a + \eta}{1 - a}.
\]

- Greater nominal rigidity (larger \( \omega \)) reduces \( \lambda \) (recall \( \tilde{\kappa} = (1 - \omega) (1 - \omega \beta) / \omega \).

- Loss function endogenous.

- Calvo specification implies \( \lambda \) is small – Taylor specification leads to larger weight on output gap.
Policy implications of price stickiness

- When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.

  1. The relative price of firms who have not set their prices for a while falls. They experience in increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.

  2. Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.

- The solution is to prevent price dispersion by stabilizing the price level by keeping inflation equal to zero.
Woodford versus Friedman

- The basic new Keynesian model suggests price stability (i.e., zero inflation) is optimal.
  - Zero inflation eliminates inefficient price dispersion.

- Milton Friedman argued that a zero nominal rate of interest is optimal.
  - Zero nominal rate eliminates inefficiency in money holdings.
  - Optimal inflation is negative (deflation) at rate equal to real rate of interest.

- Khan, King, and Wolman (REStud 2003) analysis model with both distortions and conclude optimal inflation is closer to zero than to the Friedman rule.

- Schmidt-Grohe and Uribe (Handbook 2010), Coibion, Gorodnichenko, and Wieland (REStud 2012): optimal $\pi$ still small even when ZLB taking into account.
Basic model – eliminating the steady-state distortion

- Assume objective is to minimize

\[ \frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) . \]

- Note that \( x^* \) has been set equal to zero in loss function
- Fiscal subsidy to offset distortion from monopolistic competition.
- If \( x^* \neq 0 \), can’t use first order approximations to structural equations to obtain a correct second order approximation to the representative agent’s welfare.
Policy Implication of forward-looking models

- The basic new Keynesian inflation adjustment equation took the form

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \]

where \( x \) is output relative to flexible-price output and real marginal cost is \( (\sigma + \eta) x_t \).

- That is, there is no additional disturbance term.

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \Rightarrow \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} \]

- The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.

- Just set \( x_{t+i} = 0 \) for all \( i \); keeps inflation equal to zero: Blanchard and Galí’s “divine coincidence”.

- Productivity shocks don’t appear – with only prices sticky, real wage is flexible.
Cost shocks

- Divine coincidence assumes $\hat{y}_t^f = \hat{y}_t^e$ once $x^*$ eliminated, so $x_t = \hat{y}_t - \hat{y}_t^f = \hat{y}_t - \hat{y}_t^e = x_t^e$.
- With stochastic variations in markups, $\hat{y}_t^f \neq \hat{y}_t^e$ and $x_t \neq x_t^e$. So

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - \hat{y}_t^e) + \kappa \left( \hat{y}_t^e - \hat{y}_t^f \right)$$

$$= \beta E_t \pi_{t+1} + \kappa x_t^e + e_t$$

where $e$ represents an inflation or cost shock arising from fluctuations in markups.
- Then

$$\pi_t = \kappa E_t \sum_{i=0}^{\infty} \beta^i x_{t+i} + E_t \sum_{i=0}^{\infty} \beta^i e_{t+i}$$

- Cannot keep both $x^e$ and $\pi$ equal to zero: trade-offs must be made.
Sources of cost shocks

- Wedge between flexible-price output and efficient level of output.
  - Stochastic markups in wages and/or prices.
  - Time varying taxes.

- Endogenous sources
  - Sticky nominal wages.
  - Cost channel.
  - Exchange rate movements, imperfect pass through.
Policy regime: discretion vs. commitment

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.

- Minimize

\[-\frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2)\]

subject to

\[\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.\]

- Notation – let \(x\) denote \(x^e\), i.e., efficiency gap.

- Notice the Euler condition imposes no constraint – use it to solve for \(i_t\) once optimal \(\pi_t\) and \(x_t\) have been determined.

- This would not be case if central bank cares about interest rate volatility or the ZLB binds.
Discretion

The policy problem

- Policy maker takes expectations as given – leads to period by period maximization.
- Problem is to pick $\pi_t$ and $x_t$ to minimize

$$\frac{1}{2} (\pi_t^2 + \lambda x_t^2) + \psi_t (\pi_t - \beta E_t \pi_{t+1} - \kappa x_t - e_t)$$

taking $E_t \pi_{t+1}$ as given.

- The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (2)$$

$$\lambda x_t - \kappa \psi_t = 0. \quad (3)$$

- Eliminating $\psi_t$, $\lambda x_t + \kappa \pi_t = 0$ – this is a targeting rule.
Discretion

Behavior of the interest rate

- From the IS equation,

\[ i_t = E_t \pi_{t+1} + \sigma (E_t x_{t+1} - x_t) + r^n_t. \]

- Using solution,

\[ i_t = \left[ A \rho - \sigma \left( \frac{\kappa}{\lambda} \right) (\rho - 1) \right] e_t + r^n_t = B e_t + r^n_t. \]

- Shifts in natural rate of interest \( r^n \) are fully offset.
- So optimal policy involves \( i \) responding to shocks, but adopting a rule of the form

\[ i_t = B e_t + r^n_t \]

does not ensure a unique rational expectations equilibrium.
Precommitment

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.
- Under optimal commitment, central bank at time $t$ chooses both current and expected future values of inflation and the output gap.
- Minimize
  \[-\frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right)\]
  subject to
  \[\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.\]
- Notice the Euler condition imposes no constraint – use it to solve for $i_t$ once optimal $\pi_t$ and $x_t$ have been determined.
Optimal precommitment

- The central bank’s problem is to pick $\pi_{t+i}$ and $x_{t+i}$ to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{2} (\pi_{t+i}^2 + \lambda x_{t+i}^2) + \psi_{t+i} (\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}) \right].$$

- The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (4)$$

$$E_t (\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0 \quad i \geq 1 \quad (5)$$

$$E_t (\lambda x_{t+i} - \kappa \psi_{t+i}) = 0 \quad i \geq 0. \quad (6)$$

- Dynamic inconsistency – at time $t$, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = -\left( E_t \psi_{t+1} - \psi_t \right)$. When $t + 1$ arrives, a central bank that reoptimizes will again obtains $\pi_{t+1} = -\psi_{t+1}$ – the first order condition (4) updated to $t + 1$ will reappear.
An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (5) and (6) for all periods, including the current period so that

\[ \pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} = 0 \quad i \geq 0 \]

\[ \lambda x_{t+i} - \kappa \psi_{t+i} = 0 \quad i \geq 0. \]

Woodford (2003) has labeled this the “timeless perspective” approach to precommitment.
Timeless precommitment

- Under the timeless perspective optimal commitment policy, inflation and the output gap satisfy

\[ \pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right) (x_{t+i} - x_{t+i-1}) \]  \hspace{1cm} (7)

for all \( i \geq 0 \).

- Even if \( e \) is serially uncorrelated so that there is no natural source of persistence, the optimal commitment policy introduces inertia into the output gap and inflation processes.

- This commitment to inertia implies that the central bank’s actions at date \( t \) allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap.
Illustrating commitment versus discretion in the simple NK model
Improved trade-off under commitment

- The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion.
- Consider a positive inflation shock, \( e > 0 \).
- A given change in current inflation can be achieved with a smaller fall in \( x \) if expected future inflation can be reduced:

\[
\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t
\]

- Requires a commitment to future deflation.
- By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in \( \mathbb{E}_t \pi_{t+1} \) at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.
The case of a persistent cost shock
Illustrating policy effects in S-W model: inflation shock
Dynamic Stochastic General Equilibrium (DSGE) models: building a model to understand the facts

1. Consumption Euler equation: external habit persistence and non-separable utility with an exogenous wedge between return on household assets and the policy interest rate.

2. Investment: adjustment costs, capital utilization costs, and an investment specific technology shock.

3. Aggregate production function: labor, capital, and a productivity shock.


6. Monetary policy: an interest rate rule depending on inflation, an output gap and its change, the lagged policy rate, and an exogenous shock.
Summing up – issues to keep in mind

- Policy analysis – what are good shocks, what are bad shocks?
  - Identification is difficult.

- Inflation and wage dynamics critical for policy trade offs
  - Empirical models generally assume some form of indexation not observed in micro data
  - Focus is on deviations of inflation from trend rather than on trend inflation.
Trend inflation

- Basic NK Phillips curve linearized around a zero steady-state rate of inflation.
- But going to the data, no country has an average rate of inflation of zero. Need to deal with non-zero trend inflation.
- Two issue:
  - How does a non-zero trend affect the NKPC?
  - If the ZLB is accounted for, is optimal trend inflation positive?
  - Are any of the basic model’s conclusions altered with non-zero trend inflation?
Trend and cyclical inflation

Figure: U.S. inflation (pce), trend inflation (hp), and cyclical inflation
Trend inflation

- Standard NKPCs in DSGE models add ad hoc assumptions about indexation.
- Example: Justiniano, Primiceri, and Tambalotti (2011) assume non-optimizing firms set
  \[ P_t(j) = P_{t-1}(j) \pi_{t-1}^{l_p} \pi^{1-l_p} \]
  where \( \pi \) is steady-state inflation (same for households setting wages).
- Estimated values of \( \iota_p, \iota_w \approx 0 \) but this still imposes indexation to steady-state inflation.
- Costly relative price and wage dispersion depends on
  \[ [\pi_t - \iota_p \pi_{t-1} - (1 - \iota_p) \pi]^2 \approx (\pi_t - \pi)^2 \]
- Trend inflation doesn’t matter for welfare.
If Calvo model (and micro evidence) taken seriously, one can’t rely on indexation.


Nonzero trend inflation means long-run Phillips curve is not vertical under price adjustment models such as Calvo.
Calvo with trend inflation: Ascari and Sbordone (2013)

- First order condition for optimal relative price to set:

\[ p_t^* = \mu \frac{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} C_{t+i} \Pi_{t,t+i}^\theta MC_{t+i}}{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} C_{t+i} \Pi_{t,t+i}^{\theta-1}} \]

where \( \Pi_{t,t+i} \) is cumulative inflation from \( t \) to \( t+i \):

\[ \Pi_{t,t+i} = \left\{ \begin{array}{ll}
\left( \frac{P_{t+1}}{P_t} \right) \times \cdots \times \left( \frac{P_{t+i}}{P_{t+i-1}} \right) = \left( \frac{P_{t+i}}{P_t} \right) & \text{for } i = 0 \\
\left( \frac{P_{t+j}}{P_t} \right) & \text{for } j = 2, 3, \ldots
\end{array} \right. \]

- Higher future expected inflation makes firms put more weight on expected future marginal costs; firms become more forward looking.
Calvo with trend inflation

- In the steady state,

\[ p^* = \mu \frac{E_t \sum_{i=0}^{\infty} (\beta \omega^i \bar{\Pi}^\theta)^i}{E_t \sum_{i=0}^{\infty} (\beta \omega^i \bar{\Pi}^{\theta-1})^i} MC \]

where \( \bar{\Pi} \) is the gross trend inflation rate.

- If \( \bar{\Pi} > 1 \) (positive inflation), for terms in \( P^* / P \) to converge requires

\[ \bar{\Pi} < \max \left[ \left( \frac{1}{\omega \beta} \right)^{\frac{1}{\theta-1}}, \left( \frac{1}{\omega \beta} \right)^{\frac{1}{\theta}} \right]. \]

- For standard calibrations (\( \omega = 0.75, \beta = 0.99, \theta = 10 \)), annual inflation must be less than 14.1%. 

Calvo with trend inflation

- Linearizing the FOC for the optimal reset price,

\[
(1 + \omega \eta^{-1}) p_t^* = (1 + \eta^{-1}) (1 - \gamma_2) \sum_{j=0}^{\infty} \gamma_2^j E_t x_{t+j} + E_t \sum_{j=0}^{\infty} (\gamma_2^j - \gamma_1^j) (g_{t+j} - i_{t+j-1}) + E_t \sum_{j=0}^{\infty} \left\{ \gamma_2^j [1 + \omega (1 + \eta^{-1})] - \gamma_1^j \omega \right\} \pi_{t+j}
\]

where \( \gamma_2 = \omega R^{-1} g \bar{\Pi} \bar{\Theta} \), \( \gamma_2 = \gamma_1 \bar{\Pi}^{1+\Theta} \eta \), \( g \) is the growth rate of output, \( x \) is the output gap, and \( i \) is the nominal interest rate.

- When combined with equation for evolution of aggregate price level, one does not get the simple NKPC.

- Current marginal cost and expected \( t + 1 \) inflation still appear, but so do expected inflation rates further in the future and expected future discount rates.
Calvo with trend inflation

- In zero inflation case, all firms charge the same price in the steady state: \( P^* = P \) and \( \mu MC = 1 \).
- With nonzero trend inflation,

\[
p^* = \mu \frac{E_t \sum_{i=0}^{\infty} (\beta \omega^i \bar{\Pi}^\theta)^i MC}{E_t \sum_{i=0}^{\infty} (\beta \omega^i \bar{\Pi}^{\theta-1})^i} = \mu MC \left[ \frac{1 - \beta \theta \bar{\Pi}^{\theta-1}}{1 - \beta \theta \bar{\Pi}^\theta} \right] \neq 1.
\]

- Ascari and Sbordone show that in steady state,

\[
\mu MC = \left( \frac{1 - \omega \bar{\Pi}^{\theta-1}}{1 - \omega} \right)^{\frac{1}{1-\omega}} \left( \frac{1 - \omega \beta \bar{\Pi}^\theta}{1 - \omega \beta \bar{\Pi}^{\theta-1}} \right).
\]

- If steady state inflation is zero (\( \bar{\Pi} = 1 \)), \( \mu MC = 1 \). Has implications for steady-state output.
Calvo with trend inflation

- Coibion and Gorodnichenko (AER 2011) allow indexation to trend inflation:
  \[ P_t^{1-\theta} = (1 - \omega) \left( P_t^* \right)^{1-\theta} + \omega P_{t-1}^{1-\theta} \Pi^\delta(1-\theta) \]
  where \( \delta \) is the degree of indexation.

- "Standard" case in DSGE models is \( \delta = 1 \).

- Steady-state output gap is not equal to zero if \( \delta < 1 \). Coibion, Gorodnichenko and Wieland show that
  \[ \left( \frac{\bar{Y}}{\bar{Y}^f} \right)^{1+\eta} = \frac{1 - \omega \beta^{-1} \Pi^{1-\delta}(1+\eta)}{1 - \omega \beta^{-1} \Pi^{1-\delta}(\theta-1)} \left( \frac{1 - \omega}{1 - \omega \Pi^{1-\delta}(\theta-1)} \right)^{\frac{1+\eta\theta}{\theta-1}}. \]

- Higher trend inflation yields higher average markup and so a longer distortion due to imperfect competition.
Nonlinear effects and trend inflation (Levin and Yun JME 2007)

Fig. 1. Output costs of steady-state inflation (relative to zero-inflation steady state). Note: this figure depicts the output cost of steady-state inflation in the exogenous contract duration model, where the inflation rate is expressed at annual rates in percentage points and the output cost is expressed as a percent deviation from the level of output in the zero-inflation steady state. The solid line corresponds to the exact non-linear solution of the model, while the dashed line shows the implications of the log-linearized New Keynesian Phillips curve.
Trend inflation and Calvo parameter

Fig. 3. Frequency of price adjustment in the endogenous contract duration model. Note: the dashed line denotes the relationship between steady-state inflation and the frequency of price adjustment in the endogenous contract duration model, where inflation is expressed at an annual rate in percentage points, and the adjustment frequency indicates the percentage of firms that modify their prices in a given quarter. Each symbol denotes the empirical frequency of price adjustment for the specified economy and sample period.
Hyperinflation and the Calvo parameter

Note: Frequencies for Argentina are $1 - e^{-\lambda}$, where $\lambda$ is the simple pooled estimator. Data for the Euro area is from Alvarez et al. (2006), for the US from Bils and Klenow (2004), Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008), for Mexico from Gagnon (2009), for Israel from Baharad and Eden (2004), for Norway Wulfsberg (2010), Lach and Tisdon (1992), for Poland from Konieczny and Skrzypacz (2005), and for Brazil Barros et al. (2009). Logarithmic scale for the horizontal axis.
Endogenous frequency

- Levin and Yun (JME 2007) allow firms to optimally choose frequency of price adjustment in face of menu costs.
- Natural rate hypothesis reemerges as trend inflation rises.
- Other implications of trend inflation:
  - Taylor Principle – minimum response to inflation needed for determinacy increases with trend inflation.
  - Ascari and Sbordone show classic Taylor Rule results in indeterminacy for trend inflation greater than 4%.
Sticky wages ala Calvo

- Erceg, Henderson, and Levin (JME 2000) have employed the Calvo specification to incorporate sticky wages and sticky prices into an optimizing framework.
- The goods market side of their model is identical in structure to the one developed earlier.
- In the labor market, however, they assume individual households supply differentiated labor services; firms combine these labor services to produce output.
Households, labor supply, and wage setting

- Households supply a differentiated labor service; these are imperfect substitutes in production.
- Labor services are purchased by an “employment agency” that aggregates labor into a labor index $L_t$ given by

$$L_t = \left[ \int_0^1 H_t(h) \frac{\theta_n^{-1}}{\theta_n} \, dh \right]^{\frac{\theta_n}{\theta_n-1}}, \quad \theta_n > 1.$$  

- The labor index $L_t$ is used by goods-producing firms in production (i.e., firm $j$ employs $L_{jt}$ of labor aggregate).
Households, labor supply, and wage setting

- The problem of the employment agency:

\[
\min \int_0^1 W_t(h)H_t(h)\,dh \text{ subject to } \left[ \int_0^1 H_t(h) \frac{\theta_n-1}{\theta_n} \,dh \right]^{\frac{\theta_n}{\theta_n-1}} \geq L_t.
\]

- Let \( \phi_t \) denote the Lagrangian multiplier on the constraint. Then the agency’s first order condition for type \( k \) labor takes the form

\[
W_t(k) - \phi_t \left[ \int_0^1 H_t(h) \frac{\theta_n-1}{\theta_n} \,dh \right]^{\frac{1}{\theta_n-1}} H_t(k)^{-\frac{1}{\theta_n}} = 0
\]

or

\[
H_t(k) = \left[ \frac{W_t(k)}{W_t} \right]^{-\theta_n} L_t
\]

where

\[
W_t \equiv \phi_t = \left[ \int_0^1 W_t(h)^{1-\theta_n} \,dh \right]^{\frac{1}{1-\theta_n}}.
\]
The budget constraint of household $h$ is given by

$$W_t(h)H_t(h) + M_{t-1}(h) + (1 + i_{t-1})B_{t-1}(h) + \Pi_t(h) + T_t(h)$$

$$\geq P_tC_t(h) + M_t(h) + B_t(h).$$

In real terms,

$$\frac{W_t(h)}{P_t}H_t(h) + \frac{m_{t-1}(h)}{1 + \pi_t} + \left(\frac{1 + i_{t-1}}{1 + \pi_t}\right)b_{t-1}(h) + \frac{\Pi_t(h) + T_t(h)}{P_t}$$

$$\geq C_t(h) + m_t(h) + b_t(h),$$

where $m_t = M_t/P_t$, $b_t = B_t/P_t$. 
Households

- Household maximizes

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}(h)^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{H_{t+i}(h)^{1+\eta}}{1+\eta} \right].
\]  

- Utility is maximized subject to the budget constraint, the demand by the employment agency for labor services of type \( h \), and a restriction on the household’s ability to adjust its wage.

- Assume complete contingent claims market for consumption so that \( C_t(h) = C_t(j) = C_t \) for all \( h, j \).
Households and wage setting

- A fraction $1 - \omega_w$ of households randomly selected each period and allowed to adjust their wage.
- Choice of wage depends on demand for household’s labor type and $\omega_w$.
- If household allowed to adjust, it picks $W_t^*$ (all adjusting households pick same wage) to

$$\max E_t \sum_{i=0}^{\infty} \beta^i \omega^i_w \left[ -\chi \frac{1}{1+\eta} \left\{ \left( \frac{W_{t+i}^*}{W_{t+i}} \right)^{-\theta_n} L_{t+i} \right\}^{1+\eta} + \lambda_{t+i} \left( \frac{W_t^*}{P_{t+i}} \right) \left( \frac{W_{t+i}^*}{W_{t+i}} \right)^{-\theta_n} L_{t+i} \right]$$

where first term is disutility of hours and second term is value of labor income, where $\lambda_t$ is the marginal utility of consumption.
Households and wage setting

- First order condition is

\[ E_t \sum_{i=0}^{\infty} \beta^i \omega^i_w \left[ \theta_n \chi (W_t^*)^{-\theta_n(1+\eta)} (\frac{1}{W_{t+i}})^{-(1+\eta)\theta_n} L_{t+i}^{1+\eta} (1 - \theta_n) \lambda_{t+i} \left( \frac{1}{P_{t+i}} \right) \left( \frac{1}{W_{t+i}} \right)^{-\theta_n} (W_t^*)^{-\theta_n} L_{t+i}^{-\theta_n} \right] = 0 \]

- Simplifying, letting \( MRS = \chi H^\eta / C^{-\sigma} \),

\[ \frac{W_t^*}{W_t} = \left( \frac{\theta_n}{\theta_n - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \beta^i \omega^i_w \lambda_{t+i} MRS_{t+i} \left( \frac{W_t}{W_{t+i}} \right)^{-\theta_n} L_{t+i}}{E_t \sum_{i=0}^{\infty} \beta^i \omega^i_w \lambda_{t+i} \left( \frac{W_t}{P_{t+i}} \right) \left( \frac{W_t}{W_{t+i}} \right)^{-\theta_n} L_{t+i}} \]

- Aggregate wage evolves as

\[ W_t^{1-\theta_n} = (1 - \omega_w) (W_t^*)^{1-\theta_n} + \omega_n W_t^{1-\theta_n} \]
Households and wage setting: flexible wage case

- With flexible wages, \( \omega_w = 0 \) and

\[
\frac{W_t^*}{W_t} = \left( \frac{\theta_n}{\theta_n - 1} \right) \frac{\lambda_t MRS_t L_t}{\lambda_t \left( \frac{W_t}{P_t} \right) L_t} = \mu^w \frac{MRS_t}{W_t/P_t}
\]

where

\[
\mu^w \equiv \left( \frac{\theta_n}{\theta_n - 1} \right) \geq 1
\]

is the wage markup.

- Since all households set the same wage, \( W_t^* = W_t \) and

\[
\frac{W_t}{P_t} = \mu^w MRS_t \geq MRS_t.
\]
Labor market clearing

- Total hours of labor worked is

\[ H_t = \int H_t(h) dh = L_t \int \left[ \frac{W_t(h)}{W_t} \right]^{-\theta_n} \, dh = L_t \Delta_t^w, \]

where

\[ \Delta_t^w = \int \left[ \frac{W_t(h)}{W_t} \right]^{-\theta_n} \, dh \geq 1 \]

is a measure of wage dispersion.

- Therefore,

\[ (\Delta_t^w)^{-1} H_t = L_t \leq H_t. \]

- Since \( \Delta_t^w \geq 1 \), wage dispersion means more hours need to be worked to achieve a given aggregate level of labor input \( L_t \).
Sticky wages and prices

Implications

- The model of inflation adjustment based on the Calvo specification implied that inflation depended on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equaled the gap between the real wage and the marginal product of labor ($mpl_t$). Thus, letting $\omega_t$ denote the real wage,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (\omega_t - mpl_t).$$

(9)

- With flexible wages, workers were always on their labor supply curves; despite price stickiness, nominal wages could adjust to ensure the real wage equaled the marginal rate of substitution between leisure and consumption ($mrs$).
Policy objectives with sticky prices and wages

- When both wages and prices display stickiness, can’t keep $x = 0$ when there are productivity shocks
  - If price level is stabilized, sticky wages prevent the real wage from adjusting, so inefficient output variability results.
  - Similarly, stabilizing the wage level still leaves prices sticky so real wages cannot jump.

- Welfare costs arise from price dispersion and from wage dispersion.
- Approximation to the welfare of representative agent is now equal to the expected present discounted value of

$$L_t = \frac{1}{2} \left( \lambda_p \pi_t^2 + \lambda_w \pi_{wt}^2 + \lambda_x x_t^2 \right)$$

where $\pi_w$ is wage inflation, $\lambda_p + \lambda_w = 1$, and

$$\frac{\lambda_w}{\lambda_p} \propto \frac{\kappa_p}{\kappa_w}.$$  

- Place more weight on stabilizing wage inflation if wages are stickier than prices.
Is the ZLB a constraint?
The zero lower bound

- Causes
  - Non-fundamentals-based liquidity traps
    - Expectationally driven
  - Fundamentals-based liquidity traps (will address later)
    - Negative shock to the equilibrium (Wicksellian) real rate of interest

- Potential constraint on monetary policy
The Taylor principle and liquidity traps

Non-fundamentals-based liquidity traps

- Benhabib, Schmitt-Grohé, and Uribe (JET 2001, JPE 2002) have argued that deflationary paths cannot be ruled out if policy satisfies the Taylor Principle.
  - Multiple steady-state equilibria in forward-looking expectational models.
  - The argument is based on the observation that the nominal rate of interest cannot fall below zero.

- Explosive deflations would eventually force the nominal interest rate to zero, but the nominal rate is then prevented from falling further.

- They argue that simple and seemingly reasonable monetary policy rules that follow the Taylor Principle in changing the nominal interest rate more than one-for-one is response to changes in inflation can introduce the possibility the economy will be caught in a deflationary liquidity trap.
The Taylor principle and liquidity traps

A simple example
The zero lower bound
Fundamentals-based liquidity traps

- Optimal policy in a basic new Keynesian model implies the policy interest rate moves one-for-one with the equilibrium real (natural, Wicksellian) interest rate:

\[ i_t = r^n_t + \pi^* + \delta (\pi_t - \pi^*) \geq 0. \]

- Negative shock to \( r^n_t \) could require \( i_t \) to be negative – ZLB prevents this.

- Deflationary trap:

\[ r_t = i_t - E_t \pi_{t+1} = -E_t \pi_{t+1} \]
Demand shock in the face of the ZLB

Ignoring ZLB

With ZLB constraint
Is the ZLB a constraint?

- Is the ZLB a constraint on monetary policy?
- Conventional model based on an expectational IS relationship:

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r^n_t) \]

- Interest rates – both current and expected future matter:

\[ x_t = - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) - \left( \frac{1}{\sigma} \right) E_t \sum_{i=1}^{\infty} (i_{t+i} - \pi_{t+1+i}) \]

\[ + \left( \frac{1}{\sigma} \right) E_t \sum_{i=0}^{\infty} r^n_{t+i}, \]

- Reflects a narrow view of the transmission mechanism – no role for quantitative easing or credit easing policies.
Conventional instruments at the ZLB

- Even at the ZLB, policy has the potential to influence real spending if it can affect expectations of future real interest rates.
  - Eggertsson and Woodford (BPEA 2003)
- If $i_t = 0$ and is expected to remain at zero until $t + T$, then
  \[
  x_t = \left(\frac{1}{\sigma}\right) \sum_{i=0}^{T} E_t \pi_{t+1+i} - \left(\frac{1}{\sigma}\right) E_t \sum_{i=T+1}^{\infty} (i_{t+i} - \pi_{t+1+i})
  \]
  \[
  + \left(\frac{1}{\sigma}\right) E_t \sum_{i=0}^{\infty} r^n_{t+i}.
  \]
  - Raising expected future inflation or committing to lower future nominal rates can stimulate current spending.
- Cost of ZLB low in linear models when central bank is credible (Adam and Billi JMCB 2006, Nakov IJCB 2008)
Promising future inflation

- Optimal policy at the ZLB involves promising future inflation. This is done by keeping interest rates low even when the ZLB no longer binds.
- Central banks have been reluctant to promise higher future inflation.
- Contrasts with recommendations made to the Bank of Japan:
- Communicating clearly the conditional nature of future interest rate paths cited as concern.
- Commitment requires promises be fulfilled – have to deliver higher future inflation.
Promising future inflation

Four period example based on Bodenstein, Hebden, and Nunes JME 2012

- Suppose economy is described by
  \[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r^n_t) \]
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

- Policy objective is to minimize
  \[ \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) \]

- In period 1, the economy is at the zero lower bound: \( r^n_1 < 0 \) and \( i_1 = 0 \).
- In periods 2, 3 and 4, the economy is out of the ZLB so that \( x_{t+i} = \pi_{t+i} = 0 \) for \( i \geq 2 \) is feasible. Assume that \( \pi_{t+4} = x_{t+4} = 0 \). (This is what makes this a simple example.)
- The issue is what happens in periods 2 and 3 and how this affects the output gap and inflation in period 1.
Promising future inflation

- Even though the ZLB no longer binds in period 2, the optimal policy in period 1 is to promise to keep rates low and to deliver higher inflation in period 2.
- This promise of higher inflation reduces the current real interest rate and is expansionary, even though $i_1 = 0$.
- Under discretion, optimal policy is to bring inflation and the output gap back to zero once shock ends.
- Costs high under discretion (Adam and Billi JME 2007).
Promising future inflation: example
Path of interest rate relative to equilibrium real rate under discretion and full commitment

Figure: The nominal rate under discretion and full commitment when ZLB is binding only in period 1 (figure shows $i_r$)
Promising future inflation: example
Path of inflation and the output gap under discretion and full commitment
Promising future inflation: example

- A lack of credibility makes it harder to stabilize in the face of the ZLB
  - Bodenstein, Hebden, and Nunes (JME 2012).
  - With less credibility, future promises must be more extreme;
- “Promising low interest rates for an extended period of time” may be a sign of a lack of credibility;
- Promising low interest rates in the future and also promising no inflation is an inconsistent policy.
- When current credibility is low, the central bank has to promise lower future rates, but that means the cost of fulfill promises is higher, raising the incentive to deviate.
  - Central banks with low credibility face the greatest temptation to break their promises.
Simple example: Discretionary equilibrium

- Economy hit by $r^n_t = r^{bad} < 0$.
  - With probability $q$, exits the ZLB in following period. With probability $1 - q$, $r^n_{t+1} = r^{bad}$.
  - Once economy exits, optimal policy under discretion sets $i = r^n$ so that $\pi = x = 0$.

- Equilibrium given by solution to

\[
\pi^{zlb} = \beta (1 - q) \pi^{zlb} + \kappa x^{zlb}
\]

\[
x^{zlb} = (1 - q) x^{zlb} + \left( \frac{1}{\sigma} \right) \left[ (1 - q) \pi^{zlb} + r^{bad} \right]
\]
Figure: Equilibrium at the ZLB; \( q \) is the probability of exiting in following period.
Figure: Equilibrium at the ZLB; effects of a fall in $q$ (a rise in probability of remaining at the ZLB).
Persistent negative supply shock

- Shifts NKPC upwards by increasing marginal costs.
  - Expansionary at the ZLB.
- But is it?
  - J. Wieland (2014)
- There is also a type 2 (Bruan, Körber, and Waki 2012) equilibrium with IS flatter than NKPC.
Promising future inflation

- Credibly promising to keep future interest rates at zero is a powerful tool:
  - Carlstrom, Fuerst and Paustian (2012), Braun, Körber, and Waki (2012)

- Consider simple model at ZLB:

\[
\begin{align*}
\chi_t &= E_t \chi_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa \chi_t.
\end{align*}
\]

\[
i_{t+i} = \begin{cases} 
  i^* & \text{for } i = 1, \ldots, T \\
  \phi_\pi \pi_t + \phi_x \chi_t & \text{for } i = T + 1, \ldots.
\end{cases}
\]

- With no inflation shocks, \( \pi_{t+i} = \chi_{t+i} = 0 \) for \( i = T + 1, \ldots \).
Promising future inflation

- Equilibrium for \( i = 1, \ldots, T \) is solution to

\[
\left( \frac{1}{\kappa} \right) (\pi_t - \beta E_t \pi_{t+1}) = \left( \frac{1}{\kappa} \right) (E_t \pi_{t+1} - \beta E_t \pi_{t+2}) - \left( \frac{1}{\sigma} \right) (i^* - E_t \pi_{t+1})
\]

- Or

\[
\pi_t = - \left( \frac{\kappa}{\sigma} \right) i^* + \left( 1 + \beta + \frac{\kappa}{\sigma} \right) E_t \pi_{t+1} - \beta E_t \pi_{t+2}
\]

with terminal condition \( \pi_{t+T+1} = 0 \).
Promising future inflation

- Consider a perfect foresight equilibrium.
- Solve backwards:

\[
\pi_{t+T} = -\left(\frac{\kappa}{\sigma}\right)i^* + \left(1 + \beta + \frac{\kappa}{\sigma}\right)\pi_{t+T+1} - \beta\pi_{t+T+2} = -\left(\frac{\kappa}{\sigma}\right)i^*
\]
\[
\pi_{t+T-1} = -\left(\frac{\kappa}{\sigma}\right)i^* + \left(1 + \beta + \frac{\kappa}{\sigma}\right)\pi_{t+T} = -\left(2 + \beta + \frac{\kappa}{\sigma}\right)\left(\frac{\kappa}{\sigma}\right)i^*
\]
\[
\pi_{t+T-2} = -\left(\frac{\kappa}{\sigma}\right)i^* + \left(1 + \beta + \frac{\kappa}{\sigma}\right)\pi_{t+T-1} - \beta\pi_{t+T}
\]
- For \(\pi_{t+T-s}\), process is explosive as \(s\) gets larger.
Promising future inflation
Promising future inflation

output gap

inflation

Taylor rule w/o ZLB
Taylor rule w/ ZLB
Extended zero rate
Inflation expectations: instrument, anchor, or automatic stabilizer?

- Optimal commitment policies use expectations of future inflation as a policy instrument.
- Are their advantages of simply anchoring expectations?
- Suppose

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - \pi_{t+1}^e - r_n^t) \]

\[ \pi_t - \pi^T = \beta \left( \pi_{t+1}^e - \pi^T \right) + \kappa x_t + u_t \]

\[ i_t = r_n^t + \pi^T + \phi_{\pi} \left( \pi_t - \pi^T \right) + \phi_x x_t. \]

\[ \pi_{t+1} = (1 - \delta) E_t \pi_{t+1} + \delta \pi^T \]

- If \( \delta = 1 \), expectations completely anchored; if \( \delta = 0 \), expectations are rational.
- Evaluate outcomes using \( \sigma_{\pi}^2 + \lambda \sigma_{\dot{x}}^2 \) as function of \( \delta \).
Figure: Loss as a function of $\delta$, where $\delta = 0$ is rational expectations and $\delta = 1$ is completely anchored expectations.
Figure: Output gap and inflation at the ZLB under rational expectations and anchored expectations as a function of the probability of exiting the ZLB.
Replacing inflation targeting
Price level targeting and the ZLB

- Optimal policy at the ZLB can be implemented via a time-varying price-level target (Eggertsson and Woodford 2003).
- Intuition – under commitment, central bank has many tools even if current policy rate at zero.
  - Can promise future low interest rates and a boom.
  - This generates expectations of future inflation which raises current inflation and lowers current real interest rate.
- Promise to generate inflation to achieve price-level target.
Price level targeting

- Vestin (JME 2006) shows price level targeting can replicate the timeless precommitment solution if the central bank is assigned the loss function $p_t^2 + \lambda_{PL}x_t^2$ in an environment of discretion.

- Under timeless precommitment,

$$\pi_t = (1 - L)p_t = \left(\frac{\lambda}{\kappa}\right) (x_t - x_{t-1}) \Rightarrow p_t = \left(\frac{\lambda}{\kappa}\right) x_t$$

- Price level targeting makes inflation expectations act as an automatic stabilizer.

- Walsh (AER 2003) adds lagged inflation to the inflation adjustment equation and shows that the advantages of price level targeting over inflation targeting decline as the weight on lagged inflation increases.
Work on price level targeting illustrates the importance of forward-looking expectations.

A positive cost shock that raised the price level would require a deflation to bring the price level back on target, and this deflation would be costly.

However, an optimal commitment policy that focuses on output and inflation stability does induce a deflation after a positive cost shock.
Price level targeting

Figure: Price level target path and actual core PCE for the U.S.
Price level targeting

- Under timeless precommitment,

\[ \pi_t = - \left( \frac{\lambda}{\kappa} \right) (x_t - x_{t-1}) \]

or

\[ \pi_t = (1 - L)p_t = - \left( \frac{\lambda}{\kappa} \right) (x_t - x_{t-1}) \Rightarrow p_t = - \left( \frac{\lambda}{\kappa} \right) x_t. \]

- Central bank stabilizes price level.
Price level stabilization under commitment

Output Gap

Inflation

Nominal interest rate

Cost Shock

Taylor Rule
Optimal Discretion
Full Commitment Optimal Policy
Vestin (JME 2006) shows price level targeting can replicate the timeless precommitment solution if the central bank is assigned the loss function $p_t^2 + \lambda_{PL} x_t^2$.


Policy under discretion more complex – choices today affect expectations of the future via endogenous state variable ($p_{t-1}$).
Reforming IT: Using expectations as automatic stabilizers under PLT

- Central banks may find it easier to commit to objectives than to future policy actions.
- Distorting objectives in a discretionary environment can improve outcomes (Walsh AER 1995)

### Outcomes to shocks under discretion

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Advantages of PLT require that expectations act as automatic stabilizers.

- Raises issues of credibility and learning

Switching policy regimes in a crisis risks gains in credibility achieved by inflation targeters.

- When adopted, the choice of price index, the underlying trend inflation rate, and the speed with which deviations from target path are expected to be reversed are all important.

Walsh (AER 2003) adds lagged inflation to the inflation adjustment equation and shows that the advantages of price level targeting over inflation targeting decline as the weight on lagged inflation increases.
Structural form of NKPC matters
Walsh (AER 2003)
Other considerations

- Advantages of PLT or nominal income targeting require that expectations act as automatic stabilizers.
  - Raises issues of credibility and learning
  - Is it easier to commit to a policy framework such as PLT/NIT than it is to a future contingent path for policy?

- Switching policy regimes in a crisis risks gains in credibility achieved by inflation targeters.
  - Optimal commitment means doing what you had previously promised to do, even if it is not the optimal thing to do at the moment.
Reforming inflation targeting
Raising the inflation target

- Raising the inflation target raises average nominal interest rates and makes hitting the ZLB less likely.
- Trade-off – steady-state loss of higher inflation against ability to improve stabilization.
- Schmitt-Grohe and Uribe (Handbook 2010), Billi (AEJ: Macro 2011), Coibion, Gorodnichenko, and Wieland (REStud 2012)
  - optimal $\pi$ still small when ZLB taken into account.
Objective defined in terms of nominal income:

\[ \frac{1}{2} E_t \sum \beta_i (y_t^n - y^{n*})^2 \]

where

\[ y_t^n = p_t + y_t \]

Problems with NIT

- Estimating trend for real growth or level of potential GDP.
- Most of U.S. NIT gap is due to real GDP shortfall. If permanent, why should inflation be higher?
Nominal income targeting

log nominal GDP and target path

Figure: Nominal income in the U.S. and target path
The zero lower bound: other solutions for getting out

- Svensson’s “foolproof way”
  - Depreciation as visible means of committing to a higher price level
- Fiscal policy: “fiscal policy must be seen not to be committed to... conventional prescriptions for good fiscal policy...”. (Sims 2000, p. 969, italics in original) – more on this later.
- Quantitative easing/credit easing policies – more on this next week.