Advanced Topics in Monetary Economics II

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Course outline

1. New Keynesian monetary policy models, nominal rigidities, and determinacy;
2. Optimal policy away from and at the zero lower bound;
3. Extensions to the open economy;
4. Uncertainty and monetary/fiscal interactions;
5. Labor frictions, unemployment and monetary policy;
The basic new Keynesian model

1. Households purchasing consumption goods and supplying labor services.
2. Monopolistic competition characterizing goods and labor market.
3. Sticky and staggered adjustment of prices and wages.
4. A nominal interest rate employed as the instrument of monetary policy.
5. Agents behave optimally given constraints they face.
Households

- Continuum of households indexed by $h$ whose utility is a function of a composite consumption good $C_t(h)$, real money balances $M_t(h)/P_t$, and leisure $1 - H_t(h)$, where $H_t(h)$ is the time (hours) devoted to market employment.

- Household preferences given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}(h)^{1-\sigma}}{1 - \sigma} + \frac{\gamma}{1 - b} \left( \frac{M_{t+i}(h)}{P_{t+i}} \right)^{1-b} - \chi_t \frac{N_{t+i}(h)^{1+\eta}}{1 + \eta} \right].$$

- The consumption aggregate that yields utility is defined as

$$C_t(h) = \left[ \int_0^1 c_{jt}(h)^{\frac{\theta_t}{\theta_t-1}} d\theta_t \right]^{\frac{\theta_t}{\theta_t-1}} \quad \theta_t > 1.$$

where $c_{jt}(h)$ is type $j$ good.

- The parameter $\theta$ governs the degree of imperfect competition in goods market.
Household demand for individual final goods

Regardless of the level of $C_t(h)$, always optimal to purchase the least cost combination of the individual goods to achieve $C_t(h)$:

$$\min_{\{c_{jt}\}} \int_0^1 p_{jt} c_{jt}(h) \, dj + \psi_t \left\{ C_t(h) - \left[ \int_0^1 c_{jt}(h) \frac{\theta_t-1}{\theta_t} \, dj \right]^{\frac{\theta_t}{\theta_t-1}} \right\}$$

The demand for good $j$ can be written as

$$c_{jt}(h) = \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta_t} C_t(h). \quad (1)$$
Households

- From the definition of the composite level of consumption, this implies

\[
C_t(h) = \left[ \int_0^1 c_{jt}(h) \frac{\theta_t - 1}{\theta_t} \, dj \right]^{\frac{\theta_t - 1}{\theta_t - 1}} = \left[ \int_0^1 \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta_t} C_t(h) \right]^{\frac{\theta_t - 1}{\theta_t - 1}} \, dj
\]

\[
= \left( \frac{1}{\psi_t} \right)^{-\theta_t} \left[ \int_0^1 p_{jt}^{1-\theta_t} \, dj \right]^{\frac{\theta_t - 1}{\theta_t - 1}} C_t(h).
\]

- Solving for \( \psi_t \),

\[
P_t \equiv \psi_t = \left[ \int_0^1 p_{jt}^{1-\theta_t} \, dj \right]^{\frac{1}{1-\theta_t}}.
\] (2)

- The Lagrangian multiplier is the appropriately aggregated price index for consumption and hence

\[
c_{jt}(h) = \left( \frac{p_{jt}}{P_t} \right)^{-\theta_t} C_t(h).
\]
Household’s other first order conditions

1. Euler condition:

\[ C_t(h)^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}(h)^{-\sigma} \]

2. Labor supply choice (with markup):

\[ \text{MRS} = \text{real wage:} \quad \frac{\chi_t N_t(h)^{\eta}}{C_t(h)^{-\sigma}} = \frac{1}{\mu_t^W} \frac{W_t}{P_t} \]

3. Demand for money:

\[ \text{MRS} = \text{cost of holding money:} \quad \frac{\gamma \left( \frac{M_{t+i}(h)}{P_{t+i}} \right)^{-b}}{C_t(h)^{-\sigma}} = \frac{R_t - 1}{R_t} \]
Firms

Continuum of firms of measure 1. Firms maximize profits, subject to three constraints:

1. The first is the production function summarizing the technology available for production. For simplicity, ignore capital, so output is a function solely of labor input $L_{jt}$ and an aggregate productivity disturbance $Z_t$:
   \[ c_{jt} = Z_t N_{jt}, \quad \mathbb{E}(Z_t) = 1. \]

2. The second constraint on the firm is the demand curve each faces:
   \[ c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t. \]

3. The third constraint is that each period some firms are not able to adjust their price. This nominal rigidity is what leads to the Keynesian features of the new Keynesian model.
Price adjustment: micro facts

- Bils and Klenow (JPE, 2004)
  - median duration between price changes is 4.3 months for items in the U.S. CPI with wide variation in frequency across different categories of goods and services.

- Klenow and Kryvtsov (QJE 2008)
  - price changes large on average, but a significant fraction of price changes are small.
  - variations in the size of price changes, rather than variation in the fraction of prices that change, can account for most of the variance of aggregate inflation.

- Nakamura and Steinsson (QJE 2008)
  - excluding sales increases median duration between changes from 4.5 months to 10 months.
  - the probability the price of an item changes (the hazard function) declines during the first few months after a change in price.
Price adjustment

Alternative approaches

- Time dependent pricing models
  - Calvo – fixed probability of adjusting each period.
  - Taylor’s model of fixed-length contracts

- State dependent price models
  - Dotsey, King, and Wolman (QJE 1999)
  - Dotsey and King (JME 2005)
  - Golosov and Lucas (JPE 2007)
  - Gertler and Leahy (JPE 2008)
  - Costain and Nakov (JMCB 2011)
Price adjustment: the basic TDP Calvo model

- Each period, the firms that adjust their price are randomly selected: a fraction $1 - \omega$ of all firms adjust while the remaining $\omega$ fraction do not adjust.
  - The parameter $\omega$ is a measure of the degree of nominal rigidity; a larger $\omega$ implies fewer firms adjust each period and the expected time between price changes is longer.

- For those firms who do adjust their price at time $t$, they do so to maximize the expected discounted value of current and future profits.
  - Profits at some future date $t + s$ are affected by the choice of price at time $t$ only if the firm has not received another opportunity to adjust between $t$ and $t + s$. The probability of this is $\omega^s$. 
Price adjustment
The firm's decision problem

- First consider the firm's cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written as
  \[
  \min_{N_t} \left( \frac{W_t}{P_t} \right) N_t + \phi_t \left( c_{jt} - Z_t N_{jt} \right).
  \]
  where $\phi_t$ is equal to the firm's real marginal cost. The first order condition implies
  \[
  \left( \frac{W_t}{P_t} \right) = \phi_t Z_t,
  \]
  or
  \[
  \phi_t = \frac{W_t}{P_t} Z_t
  \]
The firm’s pricing decision problem then involves picking $p_{jt}$ to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,i} \Pi \left( \frac{p_{jt}}{P_{t+i}}, \varphi_{t+i}, c_{t+i} \right),$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma}$ and profits in $t+i$ if price has not been changed are

$$\Pi(p_{jt}) = \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{c_{jt+i}} - \varphi_{t+i} c_{jt+i} \right]$$

$$= \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i}$$
Price adjustment

- All firms adjusting in period $t$ face the same problem, so all adjusting firms will set the same price.

- Let $p^*_t$ be the optimal price chosen by all firms adjusting at time $t$. The first order condition for the optimal choice of $p^*_t$ is

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{1}{p_{jt}} \right) \left( \frac{p^*_t}{P_{t+i}} \right)^{1-\theta} + \theta \varphi_{t+i} \left( \frac{1}{p^*_t} \right) \left( \frac{p^*_t}{P_{t+i}} \right)^{-\theta} \right] = 0.$$

- Using the definition of $\Delta_{i,t+i}$,

$$\left( \frac{p^*_t}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} / C_t)^{-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} / C_t)^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i}}.$$
Price adjustment

The first order condition can be expressed as

\[
\left( \frac{p_t^*}{P_t} \right) = \frac{H_t}{F_t}.
\]

where

\[
H_t = \left( \frac{\theta}{\theta - 1} \right) E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} / C_t)^{-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i}
\]

\[
= \left( \frac{\theta}{\theta - 1} \right) \varphi_t C_t + \omega \beta E_t H_{t+1}
\]

and

\[
F_t = E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} / C_t)^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i} = C_t + \omega \beta E_t F_{t+1}
\]
The case of flexible prices

- With flexible prices, all firms adjust each period, so $\omega = 0$.
- This implies

$$H_t = \left( \frac{\theta}{\theta - 1} \right) \varphi_t C_t; F_t = C_t$$

- So

$$\left( \frac{p_t^*}{P_t} \right) = \frac{H_t}{F_t} = \left( \frac{\theta}{\theta - 1} \right) \varphi_t = \mu \varphi_t.$$

- But with all firms setting the same price, $p_t^* = P_t$ and

$$\varphi_t = \frac{W_t / P_t}{Z_t} = \left( \frac{\theta - 1}{\theta} \right) = \frac{1}{\mu} < 1.$$
The case of sticky prices

- When prices are sticky ($\omega > 0$), the firm must take into account expected future marginal cost as well as current marginal cost when setting $p_t^*$.  
- The aggregate price index is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period $t$ and the average of the remaining fraction $\omega$ of all firms who set prices in earlier periods.  
- Because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that was prevailing in period $t - 1$.  
- Thus, the average price in period $t$ satisfies

$$P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}.$$  

(3)
State dependent price adjustment models

- SDP models allow price behavior to be influenced by an intensive and an extensive margin
  - after a large shock, those firms that adjust will make, on average, bigger adjustments (this is the intensive margin)
  - and more firms will adjust (this is the extensive margin).

- Golosov and Lucas (2007) emphasize that firms most likely to adjust are those furthest from their desired price.
  - This selection effect makes the aggregate price level more flexible than suggested by just looking at fraction of firms changing price.

- Basic intuition – highway road repairs.
  - Caplin and Spulber (QJE 1987)
Firm specific shocks

- Most models of price adjustment developed for use in macroeconomics have assumed that firms only face aggregate shocks.
  - This generally implies that all firms that do adjust their price choose the same new price as they all face the same (aggregate) shock.

- The Dotsey, King, and Wolman model features firm-specific shocks to the menu cost, but these shocks only influence whether a firm adjusts, not how much it changes prices.

- In contrast, Golosov and Lucas and Gertler and Leahy have emphasized the role of idiosyncratic shocks in influencing which firms adjust prices and in generating a distribution of prices across firms.
  - Head, Liu, Menzio, and Wright (JEEA 2012) build on search model to account for distribution of prices without nominal rigidities.
Assessing models of nominal price rigidities

- Klenow and Kryvtsov (QJE 2008)
  - No model fits all the micro facts
  - Calvo does surprisingly well except for the declining hazard rate
    - Variations in the size of price changes, rather than variation in the fraction of prices that change, can account for most of the variance of aggregate inflation.

- Carlsson and Nordström Skans (AEJ Macro 2012)
  - Uses Swedish firm-level data to compare models of staggered price adjustment with models of flexible prices but imperfect information (sticky information, rational inattention to macro factors)
  - Concludes Calvo explains data best.
Market clear and the role of price dispersion

- Output is

\[ Y_t = \int c_{jt} \, dj = Z_t \int N_{jt} \, dj = Z_t N_t \]

- But

\[ Y_t = \int c_{jt} \, dj = C_t \int \left( \frac{p_{jt}}{P_t} \right)^{-\theta} \, dj = C_t \Delta_t \Rightarrow C_t = \Delta_t^{-1} Y_t. \]

- Since \( \Delta_t \geq 1 \), price dispersion means more has to be produced to achieve a given level of \( C_t \).
  - More work effort required to produce a given \( C_t \).
Equilibrium conditions for basic sticky price NK model

Euler condition: \( C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{\sigma} \)

MRS = real wage: \( \frac{\chi_t N_t^\eta}{C_t^{-\sigma}} = \frac{1}{\mu_t^W} \frac{W_t}{P_t} \)

Marginal cost: \( \varphi_t = \frac{W_t / P_t}{Z_t} \)

Optimal price setting: \( \left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{H_t}{F_t} \)

where \( H_t = C_t^{1-\sigma} \varphi_t + \omega \beta E_t H_{t+1}; F_t = C_t^{1-\sigma} + \omega \beta E_t F_{t+1} \)

Aggregate price index: \( P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \)

Goods market clearing: \( C_t = \Delta_t^{-1} Y_t \)
Efficient and flex-price output w/ stochastic markups

- With flexible prices, $\Delta = 1$ $Y_t = C_t$, $Y_t = Z_t N_t$ and

$$\frac{\chi_t N^\eta}{C_t^{-\sigma}} = \frac{\chi_t (Y_t^f / Z_t)^{\eta}}{(Y_t^f)^{-\sigma}} = \left( \frac{1}{\mu^w_t} \right) \left( \frac{W_t}{P_t} \right) = \left( \frac{1}{\mu^w_t} \right) \varphi_t Z_t = \left( \frac{Z_t}{\mu^w_t \mu_t} \right)$$

$$Y_t^f = \left( \frac{Z_t^{1+\eta}}{\chi_t \mu^w_t \mu_t} \right)^{\frac{1}{\sigma+\eta}} \quad : \quad Y^f = Y = \left( \frac{1}{\chi \mu^w \mu} \right)^{\frac{1}{\sigma+\eta}}$$

- Efficient level of output requires $\Delta = 1$ and $\mu_t = \mu_t^w = 1$:

$$Y_t^e = \left( \frac{Z_t^{1+\eta}}{\chi_t} \right)^{\frac{1}{\sigma+\eta}} \quad : \quad Y^e = \left( \frac{1}{\chi} \right)^{\frac{1}{\sigma+\eta}} > Y^f$$
Benchmarks: efficient and flexible-price output

- Log linearizing efficient output around the efficient steady state level yields
  \[
  \hat{y}^e_t \equiv \log \left( \frac{Y^e_t}{Y^e_e} \right) = \left( \frac{1 + \eta}{\eta + \sigma} \right) \hat{z}_t - \left( \frac{1}{\sigma + \eta} \right) \hat{\chi}_t.
  \]

- Log linearizing flexible-price output around its steady state level yields
  \[
  \hat{y}^f_t \equiv \log \left( \frac{Y^f_t}{Y^f_f} \right) = \left( \frac{1 + \eta}{\eta + \sigma} \right) \hat{z}_t - \left( \frac{1}{\sigma + \eta} \right) (\hat{\chi}_t + \hat{\mu}_t + \hat{\mu}_w^w).
  \]

- Differences in steady-state levels:
  \[
  x^* \equiv \log \left( \frac{Y^f}{Y^e} \right) = \log \left( \frac{Y}{Y^e} \right) = - \left( \frac{1}{\sigma + \eta} \right) (\mu + \mu^w) \leq 0.
  \]
Obtaining a linearized version of the model

The demand side with sticky prices (and flexible wages)

- Linearize the Euler condition around steady-state:

\[ \hat{c}_t = E_t \hat{c}_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t \pi_{t+1}) \]

- Goods market equilibrium (no capital) is \( \hat{y}_t = \hat{c}_t \) (\( \hat{\Delta}_t = 0 \) to first order) so the Euler condition becomes

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t \pi_{t+1}) \]

- This is the expectational IS curve.
Demand and the output gap

- Output gap: gap between output and flex-price output:

\[ x_t = \log \left( \frac{Y_t}{Y^f_t} \right) = \log \left( \frac{Y_t}{Y^f_t} \right) - \log \left( \frac{Y^f_t}{Y^f} \right) = \hat{y}_t - \hat{y}^f_t. \]

- So from Euler condition,

\[ \hat{y}_t - \hat{y}^f_t = E_t \left( \hat{y}_{t+1} - \hat{y}^f_{t+1} \right) - \left( \frac{1}{\sigma} \right) \left( \hat{i}_t - E_t \pi_{t+1} \right) + \left( E_t \hat{y}^f_t - \hat{y}_t^f \right), \]

or

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) \left( r_t - r_t^f \right), \]

where \( r_t = \hat{i}_t - E_t \pi_{t+1} \) and \( r_t^f \equiv \sigma \left( E_t \hat{y}^f_{t+1} - \hat{y}_t^f \right). \]
Inflation adjustment

- Using the first order condition for price setting by firms and approximating around a zero average inflation, flexible-price equilibrium,

\[ \pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} (\hat{\phi}_t + \hat{\mu}_t) \]

where

\[ \tilde{\kappa} = \frac{(1 - \omega) [1 - \beta \omega]}{\omega} \]

- This equation is often referred to as the New Keynesian Phillips curve.

- Obtain similar reduced form under Taylor (AER 1979, JPE 1980) price adjustment or quadratic costs ala Rotemberg (JPE 1982).
Real marginal cost and the output gap

- The firm’s real marginal cost is equal to the real wage it faces divided by the marginal product of labor: \( \varphi_t = \left( W_t / P_t \right) Z_t^{-1} \).

- Because nominal wages have been assumed to be completely flexible, the real wage must equal the marginal rate of substitution between leisure and consumption: \( \hat{\omega}_t = \hat{\omega}_t - \hat{\rho}_t = \eta \hat{n}_t + \sigma \hat{y}_t + \chi_t + \hat{\mu}_t^w \).

- Recalling that \( \hat{c}_t = \hat{y}_t \), \( \hat{y}_t = \hat{h}_t + \hat{z}_t \),

\[
\hat{\varphi}_t + \hat{\mu}_t = \left[ (\eta \hat{n}_t + \sigma \hat{y}_t + \hat{\chi}_t + \hat{\mu}_t^w) - \hat{z}_t \right] + \hat{\mu}_t \\
= (\sigma + \eta) \left[ \hat{y}_t - \left( \frac{1 + \eta}{\sigma + \eta} \right) \hat{z}_t + \left( \frac{1}{\sigma + \eta} \right) (\hat{\chi}_t + \hat{\mu}_t + \hat{\mu}_t^w) \right] \\
= (\sigma + \eta) \left( \hat{y}_t - \hat{y}_t^f \right) \\
= (\sigma + \eta) x_t.
\]
Real marginal cost and the output gap

- Using these results, the inflation adjustment equation becomes

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x^f_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{4} \]

where \( \kappa = (\eta + \sigma) \tilde{\kappa} = (\eta + \sigma) (1 - \omega) (1 - \beta \omega) / \omega. \)

- This inflation adjustment or forward-looking Phillips curve relates inflation to output, in the form of the deviation around the level of output that would occur in the absence of sticky prices \( x_t. \)
Real marginal cost and the output gap
Generalizing production and assuming firm specific labor markets

- Assume
  \[ Y_t(i) = Z_t N_t^{1-a}, \quad 0 < a < 1. \]
- Firms with above (below) average prices have higher (lower) costs.
- Then the inflation adjustment equation becomes
  \[ \pi_t = \beta E_t \pi_{t+1} + \hat{\kappa} x_t = \beta E_t \pi_{t+1} + \hat{\kappa} x_t \]

where

\[ \kappa = \tilde{\kappa} \left( \frac{\sigma + \psi}{1 + \psi \theta} \right) = \frac{(1 - \omega)(1 - \beta \omega)}{\omega} \left( \frac{\sigma + \psi}{1 + \psi \theta} \right) \]

where \( \psi = (a + \eta) / (1 - a). \)

The simple sticky price, flex wage model

- Two equation system

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) \left( i_t - E_t \pi_{t+1} - r_f \right) \]

- Consistent with
  - optimizing behavior by households and firms
  - budget constraints
  - market equilibrium

- Two equations but three unknowns: \( x_t, \pi_t, \) and \( i_t \) – need to specify monetary policy
Solving the model for the rational expectations equilibrium: role of policy

- Suppose $i_t$ is a function of the exogenous shocks $r_t^e$ and $\hat{\mu}_t$.
- Write system as

$$
\begin{bmatrix}
\beta & 0 \\
\frac{1}{\sigma} & 1
\end{bmatrix}
\begin{bmatrix}
E_t \pi_{t+1} \\
E_t x_{t+1}
\end{bmatrix}
= \begin{bmatrix}
1 & -\kappa \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
x_t
\end{bmatrix}
+ \begin{bmatrix}
-\tilde{\kappa} & 0 \\
0 & \frac{1}{\sigma}
\end{bmatrix}
\begin{bmatrix}
0 \\
i_t - r^f_t
\end{bmatrix}
$$

- or

$$
\begin{bmatrix}
E_t \pi_{t+1} \\
E_t x_{t+1}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\beta} & -\frac{\kappa}{\beta} \\
-\frac{1}{\sigma \beta} & 1 + \frac{\kappa}{\sigma \beta}
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
x_t
\end{bmatrix}
+ \begin{bmatrix}
-\tilde{\kappa} & 0 \\
0 & \frac{1}{\sigma}
\end{bmatrix}
\begin{bmatrix}
0 \\
i_t - r^f_t
\end{bmatrix}
$$

- or

$$
E_t \Omega_{t+1} = M \Omega_t + Ne_t, \quad e'_t = \begin{bmatrix} 0 & i_t - r^f_t \end{bmatrix}
$$
Solving the model for the rational expectations equilibrium

- There exists a locally unique, stationary rational expectations equilibrium if and only if the number of eigenvalues of $M$ outside the unit circle is equal to the number of forward-looking variables (two).
- Condition is not satisfied!
- So a policy that just sets $i_t$ as a function of exogenous shocks $r_t^e$ and $\hat{\mu}_t$ does not ensure a unique rational expectations equilibrium.
- Condition of indeterminacy.
Solving the model for the rational expectations equilibrium

- Suppose $i_t = r^f_t + \delta \pi \pi_t$.
- Write system as
  \[
  \begin{bmatrix}
  \beta & 0 \\
  \frac{1}{\sigma} & 1 \\
  \end{bmatrix}
  \begin{bmatrix}
  E_t \pi_{t+1} \\
  E_t x_{t+1} \\
  \end{bmatrix}
  =
  \begin{bmatrix}
  1 & -\kappa \\
  0 & 1 \\
  \end{bmatrix}
  \begin{bmatrix}
  \pi_t \\
  x_t \\
  \end{bmatrix}
  +
  \begin{bmatrix}
  -\tilde{\kappa} & 0 \\
  0 & \frac{1}{\sigma} \\
  \end{bmatrix}
  \begin{bmatrix}
  0 \\
  \delta \pi_t \\
  \end{bmatrix}
  \]
  
  or
  \[
  \begin{bmatrix}
  E_t \pi_{t+1} \\
  E_t x_{t+1} \\
  \end{bmatrix}
  =
  \begin{bmatrix}
  \frac{1}{\beta} \\
  \frac{\beta \delta \pi - 1}{\sigma \beta} \\
  \end{bmatrix}
  \begin{bmatrix}
  \frac{-\kappa}{\beta} \\
  1 + \frac{\kappa}{\sigma \beta} \\
  \end{bmatrix}
  \begin{bmatrix}
  \pi_t \\
  x_t \\
  \end{bmatrix}
  \]

- Two eigenvalues outside the unit circle if and only if
  \[\delta > 1: \text{the Taylor Principle}.\]
The Taylor Principle

- Policy based on responding solely to exogenous disturbances does not ensure a unique equilibrium.
- Policy must respond to endogenous variables and policy must respond sufficiently strongly to inflation.
- “Reasonable” intuition: if inflation rises, increase \( i \) sufficiently to raise the real interest rate and engineer a contraction to bring inflation back down.
  - If policy also responds to the output gap, then Bullard and Mitra (JME 2002) show condition becomes
    \[
    \kappa(\delta\pi - 1) + (1 - \beta)\delta_x > 0.
    \]
The Taylor principle

- Stabilizing the economy or promising to “blow up the world” (Cochrane JPE 2011)?
- Suppose we have a model displaying superneutrality. The “monetary” side of the model is summarized by

\[ i_t = r_t + E_t \pi_{t+1}: \text{Fisher equation} \]

\[ i_t = r_t + \pi^* + \delta (\pi_t - \pi^*) + z_t: \text{Policy rule} \]

where \( z_t = \rho z_{t-1} + \nu_t \) is an exogenous stationary process and \( r_t \) is exogenous with respect to inflation and the nominal interest rate.

- Eliminating the nominal interest rate yields a difference equation for inflation:

\[ \pi_{t+1} = (1 - \delta)\pi^* + \delta \pi_t + z_t + \varepsilon_{t+1}, \quad E_t \varepsilon_{t+1} = 0 \]

- When does it imply a unique stationary (bounded) equilibrium?
The Taylor principle and liquidity traps

Figure: Dynamic Instability under the Taylor Principle
The Taylor Principle

- Stabilizing the economy or promising to “blow up the world” (Cochrane 2011)?
  - Any equilibrium other than $\pi_t = \pi^*$ blows up when $\delta > 1$.
  - Arbitrary number of equilibria when $\delta < 1$.

- Separate issue – Is $\delta$ identifiable?
  - Empirical work has focused on estimating $\delta$ to determine whether it exceeds 1.
  - But what matters for determinacy is what the central bank would do in situations not observed.
The Taylor principle and liquidity traps

Figure: Liquidity Trap Case