Advanced Topics in Monetary Economics II

Carl E. Walsh

UC Santa Cruz

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Course outline

- New Keynesian monetary models:
  - State dependent pricing models
  - Gaps and wedges and optimal policy
- Uncertainty and the ZLB
- Frictions in labor markets
  - Search and matching in the labor market
- The open economy
  - Policy in a currency union
- Monetary and fiscal interactions
  - Fiscal theories versus monetary theories of the price level
  - Optimal monetary and fiscal policy
Equilibrium conditions for basic sticky price NK model

Euler condition: \( C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{\sigma} \)

MRS = real wage: \( \frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} \)

Marginal cost: \( \frac{W_t}{P_t} = \frac{\varphi_t}{Z_t} \)

Optimal price setting: \( \left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{H_t}{F_t} \)

Aggregate price index: \( P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \)

where \( H_t = C_t^{1-\sigma} \varphi_t + \omega \beta E_t H_{t+1}; \ F_t = C_t^{1-\sigma} + \omega \beta E_t F_{t+1} \)

Goods market clearing: \( \Delta_t^{-1} Y_t = C_t \)

Price dispersion: \( \Delta_t = \int \left( \frac{p_t(i)}{P_t} \right)^{-\theta} di \geq 1 \)
The role of price dispersion

Output is

\[ Y_t = \int c_{jt} dj = Z_t \int N_{jt} dj = Z_t N_t \]

But

\[ Y_t = \int c_{jt} dj = C_t \int \left( \frac{p_{jt}}{P_t} \right)^{-\theta} dj = C_t \Delta_t \Rightarrow C_t = \Delta_t^{-1} Y_t. \]

Since \( \Delta_t \geq 1 \), price dispersion means more has to be produced to achieve a given level of \( C_t \).

• More work effort required to produce a given \( C_t \).
The basic linearized new Keynesian model

1. An expectational IS curve:

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) \left( i_t - E_t \pi_{t+1} - r_t^{\text{flex}} \right) \]

2. An inflation adjustment equation:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \]

3. A specification of policy behavior:

\[ i_t = r_t^{\text{flex}} + \phi_\pi \pi_t + \phi_x x_t; \phi_\pi > 1. \]
Price adjustment: micro facts

- Bils and Klenow (JPE, 2004)
  - median duration between price changes is 4.3 months for items in the U.S. CPI with wide variation in frequency across different categories of goods and services.

- Klenow and Kryvtsov (QJE 2008)
  - price changes large on average, but a significant fraction of price changes are small.
  - variations in the size of price changes, rather than variation in the fraction of prices that change, can account for most of the variance of aggregate inflation.

- Nakamura and Steinsson (QJE 2008) – five facts:
  - sales have a significant effect on estimates of the median duration between price changes (US CPI)
    - excluding sales increases median duration between changes from 4.5 months to 10 months.
  - the frequency of price changes follows a seasonal pattern.
  - the probability the price of an item changes (the hazard function) declines during the first few months after a change in price.
Price adjustment

Alternative approaches

- **Time dependent pricing models**
  - Calvo – fixed probability of adjusting each period.
  - Taylor’s model of fixed-length contracts

- **State dependent price models**
  - Dotsey, King, and Wolman (1999, 2006)
  - Dotsey and King (2005)
  - Golosov and Lucas (2007)
  - Gertler and Leahy (2008)
  - Costain and Nakov (2011)
Price adjustment: the basic TDP Calvo model

- Each period, the firms that adjust their price are randomly selected: a fraction $1 - \omega$ of all firms adjust while the remaining $\omega$ fraction do not adjust.
  - The parameter $\omega$ is a measure of the degree of nominal rigidity; a larger $\omega$ implies fewer firms adjust each period and the expected time between price changes is longer.

- For those firms who do adjust their price at time $t$, they do so to maximize the expected discounted value of current and future profits.
  - Profits at some future date $t + s$ are affected by the choice of price at time $t$ only if the firm has not received another opportunity to adjust between $t$ and $t + s$. The probability of this is $\omega^s$. 
First consider the firm’s cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written as

$$\min_{N_t} W_t N_t + \varphi^n_t (c_{jt} - Z_t N_{jt}).$$

where $\varphi^n_t$ is equal to the firm’s nominal marginal cost. The first order condition implies

$$W_t = \varphi^n_t Z_t,$$

or $\varphi^n_t = W_t / Z_t$. Dividing by $P_t$ yields real marginal cost as $\varphi_t = W_t / (P_t Z_t)$. 
The firm’s decision problem

The firm’s pricing decision problem then involves picking $p_{jt}$ to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \Pi \left( \frac{p_{jt}}{P_{t+i}}, \varphi_{t+i}, c_{t+i} \right) = E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i},$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$ and profits are

$$\Pi(p_{jt}) = \left[ \left( \frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right]$$
Price adjustment

- All firms adjusting in period $t$ face the same problem, so all adjusting firms will set the same price.

- Let $p_t^*$ be the optimal price chosen by all firms adjusting at time $t$. The first order condition for the optimal choice of $p_t^*$ is

$$
E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{1}{p_{jt}} \right) \left( \frac{p_t^*}{P_{t+i}} \right)^{1-\theta} + \theta \phi_{t+i} \left( \frac{1}{p_t^*} \right) \left( \frac{p_t^*}{P_{t+i}} \right)^{-\theta} \right]
$$

- Using the definition of $\Delta_{i,t+i}$,

$$
\left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i}}.
$$
Price adjustment

The first order condition can be expressed as

\[
\left( \frac{p^*_t}{P_t} \right) = \frac{H_t}{F_t}.
\]

where

\[
H_t = \left( \frac{\theta}{\theta - 1} \right) E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i}/C_t)^{-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i} = \left( \frac{\theta}{\theta - 1} \right)
\]

and

\[
F_t = E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i}/C_t)^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i} = C_t + \omega \beta E_t F_{t+1}
\]
The case of flexible prices

- With flexible prices, all firms adjust each period, so $\omega = 0$.
- This implies
  \[ H_t = \left( \frac{\theta}{\theta - 1} \right) \varphi_t C_t; \quad F_t = C_t \]
- So
  \[ \left( \frac{p_t^*}{P_t} \right) = \frac{H_t}{F_t} = \left( \frac{\theta}{\theta - 1} \right) \varphi_t = \mu \varphi_t. \]
- But with all firms setting the same price, $p_t^* = P_t$ and
  \[ \varphi_t = \frac{W_t / P_t}{Z_t} = \left( \frac{\theta - 1}{\theta} \right) < 1. \]
The case of sticky prices

- When prices are sticky ($\omega > 0$), the firm must take into account expected future marginal cost as well as current marginal cost when setting $p_t^*$.

- The aggregate price index is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period $t$ and the average of the remaining fraction $\omega$ of all firms who set prices in earlier periods.

- Because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that was prevailing in period $t - 1$.

- Thus, the average price in period $t$ satisfies

$$P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \quad (1)$$
Alternatives: SDP models

- SDP models allow price behavior to be influenced by an intensive and an extensive margin
  - after a large shock, those firms that adjust will make, on average, bigger adjustments (this is the intensive margin)
  - and more firms will adjust (this is the extensive margin).
  - Size of price changes among firms adjusting can vary and fraction of firms adjusting can vary.

- Basic intuition – highway road repairs.
  - Caplin and Leahy (1987)

- Golosov and Lucas (2007) emphasize that firms most likely to adjust are those furthest from their desired price – this is called the selection effect.
  - The selection effect acts to make the aggregate price level more flexible than might be suggested by simply looking at the fraction of firms that change price.
Firm specific shocks

- Most models of price adjustment developed for use in macroeconomics have assumed that firms only face aggregate shocks.
- This generally implies that all firms that do adjust their price choose the same new price as they all face the same (aggregate) shock.
- The Dotsey, King, and Wolman model features firm-specific shocks to the menu cost, but these shocks only influence whether a firm adjusts, not how much it changes prices.
- In contrast, Golosov and Lucas (2007) and Gertler and Leahy (2008) have emphasized the role of idiosyncratic shocks in influencing which firms adjust prices and in generating a distribution of prices across firms.
Assessing models of nominal price rigidities

- Klenow and Kryvtsov (QJE 2008)
  - No model fits all the micro facts
  - Calvo does surprisingly well except for the declining hazard rate
    - Variations in the size of price changes, rather than variation in the fraction of prices that change, can account for most of the variance of aggregate inflation.

- Carlsson and Nordström Skans (AEJ Macro 2012)
  - Uses Swedish firm-level data to compare models of staggered price adjustment with models of flexible prices but imperfect information (sticky information, rational inattention to macro factors)
  - Concludes Calvo explains data best.
Trend inflation

- Standard NKPCs in DSGE models add ad hoc assumptions about indexation.
- Example: Justiniano, Primiceri, and Tambalotti (2011) assume non-optimizing firms and households set

\[ P_t(j) = P_{t-1}(j) \pi_{t-1}^{l_p} \pi^{1-l_p}; \ W_t(j) = W_{t-1}(j) (\pi_{t-1} e^{z_t})^{l_w} \pi^{1-l_w} \]

where \( \pi \) is steady-state inflation and \( z_t \) is the productivity shock.
- Estimated values of \( l_p, l_w \approx 0 \) but this still imposes indexation to steady-state inflation.
- Costly relative price and wage dispersion depends on

\[ \left[ \pi_t - l_p \pi_{t-1} - (1 - l_p) \pi \right]^2 \approx (\pi_t - \pi)^2 \]

and

\[ \left[ \pi_{w,t} - l_w (\pi_{t-1} + z_t) - (1 - l_w) \pi \right]^2 \approx (\pi_{w,t} - \pi)^2. \]
- Trend inflation doesn’t matter.
Trend and cyclical inflation

Figure: U.S. inflation (pce), trend inflation (hp), and cyclical inflation
Calvo with trend inflation

- Coibion and Gorodnichenko (AER 2011)

\[ \pi_t = \left( \frac{1 - \omega \Pi^{\theta-1}}{\omega \Pi^{\theta-1}} \right) p_t^* \]

where \( p_t^* \) is the relative price set by adjusting firms, \( \Pi \) is the steady-state inflation rate, \( \omega \) is the Calvo parameter (fraction of firms not resetting prices), and \( \theta \) is the elasticity of substitution across goods.
The optimal reset price is

\[(1 + \theta \eta^{-1}) p_t^* = (1 + \eta^{-1})(1 - \gamma_2) \sum_{j=0}^{\infty} \gamma_2^j E_t x_{t+j} + E_t \sum_{j=0}^{\infty} \left( \gamma_2^j - \gamma_1^j \right) (g_{t+j} - i_{t+j-1}) + E_t \sum_{j=0}^{\infty} \left\{ \gamma_2^j \left[ 1 + \theta (1 + \eta^{-1}) \right] - \gamma_1^j \theta \right\} \pi_{t+j} \]

where \( \gamma_2 = \omega R^{-1} g \Pi^\theta \), \( \gamma_2 = \gamma_1 \Pi^{\frac{1+\theta}{\eta}} \), \( g_t \) is the growth rate of output, \( x \) is the output gap, and \( i \) is the nominal interest rate.
Trend inflation

- If Calvo model (and micro evidence) taken seriously, one can’t rely on indexation.
- Indexation to trend inflation:

  \[ P_t^{1-\theta} = (1 - \omega) (P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \bar{\Pi}^\delta (1-\theta) \]

  where \( \delta \) is the degree of indexation.
- Steady-state output gap is not equal to zero if \( \delta < 1 \). Coibion, Gorodnichenko and Wieland show that

  \[ \left( \frac{\bar{Y}}{\bar{Y}^f} \right)^{1+\eta} = \frac{1 - \omega \beta^{-1} \bar{\Pi}^{(1-\delta)\theta(1+\eta)}}{1 - \omega \beta^{-1} \bar{\Pi}^{(1-\delta)(\theta-1)}} \left( \frac{1 - \omega}{1 - \omega \bar{\Pi}^{(1-\delta)(\theta-1)}} \right)^{\frac{1+\eta \theta}{\theta-1}}. \]
Trend inflation

• Steady-state output gap is not equal to zero. Coibion, Gorodnchenko and Wieland show that

\[ \bar{X}_{1+\eta} = \left( \frac{\bar{Y}}{\bar{Y}^f} \right)^{1+\eta} = \frac{1 - \omega \beta^{-1} \bar{\Pi}^{(1-\delta)\theta(1+\eta)}}{1 - \omega \beta^{-1} \bar{\Pi}^{(1-\delta)(\theta-1)}} \left( \frac{1 - \omega}{1 - \omega \bar{\Pi}^{(1-\delta)(\theta-1)}} \right)^{\frac{1+\eta\theta}{\theta-1}} \]

• \( \bar{X} \) is increasing for very low but positive levels of trend inflation but then decreases.

• Optimal rate of inflation is positive but small.

• Interestingly, downward wage rigidity lowers the optimal inflation rate and makes ZLB less likely (will return to this point).
Optimal policy: Topics to cover

1. Policy objectives
2. Optimal policy under discretion and commitment
3. The zero lower bound
4. Uncertainty
Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

\[ E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2] . \] (2)

\( x_t \) is the gap between output and the output level that would arise under flexible prices, and \( x^* \) is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the steady-state level of output.
Policy weights

- Theory says something about the weights in the loss function:

\[
E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi^2_{t+i} + \lambda (x_{t+i} - x^*)^2 \right],
\]

where

\[
\Omega = \bar{Y} U_c \left[ \frac{\omega}{(1 - \omega)(1 - \omega \beta)} \right] (\theta^{-1} + \eta) \theta^2
\]

and

\[
\lambda = \left[ \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \right] \frac{(\sigma + \eta)}{1 + \eta \theta} \theta.
\]

- Greater nominal rigidity (larger \(\omega\)) reduces \(\lambda\).
- Loss function endogenous.
- Calvo specification implies \(\lambda\) is small – Taylor specification leads to larger weight on output gap.
Policy implications of price stickiness

- When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.

  1. The relative price of firms who have not set their prices for a while falls. They experience an increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.

  2. Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.

- The solution is to prevent price dispersion by stabilizing the price level by keeping inflation equal to zero.
Woodford versus Friedman

- The basic new Keynesian model suggests price stability (i.e., zero inflation) is optimal.
  - Zero inflation eliminates inefficient price dispersion.
- Milton Friedman argued that a zero nominal rate of interest is optimal.
  - Zero nominal rate eliminates inefficiency in money holdings.
  - Optimal inflation is negative (deflation) at rate equal to real rate of interest.
- Khan, King, and Wolman (2000) analysis model with both distortions and conclude optimal inflation is closer to zero than to the Friedman rule.
- Schmidt-Grohe and Uribe (2009), Coibion, Gorodnichenko, and Wieland (2012, REStudies): optimal $\pi$ still small even when ZLB taking into account.
Basic model – eliminating the steady-state distortion

- Assume objective is to minimize

$$\frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2).$$

- Note that $x^*$ has been set equal to zero in loss function
- Fiscal subsidy to offset distortion from monopolistic competition.
- If $x^* \neq 0$, can’t use first order approximations to structural equations to obtain a correct second order approximation to the representative agent’s welfare.
Policy Implication of forward-looking models

- The basic new Keynesian inflation adjustment equation took the form

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \]

where \( x \) is output relative to flexible-price output and real marginal cost is \( (\sigma + \eta) x_t \).
- That is, there is no additional disturbance term.

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \Rightarrow \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} \]

- The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.
- Just set \( x_{t+i} = 0 \) for all \( i \); keeps inflation equal to zero: Blanchard and Galí’s “divine coincidence”.
- Productivity shocks don’t appear – with only prices sticky, real wage is flexible.
Cost shocks

- Assume
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \]
  where \( e \) represents an inflation or cost shock.

- Then
  \[ \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i} \]

- Cannot keep both \( x \) and \( \pi \) equal to zero: trade-offs must be made.
Sources of cost shocks

- Stochastic wedge between marginal rate of substitution and real wage.
- Wedge between flexible-price output and efficient level of output.
  - Stochastic markups
- Endogenous sources
  - Sticky nominal wages.
  - Cost channel.
  - Exchange rate movements, imperfect pass through.
Decompositing output

- Consider the following decomposition of output:

\[ Y_t = \left( \frac{Y_t}{Y^f_t} \right) \left( \frac{Y^f_t}{Y^\text{pot}_t} \right) \left( \frac{Y^\text{pot}_t}{Y^e_t} \right) Y^e_t \]

where \( Y^e_t \) is the efficient level of output and \( Y^f_t \) is the level of output under flexible prices and wages, \( Y^\text{pot}_t \) is potential output (output with constant markups and flexible prices).

- In terms of log deviations around the steady state,

\[ y_t - y^e_t = x_t + x^\text{fpot}_t + x^\text{pot}_t. \]

- In baseline NK model, \( x^\text{fpot}_t = 0 \) and \( x^\text{pot}_t \) is a constant (related to steady-state markups). So

\[ \sigma^2_{y-y^e} = \sigma^2_x \]

and minimizing output around the efficient level is the same as minimizing \( x_t \) (Blanchard-Galí ‘divine coincidence’).
Decomposing gaps

- In general,
  \[ \sigma_{y-y^e}^2 = \sigma_x^2 + 2\sigma_{xx} f_{pot} + t.i.p. \]
  and minimizing \( \sigma_x^2 \) does not necessarily minimize \( \sigma_{y-y^e}^2 \). Covariances can matter.

- Optimal policy may not involve minimizing \( y_t - y^e_t \) since resulting fluctuations in price and wage inflation may be too costly.
  - Theory of second best in the face of multiple distortions.
Decomposing gaps

- Which fluctuations in $y_t$ are due to fluctuations in $x_t$, which due to $x_t^{fpot}$, which to $x_t^{pot}$, which to $y_t^e$?
- Which fluctuations in $y_t$ are efficient? Which are inefficient?
- Chari, Kehoe, and McGrattan (2009) emphasized need to distinguish between efficient and inefficient sources of fluctuations.
  - May be hard to identify sources of fluctuations.
Gaps and wedges

- Efficient requires that
  \[ mrs_t = mpl_t \]

- Shocks that cause wedge between these two can be
  - Exogenous
    - Efficient: preference shifts (\( \chi_t \))
    - Inefficient: shifts in desired markups to wages and prices (\( \hat{\mu}_t^w, \hat{\mu}_t^p \))
  - Endogenous
    - Inefficient: nominal wage and price rigidities (\( \phi_t^w, \phi_t^p \)).
The case of flexible prices and wages

Flex-price/wage output

- With flexible prices and wages,
  \[ \omega_t = mpl_t - \hat{\mu}_t^p; \quad \omega_t = mrs_t + \hat{\mu}_t^w \]

- Suppose marginal rate of substitution is
  \[ mrs_t = \eta \hat{n}_t + \sigma \hat{c}_t + \hat{\chi}_t \]
  and marginal productivity is
  \[ mpl_t = \hat{y}_t - \hat{n}_t = \hat{z}_t. \]

- Then, the steady-state labor equilibrium condition yields
  \[ \eta \hat{n}_t + \sigma \hat{c}_t + \hat{\chi}_t + \hat{\mu}_t^w = \omega_t = \hat{z}_t - \hat{\mu}_t^p. \]

- Now using the fact that \( \hat{y}_t = \hat{z}_t + \hat{n}_t \) and \( \hat{y}_t = \hat{c}_t \), the flexible-price equilibrium output \( \hat{y}_t^f \) can be expressed as
  \[ \hat{y}_t^f = \left( \frac{1 + \eta}{\eta + \sigma} \right) \hat{z}_t - \left( \frac{1}{\eta + \sigma} \right) (\hat{\mu}_t^p + \hat{\mu}_t^w + \hat{\chi}_t). \]
The wedges $\varphi^w_t$ and $\varphi^p_t$ are defined by

$$\omega_t = mpt - \hat{\mu}_t^p - \varphi^p_t$$

$$\omega_t = mrs_t + \hat{\mu}_t^w + \varphi^w_t$$

The efficiency wedge is

$$mrs_t - mpt = (\hat{\omega}_t - \hat{\mu}_t^w - \varphi^w_t) - (\hat{\omega}_t + \hat{\mu}_t + \varphi^p_t)$$

$$= - (\hat{\mu}_t^w + \hat{\mu}_t) - (\varphi^w_t + \varphi^p_t)$$
Measuring wedges

The observed wedge:

\[(\eta \hat{n}_t + \sigma \hat{c}_t) - (\hat{y}_t - \hat{n}_t) = - (\hat{\mu}_t + \hat{\mu}_w + \hat{\chi}_t) + (\varphi^w + \varphi^p)\]

- LHS is observable: RHS isn’t.

With log utility and \(\hat{y} = \hat{c}\),

\[\hat{n}_t = - \left( \frac{1}{1 + \eta} \right) (\hat{\mu}_t + \hat{\mu}_w + \hat{\chi}_t - \varphi^w - \varphi^p)\]

so employment gives a direct measure of the labor wedge but not its sources.
Efficiency gaps

- Galí, Gertler, and López-Salido (2002) define the “inefficiency gap” as the gap between the household’s marginal rate of substitution between leisure and consumption ($mrs_t$) and the marginal product of labor ($mpl_t$).
- They divide this gap into the wedge between the real wage and the marginal rate of substitution, which they label the wage markup, and the wedge between the real wage and the marginal product of labor (the price markup).

\[ mrs_t - mpl_t = (mrs_t - \omega_t) + (\omega_t - mpl_t) \]

- Based on United States data, they conclude the wage markup accounts for most of the time series variation in the inefficiency gap.
- Consistent with the importance of nominal wage rigidity.
Empirical estimates of the wedges: Galí, Gertler, and Lopez-Salido (REStat 2007)

- Assume $\sigma = \eta = 1$, so $lw_t = (\hat{n}_t + \hat{c}_t) - (\hat{y}_t - \hat{n}_t)$.

Figure: Galí, Gertler, and Lopez-Salido’s price gap and wage gap.
Empirical estimates of the wedges: Galí, Gertler, and Lopez-Salido (REStat 2007)

Figure: Galí, Gertler, and Lopez-Salido’s efficiency, wage, and price gaps and US unemployment rate (right axis).
Wedges in the simple example
The Justiniano and Giorgio Primiceri (2009) model gap and the price wedge

Figure: The model gap, \( y \) and the price wedge from the simple example
Wedges in the simple example

The model gap, the price wedge, and the labor wedge
Decomposing gaps using DSGE models

Is it a good shock or a bad shock? (Chari, Kehoe, and McGrattan 2009)

Suppose there are shocks to the disutility of labor and to the wage markup:

\[ mrs_t = \frac{\chi_t N_t^{\eta}}{C_t^{-\sigma}} = \left( \frac{W_t}{P_t} \right) \mu_t^w \implies \frac{\mu_t^w \chi_t N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \]

What matters is \( \mu_t^w \chi_t \) – one is a bad shock (\( \mu_t^w \)) and one is a good shock (\( \chi_t \)).

Linearized:

\[ \eta \hat{n}_t + \sigma \hat{c}_t - (\hat{w}_t - \hat{p}_t) = - (\hat{\chi}_t + \hat{\mu}_t^w) \]

Can we identify the two shocks?
Explaining the labor wedge

- Galí, Smets and Wouter (2011): using unemployment as an observable (see exercise).
- Sala, Söderström, and Trigari (2010): alternatives with markups and preferences as source of persistence.
- Justiniano, Primiceri, and Tambalotti (2012): measurement error and persistence:
  - If wage series \((\hat{w}_m^t - \hat{p}_t)\) measures \((\hat{w}_t - \hat{p}_t)\) with error, then
    \[
    \eta \hat{n}_t + \sigma \hat{c}_t = (\hat{w}_m^t - \hat{p}_t) - (e_t^m + \hat{\chi}_t + \hat{\mu}_t^w)
    \]
  - Use multiple (2) series \(\hat{w}_{1,t}^m\) and \(\hat{w}_{2,t}^m\) to reduce measurement error.
  - Assume \(\chi_t\) is AR(1) and \(\mu_t^w\) is i.i.d.
Gas and wedges in an estimated DSGE model

The Justiniano, Primiceri and Tambalotti (2012) model gap:

**Figure 1.** GDP, potential GDP and the GDP gap. In panel (b), the solid line is the median of the posterior distribution of the gap between actual and potential GDP, and the four shades of green denote 60, 70, 80, and 90 percent posterior probability intervals; the dashed line is the median of the posterior of the gap between actual and efficient GDP, in deviation from its steady state.
Gas and wedges in an estimated DSGE model

\[ y_t = (y_t - y_{t}^{pot}) + y_{t}^{pot} \]
Justiniano, Primiceri and Tambalotti (2012)

- Using one wage series, wage markups are important.
Justiniano, Primiceri and Tambalotti (2012)

- Using two wage series, wage markups are small.

**Figure 3.** Actual and optimal GDP (in deviation from potential), price and wage inflation.
Figure 3: Estimated conditional potential output and output gap

(a) Potential output, AR(1) wage markup shock

(b) Potential output, AR(1) labor disutility shock

(c) Output gap, AR(1) wage markup shock

(d) Output gap, AR(1) labor disutility shock

This figure shows the estimated paths for the conditional potential GDP in the U.S. and the output gap (the percent deviation of actual GDP from potential GDP) in the model with measurement errors under different interpretations of the labor market shocks. For potential GDP, parameters are set to their posterior median values. For the output gap, the solid lines show the median estimate and the shaded intervals represent 90 percent probability intervals over 1,000 draws from the estimated posterior distributions of parameters. The vertical shaded bands represent recessions dated by the National Bureau of Economic Research.
Sala, Söderström, and Trigari (2010)

Figure 5: Estimated output gap and labor wedge

This figure shows the estimated paths for the U.S. output gap (the percent deviation of actual GDP from potential GDP) and labor wedge (the wedge between households’ marginal rate of substitution and firms’ marginal product of labor) in the model with measurement errors under different interpretations of the labor market shocks. The solid lines show the median estimate and the shaded intervals represent 90 percent probability intervals over 1,000 draws from the estimated posterior distributions of parameters. The vertical shaded bands represent recessions dated by the National Bureau of Economic Research.
Figure 7: The labor wedge and its components

(a) AR(1) wage markup shock
(b) AR(1) wage markup shock
(c) AR(1) labor disutility shock
(d) AR(1) labor disutility shock

This figure shows the estimated paths for the labor wedge (the wedge between the marginal rate of substitution and the marginal product of labor), the marginal rate of substitution, the marginal product of labor, and the real wage in the model with measurement errors under different assumptions about the labor market shocks. The marginal rate of substitution, the marginal product of labor, and the real wage are measured as deviations from model trend. Parameters are set to their posterior median values. The shaded areas represent recessions dated by the National Bureau of Economic Research.
Figure 10: Decomposing the fundamental wedge into its inefficient and efficient components

This figure shows the estimated paths for the fundamental wedge and its inefficient and efficient components in the model with measurement errors under different assumptions about the labor market shocks. Parameters are set to their posterior median values.
Summary

- Gaps/wedges can arise from efficient shocks and inefficient shocks.
- Implications for policy are different, so identifying nature of shocks is important.
- Serious identification issues:
  - Is the labor wedge due to preference shocks, wage markup shocks, or measurement error?
Optimal policy in the basic model

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.

- Minimize

\[- \frac{1}{2} \Omega E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2)\]

subject to

\[\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.\]

- Notice the Euler condition imposes no constraint – use it to solve for \(i_t\) once optimal \(\pi_t\) and \(x_t\) have been determined.
  - This would not be case if central bank cares about interest rate volatility.
Discretion

The policy problem

- Policy maker takes expectations as given – leads to period by period maximization.
- Problem is to pick $\pi_t$ and $x_t$ to minimize
  \[ \frac{1}{2} (\pi_t^2 + \lambda x_t^2) + \psi_t (\pi_t - \beta \pi_{t+1} - \kappa x_t - e_t) \]
  taking $E_t \pi_{t+1}$ as given.
- The first order conditions can be written as
  \[ \pi_t + \psi_t = 0 \]  \[ \lambda x_t - \kappa \psi_t = 0. \]
- Eliminating $\psi_t$, $\lambda x_t + \kappa \pi_t = 0$ – this is a targeting rule.
Discretion

Behavior of the interest rate

- From the IS equation,

\[ i_t = E_t \pi_{t+1} + \sigma (E_t x_{t+1} - x_t) + r^n_t. \]

- Using solution,

\[ i_t = \left[ A\rho - \sigma \left( \frac{\kappa}{\lambda} \right) (\rho - 1) \right] e_t + r^n_t = B e_t + r^n_t. \]

- Shifts in natural rate of interest \( r^n \) are fully offset.

- So optimal policy involves \( i \) responding to shocks, but adopting a rule of the form

\[ i_t = B e_t + r^n_t \]

does not ensure a unique rational expectations equilibrium.
Precommitment

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.
- Under optimal commitment, central bank at time $t$ chooses both current and expected future values of inflation and the output gap.
- Minimize
  
  $$
  -\frac{1}{2}\Omega E_t \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right)
  $$

  subject to

  $$
  \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.
  $$

- Notice the Euler condition imposes no constraint – use it to solve for $i_t$ once optimal $\pi_t$ and $x_t$ have been determined.
Optimal precommitment

- The central bank’s problem is to pick $\pi_{t+i}$ and $\chi_{t+i}$ to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{2} (\pi_{t+i}^2 + \lambda \chi_{t+i}^2) + \psi_{t+i} (\pi_{t+i} - \beta \pi_{t+i+1} - \kappa \chi_{t+i} - e_{t+i}) \right].$$

- The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (6)$$

$$E_t (\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0 \quad i \geq 1 \quad (7)$$

$$E_t (\lambda \chi_{t+i} - \kappa \psi_{t+i}) = 0 \quad i \geq 0. \quad (8)$$

- Dynamic inconsistency – at time $t$, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = - (E_t \psi_{t+1} - \psi_t)$. When $t + 1$ arrives, a central bank that reoptimizes will again obtains $\pi_{t+1} = -\psi_{t+1}$ – the first order condition (6) updated to $t + 1$ will reappear.
Timeless precommitment

- An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (7) and (8) for all periods, including the current period so that

\[
\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} = 0 \quad i \geq 0
\]

\[
\lambda x_{t+i} - \kappa \psi_{t+i} = 0 \quad i \geq 0.
\]

- Woodford (1999) has labeled this the “timeless perspective” approach to precommitment.
Timeless precommitment

- Under the timeless perspective optimal commitment policy, inflation and the output gap satisfy

\[
\pi_{t+i} = - \left( \frac{\lambda}{\kappa} \right) (x_{t+i} - x_{t+i-1})
\]  

for all \( i \geq 0 \).

- Woodford (1999) has stressed that, even if \( \rho = 0 \), so that there is no natural source of persistence in the model itself, \( a > 0 \) and the precommitment policy introduces inertia into the output gap and inflation processes.

- This commitment to inertia implies that the central bank’s actions at date \( t \) allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap.
Illustrating commitment versus discretion in the simple NK model

MATLAB\dynare\NKmodels\Gerzensee2012\NKM_basic\Graphs_Compare.m
Improved trade-off under commitment

- The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion.
- Consider a positive inflation shock, $e > 0$.
- A given change in current inflation can be achieved with a smaller fall in $x$ if expected future inflation can be reduced:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

- Requires a commitment to future deflation.
- By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in $E_t \pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.
The case of a persistent cost shock
Illustrating policy effects in S-W model: inflation shock
Uncertainty

- Standard analysis with additive errors leads to certainty equivalence (linear structural equations, quadratic objective function)
  - Optimal policy depends on future forecasts but not the uncertainty surrounding those forecasts.
- Suppose the true model of the economy is given by

\[ y_{t+1} = A_1 y_t + A_2 y_{t/t} + B i_t + u_{t+1}, \tag{10} \]

where \( y_t \) is a vector of macroeconomic variables (the state vector), \( y_{t/t} \) is the optimal, current estimate of \( y_{t/t} \), and \( i_t \) is the policy maker’s control instrument.
- \( u_{t+1} \) represents a vector of additive, exogenous stochastic disturbances, assumed equal to \( Ce_{t+1} \) where the vector \( e \) is a set of mutually and serially uncorrelated disturbances with unit variances.
- \( A_1, A_2, \) and \( B \) are matrices of the model parameters.
Sources of model specification error

- Suppose the policy maker’s estimates of $A_1$, $A_2$, and $B$ are denoted $\bar{A}_1$, $\bar{A}_2$, and $\bar{B}$, while $\bar{y}_{t/t}$ denotes the policy maker’s estimate of the the current state $y_t$.

- Then, letting $A = A_1 + A_2$ and $\bar{A} = (\bar{A}_1 + \bar{A}_2)$, we can write the policy maker’s perceived model in the form

$$y_{t+1} = \bar{A}\bar{y}_{t/t} + \bar{B}i_t + \bar{C}(e_{t+1} + w_{t+1}) \quad (11)$$

where

$$w_{t+1} = \bar{C}^{-1}[(A - \bar{A})\bar{y}_{t/t} + (B - \bar{B})i_t + (C - \bar{C})e_{t+1}]$$

$$+ \bar{C}^{-1}A_1(y_t - y_{t/t}) + \bar{C}^{-1}\bar{A}(y_{t/t} - \bar{y}_{t/t}). \quad (12)$$
Sources of model specification error

\[ w_{t+1} = \bar{C}^{-1} [(A - \bar{A}) \bar{y}_{t/t} + (B - \bar{B}) i_t + (C - \bar{C}) e_{t+1}] + \bar{C}^{-1} A_1 (y_t - \bar{y}_{t/t}) + \bar{C}^{-1} \bar{A} (y_{t/t} - \bar{y}_{t/t}). \]

1. **Model mis-specification**: errors that arise if the policy maker’s estimate of the parameters of the model differs from their true values. This term also captures errors in modelling the structural impacts of exogenous disturbances.

2. **Imperfect information**: errors the policy maker incurs in estimating the current state of the economy.

3. **Asymmetric and/or inefficient forecasting**: informational asymmetries such as occur when the private sector has different information than the policy maker does.
Multiplicative uncertainty: Brainard (1967)

- Model ($y$ is the output gap, $\pi$ is inflation):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_t x_t + e_t$$

- Assume $\kappa_t$ is stochastic, $\kappa_t = \bar{\kappa} + \nu_t$, where $\nu_t$ is a white noise process.

- Under discretion, the policy maker takes $E_t y_{t+1}$ and $E_t \pi_{t+1}$ as given. The first order condition yields

$$x_t = - \left( \frac{\bar{\kappa}}{\lambda + \sigma^2} \right) e_t$$

- Reaction with more caution.

Robust control: The reference model

- Policy maker has reference model of form

\[ y_{t+1} = \bar{A}y_t + \bar{B}i_t + \bar{C}e_{t+1} \]

- True model is

\[ y_{t+1} = \bar{A}y_t + \bar{B}i_t + \bar{C} (e_{t+1} + w_{t+1}), \quad (13) \]

- In the robust control literature, \( w_{t+1} \) represents unknown specification errors.
- \( w_{t+1} \) is not simply an exogenous disturbance like \( e_{t+1} \) but may depend on the history of \( y_t \).

- The policy maker views \( \bar{A}\bar{y}_t/t + \bar{B}u_t + \bar{C}e_{t+1} \), i.e., the case \( w_{t+1} = 0 \), as a good “approximating model” to the true but unknown model.

- It is a good approximation in the sense that

\[ \sum_{i=0}^{\infty} \beta^i w'_{t+i} w_{t+i} \leq \eta_0, \quad (14) \]
The intuition

- Strategic game involving the policy maker and an evil agent who attempts to make life hard for the policy maker.
- Leads to a min-max strategy by the policy maker, with the policy instrument chosen to minimize the worst-case outcome.
- Equilibrium is given by the solution to

$$\min_w \max_u \E_t \left[ \sum_{t=0}^{\infty} \beta^t \left\{ -r(y_t, u_t) + \theta \beta w_{t+1} \right\} \right]$$

where $y_t$ is the state, $u_t$ is the policy maker’s control, $r(y_t, u_t)$ is the quadratic loss, and $\beta$ is the discount factor.

- As $\theta \to \infty$, evil agent is more constrained. Standard case when $\theta = \infty$. 
Using a distorted model

- The policy maker replaces the model of the economy with a distorted model, one that incorporates the worst-case process for $w_{t+1}$.
- The evil agent sets $w_{t+1}$ to maximize the policy maker's loss function.
  - The value of $w_{t+1}$ for which the worst-case outcome occurs can be expressed as a function of the state vector, $Ky_t$.
- Substituting $w_{t+1} = Ky_t$ into (13) yields the distorted model:
  \[ y_{t+1} = (\bar{A} + \bar{C}K)y_t + \bar{B}_i + \bar{C}e_{t+1}. \] (15)
- The policy maker now treats this distorted model as the true model of the economy and minimizes loss subject to (15).
  - Once the policy maker has substituted in $Ky_t$ for $w_{t+1}$, the policy problem is reduced to a standard one – certainty equivalence holds.
Using a distorted model

- A Bayesian policy maker, faced with model uncertainty, assigns a probability to each possible model, where these probabilities reflect the policy maker’s assessment of the likelihood of each model.
- A policy maker concerned with robustness bases policy on a distorted model but then proceeds to act as if there were no longer any model uncertainty.
- An example: Does it pay to underestimate inflation persistence or over estimate it? Yes.
  - Coenen (2003), Angeloni, Coenen, and Smets (2003), and Walsh (2003).
Preference for robustness

- The same policy that results from a policy maker employing the distorted model is also obtained when the policy maker believes the true model is given by

\[ y_{t+1} = \tilde{A}y_t + \tilde{B}i_t + \tilde{C}e_{t+1}, \]

and she maximizes an objective function that contains an additional adjustment for risk.

- Specifically, the policy maker’s preferences incorporate an additional sensitivity to risk. Similar risk sensitive preferences have been studied by Epstein and Zin (1989) and Weil (1990).

- Preferences are of the form

\[ V_t = U [c_t, V_{t+1}] \]

- Allows risk aversion and elasticity of intertemporal substitution to be separated.
Comparing the standard optimal rules and robust control

- Both approaches lead to same instrument or targeting rule.
- Consider simple example – new Keynesian model with $\psi_t$ the Langrangian multiplier on the NKPC.
- Standard approach – first order conditions:

$$\pi_t + \psi_t - \psi_{t-1} = 0$$

and

$$\lambda x_t - \kappa \psi_t = 0.$$ 

- Combine to yield robustly optimal targeting rule:

$$\pi_t = - \left( \frac{\lambda}{\kappa} \right) (x_t - x_{t-1}).$$
Comparing the two approaches
Equilibrium under the robustly optimal rule

- Together with the inflation adjustment equation, this yields

$$\left[1 + \beta + \left(\frac{\kappa^2}{\lambda}\right)\right] x_t = \beta E_t x_{t+1} + x_{t-1} - \left(\frac{\kappa}{\lambda}\right) e_t,$$  \hspace{1cm} (16)

which can be jointly solved with the process for $e_t$ given by

$$e_t = \rho_e e_{t-1} + \varepsilon_t.$$  \hspace{1cm} (17)

under rational expectations.
Robust control problem is

$$\min_{x, \pi} \max_{w, e} \mathbb{E}_{t}^{rc} \sum_{i=0}^{\infty} \beta^{i} \left[ \left( \frac{1}{2} \right) \pi_{t+i}^{2} + \left( \frac{1}{2} \right) \lambda_{x} x_{t+i}^{2} \right. \right.$$

$$\left. \left. - \left( \frac{1}{2} \right) \beta \theta w_{t+1+i}^{2} \right. \right.$$

$$+ \psi_{t+i} \left( \pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right)$$

$$+ \varphi_{t+i} \left( \rho e_{t+i} + e_{t+i+1} + w_{t+i+1} - e_{t+i+1} \right). \right.$$
Robust control

- The policy maker’s first order conditions include

  \[
  \pi + \psi_t - \psi_{t-1} = 0, \tag{18}
  \]

  \[
  \lambda x_t - \kappa \psi_t = 0, \tag{19}
  \]

- The evil agent’s first order conditions include

  \[
  -\varphi_t + \rho_e \varphi_t - \left( \frac{1}{\beta} \right) \varphi_{t-1} = 0, \tag{20}
  \]

  and

  \[
  -\theta w_{t+1} + \varphi_t = 0. \tag{21}
  \]
Robust control

Equilibrium

- Combining first two with the inflation adjustment equation yields

\[
\left[1 + \beta + \left(\frac{\kappa^2}{\lambda}\right)\right] x_t = \beta E_t^r x_{t+1} + x_{t-1} - \left(\frac{\kappa}{\lambda}\right) e_t, \quad (22)
\]

which is identical to equation obtained in standard case except for the formation of expectations.

- Evil agent’s first order conditions imply that

\[
\varphi_{t-1} = \beta \rho e \varphi_t - \beta \psi_t = \beta \rho e \varphi_t - \beta \left(\frac{\lambda}{\kappa}\right) x_t.
\]

- Advancing this expression one period, taking expectations, solving the resulting expression forward implies that

\[
\omega_{t+1} = - \left(\frac{\beta \lambda}{\kappa \theta}\right) \sum_{i=0}^{\infty} (\beta \rho e)^i E_t^r x_{t+1+i}. \quad (23)
\]
Suppose the evil agent commits to a contingency rule that makes misspecification a function of exogenous state variables.

Specifically, assume evil agent commits to \( w_{t+1} = K e_t \).

Define \( \tilde{\rho}_e = \rho_e + K \). Then, for any choice of \( K \) such that \( |\tilde{\rho}_e| < 1 \), the policy maker’s problem becomes

\[
\min_i \ E_t r^c \sum_{i=0}^{\infty} \beta^i \left\{ \left( \frac{1}{2} \right) \pi_{t+i}^2 + \left( \frac{1}{2} \right) \lambda x_{t+i}^2 - \left( \frac{1}{2} \right) \beta \theta (K e_{t+i})^2 
+ \psi_{t+i} \left( \pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right) 
+ \varphi_{t+i} \left( \tilde{\rho}_e e_{t+i} + \epsilon_{t+i+1} - e_{t+i+1} \right) \right\}.
\]
Extensions: A simple rule for the evil agent

- This is a standard problem with the shock process replaced by the distorted process
  \[ e_{t+1} = \tilde{\rho}_e e_t + \varepsilon_{t+1}. \] (24)

- The policy maker (and the public) takes \( \tilde{\rho}_e \) (i.e., \( K \)) as given.

- The optimal targeting criterion is \( x_t = x_{t-1} - (\kappa/\lambda) \pi_t \), which is independent of \( \tilde{\rho}_e \) (and therefore \( K \)).

- This independence reflects the fact that the standard targeting criterion is designed to be robust with respect to exactly the type of model mis-specification of the disturbance process that is reflected in (24). (see Giannoni and Woodford 2003).
Using multiple models

- McCallum has long argued for evaluating simple rules in multiple models.
- Levin and Williams (2003).
- Forward-looking models too easy to control – rules optimal in forward-looking models tend to do poorly in backward-looking models.
- Standard approach – use different models, but outcomes are evaluated using a fixed loss function.
  - What if loss function for model A is different from loss function for model B?
  - Theory says this will be the case.

- Example: greater nominal rigidity reduces output elasticity of inflation.
  - requires greater output gap variability to control inflation;
  - but theory says weight on gap fluctuations in loss function should fall.
Structural inflation inertia

- Important for relative performance of different targeting rules (Walsh 2003).
- Suppose

\[ \pi_t - \gamma \pi_{t-1} = \beta (E_t \pi_{t+1} - \gamma \pi_t) + \kappa x_t + e_t. \]

- Woodford (2003) shows that second order approximation to welfare is proportional to

\[ - \left( \frac{1}{2} \right) \sum_{i=0}^{\infty} \left[ (\pi_t - \gamma \pi_{t-1})^2 + \lambda x_t^2 \right]. \]

- Define \( z_t \equiv \pi_t - \gamma \pi_{t-1} \). Then model becomes

\[
\min \left( \frac{1}{2} \right) \sum_{i=0}^{\infty} \left[ (\pi_t - \gamma \pi_{t-1})^2 + \lambda x_t^2 \right] \\
\text{subject to} \\
z_t = \beta E_t z_{t+1} + \kappa x_t + e_t.
\]

- Loss independent of \( \gamma \)!
Is the ZLB a constraint?

Figure: Distribution of the federal funds rate, Jan. 1960 - Mar. 2012
The zero lower bound

- **Causes**
  - Non-fundamentals-based liquidity traps
    - Expectationally driven
  - Fundamentals-based liquidity traps (will address later)
    - Negative shock to the equilibrium (Wicksellian) real rate of interest
- **Potential constraint on monetary policy**
Benhabib, Schmit-Grohé, and Uribe (2001, 2002) have argued that deflationary paths cannot be ruled out if policy satisfies the Taylor Principle.

- Multiple steady-state equilibria in forward-looking expectational models.
- The argument is based on the observation that the nominal rate of interest cannot fall below zero.

Explosive deflations would eventually force the nominal interest rate to zero, but the nominal rate is then prevented from falling further.

They argue that simple and seemingly reasonable monetary policy rules that follow the Taylor Principle in changing the nominal interest rate more than one-for-one is response to changes in inflation can introduce the possibility the economy will be caught in a deflationary liquidity trap.
The Taylor principle and liquidity traps

Suppose we have a model displaying superneutrality. The “monetary” side of the model is summarized by

\[ i_t = r^n_t + \mathbb{E}_t \pi_{t+1} \]

\[ i_t = g(\pi_t) \]

where \( r^n_t \) is exogenous with respect to inflation and the nominal interest rate and \( g(\pi) \) is the policy rule.

For simplicity, let

\[ g(\pi_t) = r^n_t + \pi^* + \delta (\pi_t - \pi^*) \]

\[ = r^n_t + (1 - \delta) \pi^* + \delta \pi_t \]

Then combining these equations,

\[ \mathbb{E}_t \pi_{t+1} = (1 - \delta) \pi^* + \delta \pi_t \]

There is a unique stationary equilibrium inflation rate equal to \( \pi^* \).

However, if the time \( t \) inflation rate is not equal to \( \pi^* \), the inflation process is unstable.
The Taylor principle and liquidity traps

A simple example

\[ \pi(t) \]

\[ \pi(t+1) \]

\[ \pi^* \]

\[ \pi^{**} \]

\[ \pi(t) \]

\[ \pi^* \]

\[ \pi^{**} \]

Figure: Liquidity trap case
The zero lower bound
Fundamentals-based liquidity traps

- Optimal policy an a basic new Keynesian model implies the policy interest rate moves one-for-one with the equilibrium real (natural, Wicksellian) interest rate:

\[ i_t = r_t^n + \pi^* + \delta (\pi_t - \pi^*) \geq 0. \]

- Negative shock to \( r_t^n \) could require \( i_t \) to be negative – ZLB prevents this.

- Deflationary trap:

\[ r_t = i_t - E_t \pi_{t+1} = -E_t \pi_{t+1} \]
Policy at the ZLB

- If $i = 0$, short-term government debt and money perfect substitutes.
  - Altering $B + M$ doesn’t cause returns to adjust.
- Consider the nominal budget constraint of a household holding money plus short-term government debt, denoted $B$ yielding nominal return $i^b$:

$$P_t Y_t + (1 + i^b_t) B_t + M_t \geq P_t C_t + P_t T_t + B_{t+1} + M_{t+1}.$$ 

- The budget constraint can be expressed in real terms as

$$Y_t + \left(1 + r^b_t\right) F_t \geq C_t + T_t + \left(\frac{i^b_t}{1 + \pi_t}\right) m_t + F_{t+1},$$

where $F_t = b_t + m_t$ equals real holds of financial assets.
- If the nominal interest is zero, $1 + r^b = 1/(1 + \pi)$,

$$Y_t + \left(\frac{1}{1 + \pi_t}\right) F_t \geq C_t + T_t + F_{t+1},$$

and $m$ drops out.
Is the ZLB a constraint?

- Is the ZLB a constraint on monetary policy?
- Conventional model based on an expectational IS relationship:

\[
x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r^n_t)
\]

- Interest rates – both current and expected future matter:

\[
x_t = - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) - \left( \frac{1}{\sigma} \right) E_t \sum_{i=1}^{\infty} (i_{t+i} - \pi_{t+1+i}) \\
+ \left( \frac{1}{\sigma} \right) E_t \sum_{i=0}^{\infty} r^n_{t+i},
\]

- Reflects a narrow view of the transmission mechanism – no role for quantitative easing or credit easing policies.
Conventional instruments at the ZLB

- Even at the ZLB, policy has the potential to influence real spending if it can affect expectations of future real interest rates.
  - Eggertsson and Woodford (2003)
- If $i_t = 0$ and is expected to remain at zero until $t + T$, then

$$
x_t = \left( \frac{1}{\sigma} \right) \sum_{i=0}^{T} E_t \pi_{t+1+i} - \left( \frac{1}{\sigma} \right) E_t \sum_{i=T+1}^{\infty} (i_{t+i} - \pi_{t+1+i})
+ \left( \frac{1}{\sigma} \right) E_t \sum_{i=0}^{\infty} r^n_{t+i}.
$$

- Raising expected future inflation or committing to lower future nominal rates can stimulate current spending.
  - Cost of ZLB low in linear models when central bank is credible (Nakov 2008)
Promising future inflation

- Optimal policy at the ZLB involves promising future inflation. This is done by keeping interest rates low even when the ZLB no longer binds (Eggertsson and Woodford 2003).
- Central banks have been reluctant to promise higher future inflation.
- Contrasts with recommendations made to the Bank of Japan:
- Communicating clearly the conditional nature of future interest rate paths cited as concern.
- Commitment requires promises be fulfilled – have to deliver higher future inflation.
- Central banks may lack the credibility to steer future expectations (Bodenstein, Hebden, and Nunes 2010).
Promising future inflation
Four-period example of Bodenstein, Hebden and Nunes (2010)

- Suppose economy is described by

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_{t}^n) \]

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

- Policy objective is to minimize

\[ \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \]

- In period 1, the economy is at the zero lower bound: \( r_1^n < 0 \) and \( i_1 = 0 \).
- In periods 2, 3 and 4, the economy is out of the ZLB so that \( x_{t+i} = \pi_{t+i} = 0 \) for \( i \geq 2 \) is feasible. Assume that \( \pi_{t+4} = x_{t+4} = 0 \). (This is what makes this a simple example.)
- The issue is what happens in periods 2 and 3 and how this affects the output gap and inflation in period 1.
Promising future inflation: example
Path of interest rate relative to equilibrium real rate under discretion and full commitment

Figure: The nominal rate under discretion and full commitment when ZLB is binding only in period 1 (figure shows $i_r^n$)

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Promising future inflation: example
Path of inflation and the output gap under discretion and full commitment

Figure: Inflation and the output gap under discretion and full commitment when the ZLB is binding only in period 1
Promising future inflation: example

Path of inflation and output gap with imperfect credibility

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Promising future inflation: example
Path of interest rate relative to equilibrium real rate with imperfect credibility

Figure: The nominal rate under discretion and full commitment when ZLB is binding only in period 1 (figure shows $r_n$)

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Promising future inflation: example

- A lack of credibility makes it harder to stabilize in the face of the ZLB;
- With less credibility, future promises must be more extreme;
- “Promising low interest rates for an extended period of time” may be a sign of a lack of credibility;
- Promising low interest rates in the future and also promising no inflation is an inconsistent policy.
- When current credibility is low, the central bank has to promise lower future rates, but that means the cost of fulfill promises is higher, raising the incentive to deviate.
  - Central banks with low credibility face the greatest temptation to break their promises.
Raising the inflation target

- Raising the inflation target raises average nominal interest rates and makes hitting the ZLB less likely.
- Trade-off – steady-state loss of higher inflation against ability to improve stabilization.
- Schmidt-Grohe and Uribe (2009), Coibion, Gorodnichenko, and Wieland (2012, REStudies): optimal $\pi$ still small when ZLB taking into account.
Reforming IT: Using expectations as automatic stabilizers under PLT

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

- Central banks may find it easier to commit to objectives than to future policy actions.
  - Distorting objectives in a discretionary environment can improve outcomes (Walsh 1995)

**Outcomes to shocks under discretion**

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Price level targeting and the ZLB

- Optimal policy at the ZLB can be implemented via a time-varying price-level target (Eggertsson and Woodford 2003).
- Intuition – under commitment, central bank has many tools even if current policy rate at zero.
  - Can promise future low interest rates and a boom.
  - This generates expectations of future inflation which raises current inflation and lowers current real interest rate.
- Promise to generate inflation to achieve price-level target.
Price level targeting

- Vestin (2006) shows price level targeting can replicate the timeless precommitment solution if the central bank is assigned the loss function $p_t^2 + \lambda_{PL} x_t^2$ in an environment of discretion.

- Under timeless precommitment,

$$\pi_t = (1 - L) p_t = \left( \frac{\lambda}{\kappa} \right) (x_t - x_{t-1}) \Rightarrow p_t = \left( \frac{\lambda}{\kappa} \right) x_t$$

- Price level targeting makes inflation expectations act as an automatic stabilizer.

- Walsh (2003) adds lagged inflation to the inflation adjustment equation and shows that the advantages of price level targeting over inflation targeting decline as the weight on lagged inflation increases.
PLT: other considerations

- Advantages of PLT require that expectations act as automatic stabilizers.
  - Raises issues of credibility and learning
- Switching policy regimes in a crisis risks gains in credibility achieved by inflation targeters.
  - When adopted, the choice of price index, the underlying trend inflation rate, and the speed with which deviations from target path are expected to be reversed are all important.
- Walsh (2003) adds lagged inflation to the inflation adjustment equation and shows that the advantages of price level targeting over inflation targeting decline as the weight on lagged inflation increases.
The zero lower bound: other solutions for getting out

- Svensson’s “foolproof way”
  - Depreciation as visible means of committing to a higher price level
- Fiscal policy: “fiscal policy must be seen not to be committed to... conventional prescriptions for good fiscal policy...”. (Sims 2000, p. 969, italics in original) – more on this later.
- Quantitative easing/credit easing policies – more on this later.
Fed’s balance sheet: Conventional

[Graph showing the Fed's balance sheet from 2007 to 2012 with different categories: Traditional Security Holdings, Lending to Financial Institutions, Liquidity to Key Credit Markets.
Fed’s balance sheet: Conventional and unconventional
Unconventional policies: Quantitative easing

- So far, monetary policy only works via the Wicksellian interest rate gap \( (i_t - E_t \pi_{t+1}) - r^n_t \).
  - Pre-crisis consensus: No direct role for money – \( M \) not a separate policy tool once \( i \) set

- At the ZLB – money and short-term assets perfect substitutes

- Quantity of money can still matter
  - Depends on the properties of money demand
  - Depends on monetary expansion being permanent

- Paying interest on reserves gives central bank two instruments:
  - Policy interest rate and rate on reserves
  - Policy interest rate and monetary aggregate.
Money in the NK model

- Assume money generates utility directly.
- Demand for money determined by equating marginal rate of substitution between real money balances and the opportunity cost of holding money.
  - The opportunity cost depends on the nominal rate of interest.
- In linearized form:
  \[ m_t - p_t = \gamma y_t - \eta i_t. \]
- Given \( i_t \) and \( y_t \) determined by rest of system, this equation just determines \( m_t \). No separate role for money.
  - Assumes separable utility.
Money in the NK model

- With interest paid on money, then \( m \) and \( i \) (or \( i \) and \( i^m \)) become separate instruments.
- In linearized form:
  \[
  m_t - p_t = \gamma y_t - \eta (i_t - i^m_t).
  \]
- Given \( i_t \) and \( y_t \) determined by rest of system, this equation no longer determines \( m_t \) unless \( i^m_t \) is also specified.
- Set \( i^m = i \) to eliminate the Friedman distortion without requiring that \( i = 0 \).
- Issue is whether the quantity of money matters.
Does money matter at the ZLB?

- Consider a standard (linearized) money demand equation:

  \[ m_t - p_t = \gamma y_t - \eta (r_t + E_t \pi_{t+1}) = \gamma y_t - \eta (r_t + E_t p_{t+1} + p_t). \]

- Solving forward:

  \[ p_t = E_t \sum_{j=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right) (m_{t+j} - \gamma y_{t+j} + \eta r_{t+j}). \]

- So the future path of money matters for prices today.
Unconventional policies: Affecting long-term interest rates

- Standard models of the term structure imply the m-period zero-coupon bond rate is related to the expected future path of short-term real rate according to

\[ r_t^m = \left( \frac{1}{m-1} \right) E_t \sum_{j=0}^{m-1} (i_{t+j} - \pi_{t+j}) + \Psi_{m,t}, \]  

where \( \Psi_{m,t} \) is a risk premium.

- When \( \Psi_{m,t} \) is viewed as exogenous, (25) implies that only by influencing expectations about future interest rates and inflation can the central bank affect the real economy at the ZLB.

- If \( \Psi_{m,t} \) is endogenous and varies with factors influenced by monetary policy, then altering the path of the future policy rate may not be the sole means of affecting the economy.

- Question: are there other transmission channels besides the expected future path of interest rates that might provide the means to affect long-term rates? (Particularly relevant at the ZLB.)
Assume all agents can freely buy and sell assets and assets are valued only because of their payouts.

Then price of asset $j$ with payoffs $x_j(s)$ in future state $s$ is

$$p_j = \sum_{s=1}^{S} \pi(s) m(s) x_j(s) = \sum_{s=1}^{S} \pi(s) \beta \frac{U_c(c(s))}{U_c(c_t)} x_j(s)$$

$$= \beta E_t \left( \frac{U_c(c(s))}{U_c(c_t)} \right) x_j(s),$$

where $\pi(s)$ is the probability of state $s$.

Asset quantities do not appear.

Money is different – usually viewed as having a non-pecuniary return.
Wallace (AER 1981) demonstrated a Modigliani-Miller result for open market operations.

- Asset prices are independent of the central bank’s balance sheet – open market operations and the form they take (short-term gov’t debt, long-term gov’t debt, private assets) are irrelevant.

- Conditions needed (Curdia and Woodford JME 2011):
  - If assets are valued only for the pecuniary returns and
  - If all investors can purchase arbitrary quantities of the same assets at the same prices.

- Government purchases of assets doesn’t alter total risk.
Budget identities

Suppose household can hold one-period bonds, two-period bonds, and base money. The budget constraint in nominal form would take the form

\[ P_t Y_t + D_{1,t-1}^P + p_{1,t} D_{2,t-1}^P + M_{t-1} \geq P_t C_t + P_t T_t + p_{1,t} D_t^P + p_{2,t} D_{2,t}^P + M_t. \]

Here \( p_{1,t} \) is the price of a one period discount bond (so \( p_{1,t} = 1/(1+i_{1,t}) \)) and \( p_{2,t} \) is the price of a two period bond. Now define financial wealth as

\[ F_t \equiv D_{1,t-1}^P + p_{1,t} D_{2,t-1}^P + M_{t-1} \]
Budget identities

- One can rewrite the budget constraint as

\[
P_t Y_t + F_t = P_t C_t + P_t T_t + p_{1,t} \left( D_{1,t}^P + E_t p_{1,t+1} D_{2,t}^P + M_t \right)
+ (p_{2,t} - p_{1,t} E_t p_{1,t+1}) D_{2,t}^P + (1 - p_{1,t}) M_t
= P_t C_t + P_t T_t + p_{1,t} F_{t+1}
+ (p_{2,t} - p_{1,t} E_t p_{1,t+1}) D_{2,t}^P + (1 - p_{1,t}) M_t
\]

- The one period interest rate is

\[
1 + i_{1,t} \equiv \frac{1}{p_{1,t}}.
\]  \(\text{(26)}\)

- The two period rate is

\[
(1 + i_{2,t})^2 \equiv \frac{1}{p_{2,t}}
\]
No arbitrage conditions

- Under the expectations hypothesis,
  \[ p_{2,t} = p_{1,t} E_t p_{1,t+1} \]  
  (27)

- To first order,
  \[ i_{2,t} = \left( \frac{1}{2} \right) \left( i_{1,t} + E_t i_{1,t+1} \right) \]

- So the budget constraint reduces to
  \[ P_t Y_t + F_t = P_t C_t + P_t T_t + p_{1,t} F_{t+1} + (1 - p_{1,t}) M_t. \]

- At the ZLB, \( p_{1,t} = 1 \) and the budget constraint becomes
  \[ P_t Y_t + F_t = P_t C_t + P_t T_t + p_{1,t} F_{t+1}. \]

- The composition of the portfolio between money, one-period bonds or two-period bonds is irrelevant.
Portfolio balance models

- Assumes imperfect asset substitutability, asset market segmentation
- Monetarists vs. Keynesians debates
  - Meltzer 1995, Tobin 1969;
  - Debate about quantitative significance

- Empirical evidence
  - Modigliani and Sutch (1967): Found little evidence that Operation Twist mattered in the 1960s;
  - Clouse, et. al. (2003). Bernanke, Reinhart, and Sack (2004): It would require extremely large open market operation in non-standard assets to have a significant impact on yields;
Wallace irrelevance result:

- Open market operations are neutral if (i) assets valued only for their pecuniary returns (ii) all investors can purchase arbitrary quantities of the assets at the same market prices.

- Under these conditions, size and composition of the central bank’s balance sheet are irrelevant for the equilibrium.

- Monetary policy can still matter by affecting the nominal interest rate on overnight balances.
The Curdia-Woodford model: households

- Household preferences:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ u^{\tau_t(i)}(c_t(i) : \xi_t) - \int_0^1 v^{\tau_t(i)}(h_t(j : i) : \xi_t) \right]
\]

where \( \tau_t(i) \in [b, s] \) is the household’s type and for \( \tau = b, s \),

\[
u^\tau(c : \xi) = \frac{c^{1-\frac{1}{\sigma_{\tau}}}(\bar{C}^\tau)^{\frac{1}{\sigma_{\tau}}}}{1 - \frac{1}{\sigma_{\tau}}}
\]

and \( \bar{C} \) is an exogenous preference disturbance.

- \( c_t(i) \) is a Dixit-Stiglitz aggregator with elasticity \( \theta \).
Household supplies continuum of labor types $j$ with disutility of work given by

$$v^\tau (h : \zeta_t) = \frac{\psi_t}{1 + \nu} h^{1+\nu} \bar{H}^{-\nu}$$

and $\bar{H}$ is an exogenous disturbance.

Production of good $j$ but a monopolistically competitive supplier:

$$y_t(j) = A_t h_t(j)^{1/\phi}, \phi > 1$$
The Curdia-Woodford model: households

- Each period, with probability $1 - \delta$, new household type is drawn (so with probability $\delta$ household type remains same as in previous period).
  - When new type drawn, it is $b$ with probability $\pi_b$ and $s$ with probability $\pi_s = 1 - \pi_b$.
  - Assume
    \[ u_c^b > u_c^s \text{ for all } c \]
  - So a type $b$ is more impatient to consume.
  - With heterogeneity among households, there is a role for financial intermediation. Type $b$ wants to borrow; type $s$ wants to save.

- Portfolio options available to households – deposits at or loans from bank and government bonds.
  - Bonds perfect substitute for deposits so both yield the same return.
  - Forcing lending to go through banks is a financial frictions.
The Curdia-Woodford model: households

- Households can sign state contingent contracts to insure against aggregate risk and risk of type, but receives transfers from these insurance contracts only occasionally.
  - Implies that all households have same expectations about marginal utility of consumption far enough into the future.
- Consumption is the same for all households of same type, and is a function of $\lambda^\tau_t$, time $t$ marginal utility of type $\tau = b, s$.
- Two Euler conditions:

  $$\lambda^\tau_t = \beta E_t \left\{ \left( \frac{1 + i^\tau_t}{1 + \pi_{t+1}} \right) \left[ \delta + (1 - \delta)\pi^\tau \right] \lambda^\tau_{t+1} + (1 - \delta)(1 - \pi^\tau)\lambda^{-\tau}_t \right\}$$

  where $\lambda^{-\tau}$ is marginal utility of type not $\tau$. 

Calvo price adjustment but driving force for inflation now depends on both $\lambda^b$ and $\lambda^s$.

- Can express new Keynesian Phillips curve as a standard linearized equation with the addition of

$$\Omega_t = \lambda_t^b - \lambda_t^s,$$

the marginal utilities gap. (see Curdia and Woodford 2009).

- This in turn will be related to the credit spread.
- Leads to a type of cost channel as in Christiano, Eichenbaum, and Evans (2005) or Ravenna and Walsh (2006).
Financial intermediaries

- Credit spread is

\[
1 + \omega_t \equiv \frac{1 + i_t^b}{1 + i_t^d} \geq 0
\]

where \( i^d \) is also the rate on government debt.

- Intermediaries take in deposits and make one-period loans. They also hold reserves \( M_t \) with the central bank that pay \( i_t^m \).

- Intermediaries take interest rates as given. The loan rate exceeds the deposit rate for two reasons:
  - Curdia and Woodford assume resources need to be used in originating loans.
  - Some borrowers do not repay.

- Intermediary balance sheet:

\[
d_t = m_t + L_t + \chi_t(L_t) + \Xi^P_t(L_t, m_t) + \pi_t^{FI}
\]

where \( m \) are real reserve holdings and \( L \) are loans, \( \chi_t(L_t) \) is the volume of bad loans extended, \( \Xi^P_t(L_t, m_t) \) are real resource costs and \( \pi_t^{FI} \) are payouts to shareholders.
Financial intermediaries

- Payout on deposits:

\[
\left(1 + i_t^d\right) d_t = (1 + i_t^b) L_t + (1 + i_t^m) m_t.
\]

- Combining,

\[
\pi_t^{FI} = d_t - m_t - L_t - \chi_t(L_t) - \Xi_t^P (L_t, m_t)
\]

\[
= \frac{(1 + i_t^b) L_t + (1 + i_t^m) m_t}{1 + i_t^d} - m_t - L_t - \chi_t(L_t) - \Xi_t^P (L_t, m_t)
\]

or

\[
\pi_t^{FI} = \left(\frac{i_t^d - i_t^b}{1 + i_t^b}\right) d_t - \left(\frac{i_t^m - i_t^b}{1 + i_t^b}\right) m_t + \chi_t(L_t) + \Xi_t^P (L_t, m_t)
\]
Financial intermediaries

- This last equation implies the FOC for loans is

\[ \chi_{Lt}(L_t) + \Xi_{Lt}(L_t, m_t) = \left( \frac{i_t^b - i_t^d}{1 + i_t^d} \right) = \omega_t \quad (28) \]

- The FOC for \( m \) is

\[ -\Xi_{mt}(L_t, m_t) = \delta_m \equiv \frac{i_t^d - i_t^m}{1 + i_t^d} \quad (29) \]

- Market clearing in the loan market:

\[ b_t = L_t + L_{cb} \]

where \( L_{cb} \) represents private sector borrowing from the central bank.
Central bank policy

- Balance sheet:
  \[ m_t = L^{cb}_t + b^{cb}_t \]
  where \( b^{cb}_t \) is central bank holdings of government debt.
- Resource costs of central bank lending to private sector is \( \Xi_t^{cb}(L^{cb}_t) \).
- Central bank pays interest \( i^m_t \) on reserves and receives \( i^d_t \) on its holdings of government debt.
- Demand for reserves and spread \( \delta^m \) satisfy joint inequalities:
  \[ m_t \leq m^d_t(L_t, \delta^m_t) \]
  \[ \delta^m_t \geq 0. \]
  - At least one must hold with equality.
  - If \( m_t = m^d_t \), then \( \delta^m > 0 \) and \( i^d_t > i^m_t \).
Central bank policy

- $i^d$ is the policy rate.
  - By adjusting $m$, central bank can control $\delta^m$.
  - By adjusting $i^m_t$, it can control level of $i^d_t$ for given $\delta^m_t$:

$$0 \leq i^m_t \leq i^d_t.$$

- Three dimensions of policy: $M_t$, $i^m_t$ and composition of balance sheet between $L^{cb}_t$ and $b^{cb}_t$.
  - Or interest rate $i^d_t$, $M_t$, (which then determines $i^m_t$) and credit policy $L^{cb}_t$. 

Welfare

- Depends on the standard new Keynesian variables plus
  - $\Omega$ – an increase in the spread between marginal utilities of two household types reduces welfare because it is associated with a less efficient allocation since a social planner would equate marginal utility across the two agents.
  - $\Xi$ since this is a resource cost of financial intermediation.

- Friedman rule holds – supply reserves up to their satiation level $\bar{m}$.
  - Note that this does not require $i^d = 0$, only $\delta^m = 0$. 
Optimal policy: quantitative easing

- If $\bar{m}$ is the satiation level of $m$, then optimal money supply policy has $m = \bar{m}$ but there is no value in expanding $m$ beyond this point except
  - since $L_{ct}^{cb} \leq m_t$, it could make sense (i.e., be optimal) to expand $m$ above $\bar{m}$ if it were necessary to increase lending by the central bank to the private sector and the central bank had already reduced its holdings of government debt (recall $m = L^{cb} + b^{cb}$).

- In the Curdia-Woodford model, there is no benefit directly of expanding $m$ above $\bar{m}$ in general – i.e., no role for quantitative easing if the increase in reserves finances central bank purchases of government debt (not additional lending to the private sector).
Optimal policy: quantitative easing

- This irrelevance result is conditional on the policy paths for $i_t^d$, $i_t^m$, and $L_t^{cb}$.
  - From central bank’s balance sheet,
    \[ m_t - b_t^g = L_t^{cb} \]
    so if $L_t^{cb}$ is given, the irrelevance result involves open market operations in government debt and when $i_t^d = i_t^m$ reflects a classic liquidity trap.
  - Auerbach and Obstfeld (2005) argue a permanent increase in $M$ is effective even when $i_t^d$ is currently at the zero bound, but that is because such an increase would not be consistent with the policy rule for $i_t^d$.
  - Therefore, Auerbach and Obstfeld are considering a rise in $M$ and a change in the expected future path of the policy rate.
Optimal policy: credit policies

- Assume \( m = \bar{m} \). Credit policy involves the central bank selling government bonds and making loans to the private sector.

- How does \( L^{cb} \) affect welfare?
  - For given \( b \) (private sector borrowing) it reduces \( L \) (private lending) and therefore saves on the resource cost of private intermediation.
    - This gain only occurs because Curdia and Woodford assume the central bank can lend costlessly – i.e., the central bank has an advantage in lending to the private sector over private intermediaries. Very unlikely so they add a resource cost \( \Xi^{cb}(L^{cb}_t) \) for central bank lending.
  - Since from (28)
    \[
    \chi_{Lt}(L_t) + \Xi^P_{Lt}(L_t, m_t) = \omega_t,
    \]
    a fall in \( L \) reduces \( \omega \), the spread between \( i^b \) and \( i^d \) falls and this improves welfare.
Optimal policy: credit policies

- Assume $\Xi^{cb}(0) > 0$ then if $\Xi^{cb}(0)$ is sufficiently high, a Treasuries only policy will always be optimal. However, there can be a critical $\Xi^{cb, crit}$ such that if it exceeds $\Xi^{cb}(0)$, credit policies optimally come into play.
- They show this is more likely to be the case at the zero lower bound, especially if the central bank cannot commit to a future path for the policy rate.
Optimal policy: credit policies

- Optimal first order conditions for $L_{cb}^t$ takes for

$$
\phi_{\Xi,t} \left[ \Xi_{Lt}^p \left( b_t - L_{cb}^t \right) - \Xi_L^{cb} \left( L_{cb}^t \right) \right] + \phi_{\bar{\omega},t} \left[ \Xi_{t}^{pp} \left( b_t - L_{cb}^t \right) + \chi_t'' \right] \left( b_t - L_{cb}^t \right) + \chi_t' \left( b_t - L_{cb}^t \right) + \chi_t \left( b_t - L_{cb}^t \right) + \chi_t \left( b_t - L_{cb}^t \right)
$$

where $\phi_{\Xi,t}$ is shadow value of a reduction in intermediation costs and $\phi_{\bar{\omega},t}$ is shadow value of a reduction in the spread $\bar{\omega}$ where over bar indicates these are evaluated when $i^d = i^m = 0$ so $m = \bar{m}$.

- Results depend on nature of the financial shock.
  - Shocks to private sector intermediation costs $\Xi_{t}^P \left( L_t, m_t \right)$.
  - Shocks to loan defaults $\chi_t \left( L_t \right)$. 
Define

\[ \overline{\Xi}_{cb}^{L} \left( L_{t} - L_{t}^{cb} \right) + \frac{\phi_{\omega,t}}{\phi_{\Xi,t}} \left[ \Xi_{t}^{p''} \left( b_{t} - L_{t}^{cb} \right) + \chi_{t}'' \left( b_{t} - L_{t}^{cb} \right) \right] = L_{t}^{cb} \left( L_{t}^{cb} \right) < \Xi_{L}^{cb, crit} \]

Then if

\[ L_{t}^{cb} > 0, \text{ i.e., if marginal cost of the central bank providing credit is less than the marginal private cost then it will be optimal for the central bank to engage in credit policies.} \]
Optimal policy: credit policies

- Value of $\Xi^{cb,crit}_L$ for different types of (serially correlated) shocks: Curdia and Woodford Figure 4, p. 69.

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**Fig. 4.** Response of the critical threshold value of $\Xi^b(0)$ for a corner solution, in the case of four different types of “purely financial” disturbances, each of which increases $\omega_T(L)$ by 4 percentage points. Interest-rate policy responds optimally in each case.
Different shocks all of which increase $\bar{\omega}$ by 1.2%:

- Multiplicative shock to $\bar{\Xi}^p$ – effects first and second derivative and so works via direct cost and spread channels.
- Additive $\chi_t$ shocks actually make credit easing by the central bank less desirable because this shock reduces $L$ and so $\bar{\Xi}^p_L$ falls and $\chi'$ falls (acting to reduce $\bar{\omega}$).

Figure above assumed policy interest rates adjusted optimally – benefits of credit easing increase at the zero lower bound, especially in a discretionary policy regime or under a Taylor rule.
Optimal policy: credit policies

Fig. 6. Impulse responses under optimal credit policy compared to those under a policy of “Treasuries only,” in the case of a “multiplicative Ξ” shock of the size considered in Fig. 4, if interest-rate policy follows a Taylor rule and $\Xi^{eb}$ is exactly equal to the steady-state critical threshold.
Conclusions on Curdia-Woodford

- If interest rate policy is optimal, no role for quantitative easing.
- Credit policies might be optimal but only if cost of central bank intermediation is not too large.
- Source of shocks matter – multiplicative shocks to cost of private sector intermediation raise marginal cost and spreads and can make central bank credit lending optimal.
  - More likely to be the case if interest rate policy is discretionary or follows a Taylor rule and therefore does not display inertia.
Gertler and Kiyotaki (2011)

- Earlier work focused on frictions affecting non-financial borrowers and treated intermediaries as a veil. Current crisis centered on intermediaries.
- Earlier work did not consider the types of unconventional policies used in the current crisis.
- Introduce agency problem between borrowers and lenders – creates a wedge between external finance cost and opportunity cost of internal finance.
- Assume financial intermediaries have skills in evaluating borrowers – makes it efficient for credit to flow through intermediaries.
- Households deposit funds with intermediaries, intermediaries lend to non-financial firms.
- Agency problem limits ability of intermediary to obtain funds from depositors. This accounts for wedge between deposit and loan rates. Spread widens in a crisis which raises cost of funds to non-financial firms.
Gertler and Kiyotaki (2011)

- Current crisis suggests intermediaries also have problems raising funds from other intermediaries, for example in an interbank market. So they assume intermediaries are subject to idiosyncratic liquidity shocks. Disruptions in interbank market can affect real activity. Financial markets become segmented and generate an inefficient allocation of funds among intermediaries.

- Continuum of firms of mass unity located on a continuum of islands.

- Investment opportunities arrive randomly to fraction $\pi^i$ of islands. No opportunity on fraction $\pi^n = 1 - \pi^i$ of the islands.
  - Only firms with investment opportunities can obtain new capital.

- Households deposit funds into banks; bank deposits are riskless one period securities.

- Households can also hold riskless one period government debt (a perfect substitute for bank deposits).
The model: banks

- Each period, banks choose an island to operate on prior to the resolution of any uncertainty.
- During period, bank can only make loans to firms on its island – localized lending.
- Because banks can freely move to another island at end of period, ex ante returns will be equal across islands.
- Banks raise funds in a national financial market.
  - Consists of a retail market in which banks raise funds from households and a wholesale market in which they raise funds from other banks.
- After retail market closes, investment opportunities are determined (i.e., it is like a limited participation model).
The agency problem

- After receiving deposit funds, banker can transfer fraction $\theta$ of divertable assets to own use.
  - Divertable assets are
    $$Q^h s^h_t - \omega b^h_t, \ 0 < \omega < 1$$
  - If bank diverts funds, it fails and creditors obtain $1 - \theta$ of the funds.
  - Because of bank’s incentive to divert funds, creditors will limit the amount they lend. This means banks will face a borrowing constraint.
  - With $\omega = 1$, interbank market operates frictionlessly in that bank cannot divert any of $b^h_t$. In this case, banks will not be constrained in borrowing in the interbank market.

- Let $V(s^h_t, b^h_t, d_t)$ be value function at the end of period $t$. Then to ensure bank does not divert funds, the incentive constraint
  $$V(s^h_t, b^h_t, d_t) = v_{st} s^h_t - v_{bt} b^h_t - v_t d_t \geq \theta \left( Q^h s^h_t - \omega b^h_t \right)$$
  must hold.
Bank’s decision problem

- Let $\lambda^h_t$ be Lagrangian multiplier on the incentive constraint. Let $\bar{\lambda}_t \equiv \sum_{h=i,n} \pi^h \lambda^h_t$ be average across states and islands.
- The FOCs then can be written as
  \[
  (v_{bt} - v_t)(1 + \bar{\lambda}_t) = \theta \omega \bar{\lambda}_t
  \]
- Marginal cost of interbank borrowing exceeds marginal cost of deposits if and only $\bar{\lambda}_t > 0$ and $\omega > 0$.
  \[
  \left(\frac{v_{st}}{Q^h_t} - v_{bt}\right)(1 + \lambda^h_t) = \lambda^h_t \theta (1 - \omega)
  \]
- The marginal value of assets in terms of goods, $v_{st} / Q^h_t$ exceeds the marginal interbank borrowing cost if $\lambda^h_t > 0$ and $\omega < 1$.
  \[
  v_t n^h_t \geq \left[\theta - \left(\frac{v_{st}}{Q^h_t} - v_t\right)\right] Q^h_t s^h_t - [\theta \omega - (v_{bt} - v_t)] b^h_t
  \]
- The value of net worth must be at least as large as weighted measure of the bank’s asset holdings with weights related to the value of diverting the asset.
Credit policies: lending facilities

- Central banks, unlike financial intermediaries, not constrained in raising funds.
- Central bank can intermediate credit by direct lending.
- In constrained financial markets, this expands credit supply and total lending.
- In unconstrained markets, it simply displaces private lending.
Credit policies: discount window lending

- When $\omega = 0$ (symmetric friction), central banks can borrow from banks with funds by issuing riskless government debt and lend to constrained banks at rate $R_{m,t+1}$.

- But this just displaces private interbank lending unless central bank has advantage in supply funds.

- Gertler and Kiyotaki assume central bank is able to enforce repayment better; $\theta(1 - \omega_g) < \theta$.

- For both private and central bank interbank lending to occur, central bank should set $R_{m,t+1}$ to make excess cost of discount window borrowing equal to $\omega_g$ times excess value of assets on investing islands.
  
  - If banks cannot default to central bank ($\omega_g = 1$), then penalty rate on discount window borrowing is equal to excess return on assets on investing islands.
  
  - Central bank could expand lending to drive excess returns to zero. (GK assume a limit on central bank lending.)
Labor market frictions

1. Sticky wages and labor wedges
   1. Implications for policy objectives and efficiency
   2. Effects on the long-run Phillips curve

2. Unemployment and the extensive margin
   1. Implications for policy objectives
   2. Effects of labor market structure
Sticky wages

- Erceg, Henderson, and Levin (1999) have employed the Calvo specification to incorporate sticky wages and sticky prices into an optimizing framework.
- The goods market side of their model is identical in structure to the one developed earlier.
- In the labor market, however, they assume individual households supply differentiated labor services; firms combine these labor services to produce output.
Sticky wages and prices
Implications

- The model of inflation adjustment based on the Calvo specification implied that inflation depended on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equalled the gap between the real wage and the marginal product of labor ($mpl$). Thus, letting $\omega_t$ denote the real wage,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (\omega_t - mpl_t).$$

- With flexible wages, workers were always on their labor supply curves; despite price stickiness, nominal wages could adjust to ensure the real wage equaled the marginal rate of substitution between leisure and consumption ($mrs$).
Policy objectives with sticky prices and wages

- When both wages and prices display stickiness, can’t keep $x = 0$ when there are productivity shocks
  - If price level is stabilized, sticky wages prevent the real wage from adjusting, so inefficient output variability results.
  - Similarly, stabilizing the wage level still leaves prices sticky so real wages cannot jump.
- Welfare costs arise from price dispersion and from wage dispersion.
- Approximation to the welfare of representative agent is now equal to the expected present discounted value of
  \[
  L_t = \frac{1}{2} \left( \lambda_p \pi_t^2 + \lambda_w \pi_{wt}^2 + \lambda_x x_t^2 \right)
  \]
  where $\pi_w$ is wage inflation, $\lambda_p + \lambda_w = 1$, and
  \[
  \frac{\lambda_w}{\lambda_p} \propto \frac{\kappa_p}{\kappa_w}.
  \]
- Place more weight on stabilizing wage inflation if wages are stickier than prices.
Unemployment and the business cycle
Figure: Civilian unemployment rate minus CBO estimate of NAIRU and cyclical output (HP filtered real GDP), 1964:1-2009:4
Unemployment and optimal monetary policy

- Dominant new Keynesian model
  - Has provided an important framework for thinking about policy.
  - Offers structure for conducting welfare-based policy analysis.

- But ... the framework has many shortcomings.
  - Lack of financial frictions or role for financial intermediation.
  - All labor adjustment at the intensive (hours) margin – *under-employment* but no *unemployment*.

- Questions:
  - Does stabilizing unemployment constitute a separate objective a flexible inflation targeter should pursue?
  - The canonical monetary policy model cannot cast light on the costs of inefficient fluctuations in unemployment relative to the costs of inflation volatility.
Most cyclical adjustment is at the extensive margin

Figure: Employment (blue), hours/worker (black), total hours (green), NFB, logs, HP filtered, 1964:1 -2009:4.
Adding unemployment to DSGE models

- Early work focused on dynamics:
  - Cooley and Quadrini (1999) – limited participation
  - Walsh (2003, 2005) – price stickiness, examined effect on persistence.

- Empirical work:
  - Ravenna and Walsh (2008) – tested the unemployment-based Phillips curve;
  - Trigari (2004) – estimated DSGE;
  - Sala, Söderström, Trigari (2008) – estimated US with wage and price rigidities;
  - Christoffel, Kuester, and Linzert (2006) – EU area;

- Policy analysis:
The basic DMP model has three components:

**Beveridge curve:**

\[
\Delta u_t = \rho (1 - u_t) + \mu \theta_t^a u_t, \quad \theta_t = \frac{v_t}{u_t}, \quad 0 < a < 1
\]

\[
\Delta u_t = 0 \Rightarrow u_t = \frac{\rho}{\rho + \mu \theta_t^a}
\]
The Beveridge curve

The U.S. Beveridge Curve

Civilian unemployment rate (%)

Vacancy rate (%)

The Beveridge Curve shifted out in the Great Recession

* 2000:12-2007:12
+ 2008:1-2009:12
x 2010:1-2012:2
The basic DMP model has three components (steady state version):

1. Beveridge curve;
2. A theory of job creation:

\[ V = -\kappa + q(\theta) (\mu z - w) + [1 - q(\theta)] V \Rightarrow (\mu z - w) = \frac{\kappa}{q(\theta)} \]

\[ q(\theta) (\mu z - w) = \kappa \]
The DMP model (Hall 2012a)

- The basic DMP model has three components:
  1. Beveridge curve;
  2. A theory of job creation:
  3. A bargaining model of wages (typically Nash):

\[ w - R = \beta (\mu z - R) \Rightarrow w = (1 - \beta)R + \beta \mu z \]

- Problem – the Shimer puzzle
Ravenna and Walsh (2011)

- Basic new Keynesian model for households and retail firms
- Add a Mortensen-Pissarides search and matching model of the labor market.
- Ignore adjustment on the intensive margin to focus on the extensive margin.
- Wages are flexible and set by Nash bargaining (but bargaining share is stochastic).
  - Keeps model very similar to basic NK model, but most recent work also assumes real wage stickiness.
Final goods

- Household obtain utility from consumption:
  \[ U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \]

- Consumption consists of market goods and home productions:
  \[ C_t = C_t^m + w^u (1 - N_t). \]

- The total expenditure on final goods from households and wholesale firms is
  \[ \int_0^1 P_t(j) C_t^m(j) \, dj + \kappa \int_0^1 P_t(j) v_t(j) \, dj = P_t(C_t^m + \kappa v_t) \]

- Goods market clearing:
  \[ Y_t = C_t^m + \kappa v_t \]
Wholesale goods, employment and wages

- Production by wholesale firm $i$ is

$$Y_{it}^w = Z_t N_{it},$$

where $Z_t$ is a common, aggregate productivity disturbance with a mean equal to 1 and bounded below by zero.

- Wholesale firms sell their output in a competitive market at the price $P_{t}^w$.

- The real marginal cost of a retail firm is the inverse of the retail-price mark up:

$$\frac{1}{\mu_t} \equiv \frac{P_{t}^w}{P_t}.$$
The labor market

- Wholesale firms must post vacancies to obtain new employees.
- If a job produces output $Z_t$ and $w_t$ is the wage paid to the worker, then the value of a filled job in terms of final goods is

$$J_t = \left( \frac{P_t^w}{P_t} \right) Z_t - w_t + (1 - \rho) \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) J_{t+1},$$

- The job posting condition is $q_t J_t = \kappa$, where $\kappa$ is the vacancy posting cost and $q_t$ is the probability of filling a vacancy, so

$$\frac{Z_t}{\mu_t} = w_t + \frac{\kappa}{q_t} - (1 - \rho) \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{\kappa}{q_{t+1}} \right)$$

- If $\kappa = 0$, this yields the standard result that $1/\mu_t = P_t^w / P_t = w_t / Z_t$. 
Employment dynamics

- Each period, an exogenous fraction $\rho$ of existing matches terminate. The number of unemployed job seekers in period $t$ is

$$u_t \equiv 1 - (1 - \rho) N_{t-1}.$$  

- Unemployed workers are matched stochastically with job vacancies, with matching process represented by a CRS matching function:

$$m(u_t, v_t) = \chi v_t^\alpha u_t^{1-\alpha} = \chi \theta_t^\alpha u_t$$

where $\theta_t \equiv v_t / u_t$ is the measure of labor market tightness, and $0 < \alpha < 1$.

- Aggregate employment evolves according to

$$N_t = (1 - \rho) N_{t-1} + m(u_t, v_t).$$
Wages and the relative price

- The equilibrium real wage under Nash bargaining is

\[ w_t = (1 - b_t) w^u + b_t \left[ \frac{Z_t}{\mu_t} + (1 - \rho) \left( \frac{1}{R_t} \right) \kappa E_t \theta_{t+1} \right] \]

- The relative price of wholesale goods in terms of retail goods is equal to \( P_w^t / P_t = 1 / \mu_t = \tau_t / Z_t \). where

\[ \zeta_t \equiv w^u + \left( \frac{1}{1 - b_t} \right) \left( \frac{\kappa}{q_t} \right) \]

\[-(1 - \rho) \left( \frac{1}{R_t} \right) E_t \left( \frac{1 - b_{t+1} \rho_{t+1}}{1 - b_{t+1}} \right) \left( \frac{\kappa}{q_{t+1}} \right). \]

- Labor market tightness affects inflation through \( \zeta_t \).
Linearized model

The unemployment-based Phillips curve

- The linearized Phillips curve takes the standard form:

\[ \pi_t = \beta E_t \pi_{t+1} - \delta \hat{\mu}_t. \]

- To obtain a Phillips curve in terms of unemployment gaps, we use the fact that real marginal cost can be expressed as a function of labor market tightness and \( \hat{u}_{t+1} = \rho_u \hat{u}_t - \alpha \rho \eta \hat{\theta}_t \) to obtain

\[ \pi_t = \beta E_t \pi_{t+1} - \delta \gamma_1 \hat{u}_{t+1} + \delta \gamma_2 \tilde{r}_t + \delta B \hat{b}_t, \]

where the \( a_i \) are functions of the model’s structural parameters.

- There is also a cost channel and bargaining shocks act like cost shocks in a basic NK model.
The second order approximation to welfare is

$$\sum_{i=0}^{\infty} \beta^i U(C_{t+i}) = \frac{U(\bar{C})}{1-\beta} - \frac{\varepsilon}{2\delta} U_c \bar{C} \sum_{i=0}^{\infty} \beta^i L_{t+i} + t.i.p.$$ 

where $t.i.p.$ denotes terms independent of policy, and the period-loss function is

$$L_t = \pi_t^2 + \lambda_0 \bar{c}_t^2 + \lambda_1 \bar{\theta}_t^2.$$ 

The weight on $\bar{c}_t^2$ is the same as that obtained in a standard NK model if utility is linear in hours worked.

Weight on labor market tightness is

$$\lambda_1 = (1 - \alpha) (\delta / \varepsilon) (\kappa \bar{V} / \bar{C}).$$
Distortions: intuition

\[ U(C_t) = U \left\{ \int \left[ \left( c_t(j) \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} + C_t^{\text{non-market}} \right\} \]

1. Inefficient volatility in consumption when the consumption gap is non-zero.
2. Inefficient composition of market consumption resulting from relative price dispersion due to non-zero inflation.
3. Inefficient composition of total consumption due to search frictions when the labor market tightness gap is non-zero.
### Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exogenous separation rate</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Vacancy elasticity of matches</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Replacement ratio</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Steady state vacancy filling rate</td>
<td>$q$</td>
</tr>
<tr>
<td>Labor force</td>
<td>$N$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Markup</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Price adjustment probability</td>
<td>$1 - \omega$</td>
</tr>
</tbody>
</table>
Optimal commitment – response to a bargaining shock

Figure: Response to a one standard deviation bargaining shock under optimal commitment ($\pi$ and $\theta$ scaled in percentage point deviations from steady state; unemployment scaled as percentage points of total labor force).
Optimal commitment – role of the loss function

We consider three alternative objectives for the central bank:

1. **The welfare based objective:**

   \[ L_t = \pi_t^2 + \lambda_0 \tilde{c}_t^2 + \lambda_1 \tilde{\theta}_t^2. \]

2. **A standard inflation-consumption gap loss function:**

   \[ L_t = \pi_t^2 + \lambda_0 \tilde{c}_t^2 \]

3. **An inflation and unemployment gap loss function:**

   \[ L_t = \pi_t^2 + \lambda \tilde{u}_t^2. \]
Table 2: Alternative policy objectives (optimal commitment)

<table>
<thead>
<tr>
<th>Quadratic loss Welfare cost*</th>
<th>Welfare-based loss Welfare cost*</th>
<th>$\sigma_\pi$</th>
<th>$\sigma_{\tilde{u}}$</th>
<th>$\sigma_{\tilde{\theta}}$</th>
<th>$\sigma_\pi / \sigma_{\tilde{u}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative to Opt. Commitment (%)</td>
<td>0</td>
<td>0</td>
<td>0.24</td>
<td>0.72</td>
<td>11.82</td>
</tr>
<tr>
<td>Std. Loss in $\pi$ and $\tilde{c}$–gap, $\lambda = \lambda_0$</td>
<td>4.59</td>
<td>0.0011</td>
<td>0.02</td>
<td>0.75</td>
<td>12.36</td>
</tr>
<tr>
<td>Std. Loss in $\pi$ and $\tilde{u}$–gap, $\lambda = 0.0035$</td>
<td>0.34</td>
<td>0.0001</td>
<td>0.22</td>
<td>0.72</td>
<td>11.86</td>
</tr>
<tr>
<td>Std. Loss in $\pi$ and $\tilde{u}$–gap, $\lambda = 0.0521$</td>
<td>275.93</td>
<td>0.0683</td>
<td>1.96</td>
<td>0.51</td>
<td>8.27</td>
</tr>
</tbody>
</table>

* Relative to welfare-based optimal commitment, as percent of steady-state consumption.
Optimal commitment – role of the loss function

Figure: Responses to one std dev. bargaining shock under optimal commitment policies minimizing different loss functions: o welfare loss; * eq. 32, + eq. 33 with $\lambda = 0.0035$; x eq. 33 with $\lambda = 0.0521$. 
U.S. versus EU. calibration

- Stylized comparison of U.S. and E. U. labor markets.
- Hold all parameters the same except rate of job destruction $\rho$, steady-state employment and replacement ratio.

U.S. values  
$\rho = 0.10$  
$\bar{N} = 0.9416$  
$\phi = 0.54$

EU values  
$\rho = 0.025$  
$\bar{N} = 0.0.8989$  
$\phi = 0.65$
Implied Phillips curves: U.S. and EU

U.S. calibration: $\pi_t = \beta E_t \pi_{t+1} - 0.087 \tilde{u}_{t+1} + 0.103 \tilde{r}_t + 0.081 \hat{b}_t$

EU. calibration: $\pi_t = \beta E_t \pi_{t+1} - 0.065 \tilde{u}_{t+1} + 0.845 \tilde{r}_t + 0.099 \hat{b}_t$

- Two differences are apparent.
  - Interest rate channel is much larger with EU calibration.
  - Inflation is less sensitive to the unemployment gap with EU calibration.

- These differences reflect the higher persistence of unemployment under the EU calibration ($\rho_u = 0.798$ for EU versus $0.345$ for U.S.)
  - When $\rho_u$ is large, both current and future labor market conditions move together; impact of current unemployment offset to some degree by the co-movement of expected future unemployment.

- EU faces better trade off.
Summary

- Developed micro-founded Phillips curve and simple policy model expressed in terms of inflation and an unemployment gap;
- Developed model consistent policy objectives that highlight role of labor market frictions;
- Distortions due to labor frictions can be summarized in terms of stabilizing a labor market tightness gap;
  - This distortion can also be expressed in terms of an unemployment gap.
- Inefficient fluctuations in unemployment pose policy trade-offs
  - But price stability remains close to optimal, particularly with EU calibration.
- Focusing on the wrong welfare function can lead to significant welfare losses.
Search frictions, wedges, and price stability

- Why do many models with frictions imply price stability is close to optimal?
- Tax interpretation. Suppose economy characterized by sticky prices, search frictions in the labor market, and monopolistic competition.
- Ravenna and Walsh (2011) show that the first best allocation can be supported using three taxes and monetary policy.
  - Price stability that ensures the retail price markup $\mu_t$ is constant.
  - A steady-state subsidy $\tau^H_t$ to retail firms to eliminate distortion due to imperfect competition to ensure the markup is constant at 1.
  - A tax (subsidy) $\tau^f_t$ on intermediate firms to ensure vacancy posting is efficient.
  - A tax on household labor consumption $\tau^C_t$ to ensure hours choice is optimal.
Tax instruments

- If the Hosios condition is satisfied, then the first best can be achieved with a steady-state subsidy to retail firms to eliminate distortion due to imperfect competition plus a policy of price stability. The other two taxes are not needed.

- If the Hosios condition is not met, then
  - \( \tau_t^f \) can be used to ensure the markup varies to ensure efficient vacancy posting.
  - This distorts the choice of hours so \( \tau_t^C \) is needed to ensure hours are efficient.
  - Monetary policy ensures price stability to eliminate the distortion created by price dispersion.
  - \( \tau^\mu \) ensures \( \mu = 1 \).
Using only monetary policy

- If wage bargaining is Nash but fails the Hosios condition, the intermediate sector tax that corrects firms’ incentive to post vacancy is large but basically acyclical.

- Monetary policy can replicate this but doing so requires little movements in markups, so price stability is close to optimal.

- With a wage norm at an inefficient level, the required intermediate sector tax must vary significantly to achieve efficient vacancy posting.

- Replicating this using only monetary policy would require large deviations from price stability and distort the hours choice, so the cost of trying to eliminate inefficient vacancy postings is large. Optimal policy improves only a little relative to price stability.
Is the long-run Phillips curve vertical?

- Akerlof, Dickens, and Perry (1996)
- Bengino and Ricca (2011)
- Coibion, Gorodichenko, and Wieland (2011).
Understanding costly resource mobility is important:

- In U.S. – for understanding whether recent high unemployment is structural in nature because of the inability of labor resources to shift quickly between uses.
- In EU – for understanding how the flow of resources among member countries affects EU-wide developments and inflation

DSGE policy models:

- Costly to adjust prices but labor and capital can move between firms without cost.
  - Perfectly integrated financial markets, perfect mobility of labor within members but absolute immobility across member states.
Key questions

1. How important is resource mobility for the transmission mechanism of monetary policy?

2. How important is resource mobility for the objectives of monetary policy?

- Resource mobility will matter for both what monetary policy can do and should do.
- Focus will be on labor mobility to illustrate this conclusion.
Sectoral dispersion and unemployment was a topic of debate in the 1980s.

- Lilien (1982)
- Abraham and Katz (1986)

Lilien’s index of dispersion:

\[
\sigma_t = \left[ \sum_{i=1}^{K} \left( \frac{e_{i,t}}{e_t} \right) (\Delta \log e_{i,t} - \Delta \log e_t)^2 \right]^{1/2}.
\]

Shifting Beveridge Curve has reopened debate.
Figure: The civilian unemployment rate, the vacancy rate, and sectorial dispersion (U.S.)
Table 1A
U.S.: Monthly 2000:12-2010:09

<table>
<thead>
<tr>
<th></th>
<th>Unemployment rate</th>
<th>Vacancy rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.31**</td>
<td>−0.16</td>
</tr>
<tr>
<td>$a_1$</td>
<td>−0.29**</td>
<td>0.09</td>
</tr>
<tr>
<td>$b_1$</td>
<td>1.01**</td>
<td>0.82**</td>
</tr>
<tr>
<td>$\sum_{i=1}^4 c_i$</td>
<td>0.01</td>
<td>−0.00</td>
</tr>
</tbody>
</table>

** Significant at the 5% level; * Significant at the 10% level.
Sectoral dispersion, unemployment and vacancies
Abraham and Katz (1986) regressions

Table 1B
U.S.: Monthly 2000:12-2010:09

<table>
<thead>
<tr>
<th></th>
<th>Unemployment rate</th>
<th>Vacancy rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.24**</td>
<td>-0.04</td>
</tr>
<tr>
<td>$a_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum_{i=1}^4 c_i$</td>
<td>-0.17**</td>
<td>0.06*</td>
</tr>
</tbody>
</table>

** Significant at the 5% level; * Significant at the 10% level.
Sectoral dispersion

- Using JOLTS data, sectoral dispersion is associated with higher unemployment, consistent with Lilien’s earlier findings.
- Vacancies are negatively (but not statistically significantly) related to sectoral dispersion;
  - This is evidence that the sectoral dispersion index is just reflecting cyclical factors;
  - But, weaker evidence against Lilien’s hypothesis than found by Abraham and Katz.
- Even if sectoral shifts do not raise the natural rate of unemployment, labor reallocations across firms, sectors, and time can be costly – with implications for macro dynamics and policy.
Models of costly labor allocation: three examples

1. Quadratic costs of adjusting employment;
2. Search with skill heterogeneity – composition effects;
Example 1: Quadratic costs of adjusting labor
Lechthaler and Snower (2011)

- If it is costly for firms to adjust their employment levels, then hiring decisions and labor utilization decisions will need to be forward looking, just as price setting behavior is.
- This also means that these adjustment costs can affect marginal costs and inflation.
- This affects the way the economy responds to disturbances – i.e. economic dynamics are affected.
- Volatile employment generates costs that monetary policy can affect.
- Implies stabilizing employment changes is a legitimate objective of monetary policy along side inflation and output gap stabilization.
Response to a markup shock: optimal commitment

Figure: Optimal response under commitment to a serial correlated markup shock in the quadratic costs of adjustment model.
Example 2: Skill heterogeneity?

Decline in vacancy yield

Figure: The U.S. unemployment rate, the vacancy rate, and the hiring yield (right scale).
Decline in vacancy yield

Figure: The hiring yield and forecasted yield based on labor market tightness (V/U). Forecast obtain from an OLS regresion of the yield on a constant and V/U, 2000:12 - 2009:12.
Example 2: Skill heterogeneity
Ravenna and Walsh (2010)

- Low skill and high skill workers.
- Low skill worker more likely to experience job separation.
- In a recessions, the skill mix of the unemployed shifts towards low skill workers:
  - Reduces the vacancy yield rate as firms see more job applicants they don’t want to hire;
  - Reduces incentive for firms to post vacancies;
  - Job finding rate falls because probability of finding a job for a low-skill worker falls and because low-skill workers become a larger share of the total unemployed.
Response to a neutral TFP shock
Role of composition effect

Figure: Skill heterogeneity: response to a negative productivity shock: Job finding and screening rates
Role of skill-bias shock
Example 3: Sector heterogeneity

- Two sectors, hiring costs are higher if worker previously employed in the other sector:
  - captures the idea that workers may have sector or job specific skills;
  - implies the composition of the pool of unemployed matters for job creation.
- Burst of unemployment in one sector pushes up unemployment but may weaken incentives for firms in other sectors to create job openings.
- Efficiency implications – employment reduction in one sector imposes a negative externality on firms in other sectors as average quality of the unemployed (from the perspective of other sectors) deteriorates.
Sector heterogeneity and costly labor search
A common productivity shock

Figure: Impulse responses to a serially correlated productivity shock to both sectors: Solid line is model without composition effects.
Sector heterogeneity and costly labor search

A sector specific productivity shock

Figure: Impulse responses of hours and employment to a negative productivity shock only to sector 1: Solid line is model without composition effects.
Is the long-run Phillips curve vertical?

- Akerlof, Dickens, and Perry (1996)
- Bengino and Ricca (2011)
- Coibion, Gorodichenko, and Wieland (2011).
Summary and implications

- Current DSGE policy models minimize costs of labor reallocation.
- The Great Recession in the U.S. does not overturn earlier conclusions about the link between sectoral dispersion and unemployment.
  - Evidence from Beveridge Curve and decline in vacancy yield suggests mismatch of workers and job openings may have increased.
- When labor reallocation is costly, the economy’s dynamics and the cost of fluctuations are affected.
  - Role for labor market objectives.
  - Low turnover in labor markets can raise the importance of inflation stability.
  - Composition effects may be important for macro dynamics and therefore for policy objectives and for designing monetary policy.
- These general conclusions will apply to other factors of production and to other situations in which there are costs of adjustment that reflect the imperfect mobility of resources.

- Four different types of firms: firms producing (1) domestic good, (2) importing consumption goods, (3) importing investment goods, and (4) exporting goods.
- Within each category, continuum of firms each producing a differentiated product.
- Domestic goods firms use labor and capital to produce output which they sell to a retailer.
- Retailer transforms domestic goods into a homogeneous final good sold to households.
- Relative prices: (1) domestic goods and final good; (2) domestic goods and imported consumption goods; (3) consumption and investment goods; (4) final good and export goods.
The $C_t$ that generates utility is a basket of domestically produced goods and imported consumption goods:

$$C_t = \left[ (1 - \omega_c) \frac{1}{\eta_c} \left( C_t^d \right)^{\frac{\eta_c-1}{\eta_c}} + \omega_c \frac{1}{\eta_c} \left( C_t^m \right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}$$

So domestic and imported goods are imperfect substitutes.

Households own the capital stock:

$$K_{t+1} = (1 - \delta) K_t + Y_t \left[ 1 - \tilde{S} \left( \frac{l_t}{l_{t-1}} \right) \right] l_t$$

They also supply labor and set wages ala Calvo, indexing to the inflation target and productivity.

- Retails bundle domestic goods via standard Dixit-Stiglitz CES aggregator.
- Leads to demand functions facing domestic firm $i$ and price aggregate of form

$$Y_{i,t} = \left( \frac{P_{i,t}^d}{P_t} \right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} Y_t; \quad P_t = \left[ \int_0^1 \left( P_{i,t}^d \right)^{\frac{1}{1-\lambda_t^d}} di \right]^{1-\lambda_t^d}$$

- $\lambda_t^d$ is time varying – markup is $1/\lambda_t^d$ so markup can vary.
- Production function of domestic firms:

$$Y_{i,t} = z_t^{1-\alpha} \varepsilon_t K_{i,t}^\alpha H_{i,t}^{1-\alpha} - z_t \phi$$

where $z$ is a unit root technology shock and $\varepsilon$ is a stationary aggregate productivity shock. $K$ is capital; $H$ labor.

- ALLV assume a fraction of a firm’s wage bill must be financed through loans – ignore this.
New Keynesian small open economy

- Continuum of importing consumption and investment firms; each buys a homogeneous good at price $P_t^*$ in world markets and transforms it into a differentiated good sold in the domestic economy.
  - Marginal cost for these firms is $S_t P_t^*$, where $S_t$ is the nominal exchange rate (domestic currency per unit of foreign currency)

- Exporting firms buy the homogeneous domestic final good and convert it into a differentiated export good.
  - Marginal cost for these firms is $P_t^d / S_t$; to generate one unit of domestic currency, the exporter needs to sell goods worth $1 / S_t$ in foreign currency which cost $P_t^d / S_t$ to purchase.

- All firms have sticky prices and time varying markups.
  - Leads to four Phillips curves (for domestic goods, imported consumption goods, imported investment goods, and exported goods.
  - Allows for imperfect pass through.
Domestic firms subject to Calvo price adjustment mechanism: firms adjust optimally with probability $1 - \zeta_d$. With probability $\zeta_d$, firms partially index to a combination of the central bank’s inflation target $\bar{\pi}_t^c$ and last period’s inflation rate: $P_{i,t} = \left[\pi_{t-1}^{\kappa_d} (\bar{\pi}_t^c)^{1-\kappa_d} \right] P_{i,t-1}$.

In terms of log-deviations from steady state,

$$\hat{\pi}_t^d - \hat{\pi}_t^c = \left(\frac{\beta}{1 + \kappa_d \beta}\right) E_t \left(\hat{\pi}_{t+1}^d - \hat{\pi}_{t+1}^c\right)$$

$$+ \left(\frac{\kappa_d}{1 + \kappa_d \beta}\right) \left(\hat{\pi}_{t-1}^d - \hat{\pi}_{t-1}^c\right)$$

$$+ \left(\frac{\kappa_d \beta}{1 + \kappa_d \beta}\right) \left(E_t \hat{\pi}_{t+1}^c - \hat{\pi}_t^c\right)$$

$$+ \frac{(1 - \zeta_d) (1 - \beta \zeta_d)}{\zeta_d (1 + \kappa_d \beta)} \left(m c_t + \lambda_t^d\right)$$

- Simplified small open economy: drop investment.
- The model consists of households who supply labor, purchase goods for consumption, and hold money and bonds, and firms who hire labor and produce and sell differentiated products in monopolistically competitive goods markets.
  - consumption risk is shared internationally.
- Each firm set the price of the good it produces, but not all firms reset their price each period.
- Households and firms behave optimally: households maximize the expected present value of utility and firms maximize profits.
Household utility

- Households consume a CES composite of home and foreign goods, defined as

\[ C_t = \left[ (1 - \gamma)^{\frac{1}{a}} \left( C_t^h \right)^{\frac{a-1}{a}} + \gamma^{\frac{1}{a}} \left( C_t^f \right)^{\frac{a-1}{a}} \right]^{\frac{a}{a-1}} \]

for \( a > 1 \).

- \( C^h \) (and \( C^f \)) are Dixit-Stiglitz aggregates of differentiated goods produced by domestic (and foreign) firms.

- Demand for good produced by firm \( j \):

\[ c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} C_t^h; \quad P_t = \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \]
Household utility

- Let $P_t^h$ (or $P_t^f$) be the average price of domestically (foreign) produced consumption goods. The problem of minimizing the cost $P_t^h C_t^h + P_t^f C_t^f$ of achieving a given level of $C_t$ yields:

$$\frac{C_t^h}{C_t^f} = \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{P_t^h}{P_t^f} \right)^{-a}.$$

- The aggregate price index is

$$P_t^c = \left( (1 - \gamma) (P_t^h)^{1-a} + \gamma (P_t^f)^{1-a} \right)^{\frac{1}{1-a}}.$$

is the aggregate (consumer) price index.
Household utility

- Household utility depends on its consumption of the composite good, money holdings, and on its labor supply:

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right].
\]

- Intertemporal optimization implies the standard Euler condition,

\[
C_t^{-\sigma} = \beta E_t R_t \left( \frac{P_t^c}{P_{t+1}^c} \right) C_{t+1}^{-\sigma},
\]

- Optimal labor-leisure choice requires that the marginal rate of substitution between leisure and consumption equal the real wage. This condition takes the form

\[
\frac{N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t^c}.
\]
Assume that the law of one price holds. This implies that

\[ P_t^f = S_t P_t^* , \]

where \( P_t^* \) is the foreign currency price of foreign-produced goods and \( S_t \) is the nominal exchange rate (price of foreign currency in terms of domestic currency).

This specification assumes complete exchange rate pass-through; given \( P_t^* \), a 1% change in the exchange rate produces a 1% change in the domestic currency price of foreign produced goods \( P_t^f \).
Law of one price and terms of trade

- Given that we have assumed the law of one price holds, the price levels in the domestic and foreign countries are linked by
  \[ P_t^c = S_t P_t^c*. \]

- For the rest of the world, we ignore the distinction between the CPI and the price of domestic production, so \( P_t^c* = P_t^* \).

- The terms of trade is equal to the relative price of foreign and domestic goods:
  \[ \Delta_t \equiv \frac{P_t^f}{P_t^h} = \frac{S_t P_t^*}{P_t^h}. \] (31)

- The real exchange rate is defined as the relative price of foreign produced goods (in terms of domestic currency) relative to the home country’s consumer price index:
  \[ Q_t = \frac{S_t P_t^*}{P_t^c}. \] (32)
International risk sharing

- Assume agents in both economies having access to a complete set of internationally traded securities.
- The time $t$ home currency price of a bond that pays off one unit of the domestic currency at time $t + 1$ is $R_{t}^{-1}$, and the Euler condition can be written as

$$
\beta E_t \left( \frac{P_t^c}{P_{t+1}^c} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} = \frac{1}{R_t}.
$$

- Since residents in the rest of the world also have access to these same financial securities, intertemporal optimization implies

$$
\frac{C_{t}^{*, -\sigma}}{S_t P_t^*} = \beta E_t \left( \frac{R_t}{P_{t+1}^* S_{t+1}} \right) C_{t+1}^{*, -\sigma}.
$$
International risk sharing implies,

\[
\beta E_t \left( \frac{P_t^c}{P_{t+1}^c} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} = \frac{1}{R_t} = \beta E_t \left( \frac{P_t^c}{P_{t+1}^c} \right) \left( \frac{Q_t}{Q_{t+1}} \right) \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma}.
\]

In turn, this implies

\[
C_t = \phi Q_t^{\frac{1}{\sigma}} C_t^*.
\] (33)

where \( \phi \) is a constant of proportionality.

For convenience, adopt \( \phi = 1 \) as a normalization.

This is consistent with a symmetric initial condition with zero net foreign asset holdings.
Uncovered interest parity

- If $R_t^*$ is the foreign interest rate, then

$$\beta E_t \left( \frac{P_t^*}{P_{t+1}^*} \right) \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} = \frac{1}{R_t^*}. $$

- Linearizing this yields

$$\sigma (E_t c_{t+1}^* - c_t^*) = r_t^* - E_t \pi_t^*$$

and doing the same for the domestic Euler condition implies

$$\sigma (E_t c_{t+1} - c_t) = r_t - E_t \pi_{t+1}^c.$$
Uncovered interest parity

- Using the definition of the terms of trade, we have that

\[
(r_t - E_t \pi^c_{t+1}) - (r^*_t - E_t \pi^*_t) = \sigma (E_t c_{t+1} - c_t) - \sigma (E_t c^*_{t+1} - c^*_t) = E_t q_{t+1} - q_t.
\]

- Subtracting the inflation terms from each side and using the definition of the real exchange rate yields the condition for uncovered interest parity:

\[
r_t = r^*_t + E_t (s_{t+1} - s_t).
\]

- The domestic nominal interest rate is equal to the foreign (rest of the world) nominal interest rate plus the expected rate of depreciation in the domestic currency.
The foreign country (ROW)

- Assume the foreign country is large relative to the home country.
- This is taken to mean that it is unnecessary to distinguish between consumer price inflation and domestic inflation in the foreign country, and that domestic output and consumption are equal.
- The foreign country’s demand for the home country’s output depends on the terms of trade. Assume that foreign households have the same preferences as those of the home country (so the demand elasticity is the same).
- The Euler condition for foreign country households implies

\[ y_t^* = E_t y_{t+1}^* - \left( \frac{1}{\sigma} \right) \left( i_t^* - E_t \pi_{t+1}^* \right), \]

or

\[ \rho_t^* \equiv i_t^* - E_t \pi_{t+1}^* = \sigma \left( E_t y_{t+1}^* - y_t^* \right). \]
Open economy expectational IS curve

- IS curve becomes

\[ y_t = E_t y_{t+1} - \left( \frac{1}{\sigma \gamma} \right) \left( i_t - E_t \pi_{t+1}^h - \rho^* \right) \]

where \( \sigma \gamma \equiv [1 - \gamma(1 - w)] / \sigma \) and \( \rho^* \equiv \rho - \sigma \gamma \gamma (1 - w) E_t \Delta y_{t+1}^* \) \((\rho = 1/\beta - 1)\).

- This is the small open economy equivalent to the closed economy expectational IS curve.

- Two primary differences between open and closed economy versions.
  - The elasticity of demand with respect to real interest rate no longer equal to \( 1/\sigma \). It equals \( 1/\sigma \gamma = [1 - \gamma(1 - w)] / \sigma \) which depends on the “openness” of the economy.
  - The real rate \( \rho^* \) depends on developments in the real of the world.
Inflation in domestic goods prices

- To determine the rate of inflation of domestically produced goods,
  \[
  \pi_t^h = \beta E_t \pi_{t+1}^h + \kappa \left( w_t - p_t^h - \varepsilon_t \right).
  \]

- The real consumption wage is \( w_t - p_t^c \), and this is related to the real product wage \( w_t - p_t^h \) by the terms of trade: \( p_t^c = p_t^h + \gamma \delta_t \). Since households equate the real consumption wage to the marginal rate of substitution between leisure and consumption,
  \[
  \eta n_t + \sigma c_t = w_t - p_t^c = w_t - p_t^h - \gamma \delta_t.
  \]

- Hence, real marginal cost is \( w_t - p_t^h - \varepsilon_t = \eta n_t + \sigma c_t + \gamma \delta_t - \varepsilon_t \).
Inflation in domestic goods prices

- We can now eliminate consumption and the terms of trade to obtain an expression for real marginal cost solely in terms of domestic output and foreign variables.

- Since \( y_t = n_t + \varepsilon_t \), \( y_t = c_t + (\gamma w / \sigma) \delta_t \), and \( y_t = \gamma^* + (1 / \sigma \gamma) \delta_t \),

\[
\pi^h_t = \beta E_t \pi^h_{t+1} + \kappa (\eta + \sigma \gamma) (y_t - \tilde{y}_t),
\]

where

\[
\tilde{y}_t \equiv [(\sigma - \sigma \gamma) y^*_t - (1 + \eta) \varepsilon_t] / (\eta + \sigma \gamma)
\]
Parallels with the closed economy NK model

- Define the output gap as

\[ x_t \equiv y_t - \tilde{y}_t. \]  

(35)

- Then we can rewrite the Euler condition as

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma_\gamma} \right) \left( i_t - E_t \pi^h_{t+1} - \tilde{\rho}_t \right) \]  

(36)

\[ \pi^h_t = \beta E_t \pi^h_{t+1} + \kappa (\eta + \sigma_\gamma) x_t. \]  

(37)

where

\[ \tilde{\rho}_t = \rho^* + \sigma_\gamma E_t (\tilde{y}_{t+1} - \tilde{y}_t). \]  

(38)
Policy objectives

- The SOE model, as Clarida, R., J. Galí, and M. Gertler (2001) have emphasized, is isomorphic to the closed economy new Keynesian model.
- The parallel is even stronger if the central bank’s objective can be represented as minimizing a quadratic from of the output gap and the inflation rate of domestically produced goods $\pi^h_t$.
- In this case, the central bank’s policy involves minimizing

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ (\pi^h_{t+i})^2 + \lambda x^2_{t+i} \right]$$

subject to the inflation adj. eq. and Euler eq.

- All the conclusions about policy reached there would apply without modification to the small open economy.
- The critical requirement is that the inflation rate appearing in the central bank’s objective function must be $\pi^h$ and not the inflation rate in consumer prices $\pi^c$.
Policy implications

- Policy trade-offs only arise from inflation shocks as long as GDP inflation and not CPI inflation enters the objective function of the central bank.
  - If CPI inflation matters, trade-offs are more complicated. An appreciation reduces firms’ marginal costs and reduces GDP inflation.
  - In face of a positive shock to aggregate spending, the central bank must raise the nominal interest rate to stabilize the output gap. But this leads to an appreciation of the exchange rate and a decline in CPI inflation.
Optimal policy

- Distortions in the closed economy:
  - Steady-state markup and sticky prices
  - Use fiscal subsidy to deal with first, price stability to deal with second.

- Distortions in the open economy:
  - Steady-state markup, sticky prices, and possibility of affecting terms of trade to benefit domestic consumers affects incentives of central bank.
    - A monetary expansion that lowers domestic interest rates causes a depreciation of the currency and lowers prices of home production relative to foreign goods. This competitive devaluation increases demand for home production, and increases $C_t$ relative to $C_t^*$.
    - A rise in $s$ increases $y$ relative to $y^*$ and $c$ relative to $c^*$.
  - In special case of $\sigma = \eta = \gamma = 1$, can still use fiscal subsidy to deal with first, domestic price stability to deal with second.
Special case

In the special case of $\sigma = \eta = \gamma = 1$, the flex-price equilibrium is efficient if

$$1 - \tau = (1 - \alpha)^{-1} \left( 1 - \frac{1}{\varepsilon} \right).$$

If $\tau$ is set to satisfy this condition, then the optimal policy ensures the output gap is zero (output equals the efficient flex-price level) and from the NKPC, domestic inflation is zero.

In this special case, domestic inflation targeting is optimal.
Domestic inflation targeting

- Continuing with the special case, output moves with the flex-price output level.
- A rise in world output has two effects on domestic output:
  - A contractionary effect due to expenditure switching as a rise in world output improves the terms of trade (a real appreciation as the price of foreign goods falls as their supply increases). This reduces demand and domestic production.
  - An expansionary effect due to the direct demand effect through higher exports.
  - If $\omega > 1$ ($\omega < 1$), contractionary (expansionary) effects dominate.
  - When $\omega = 1$, terms of trade and domestic output are unchanged in the face of a change in world output under an optimal domestic inflation targeting policy.
- With domestic prices constant, the CPI is given by

$$p_t^f = \alpha \left( e_t^f + p_t^* \right) = \alpha s_t^f.$$
Second order approximation to utility

- In special case of $\sigma = \eta = \gamma = 1$, losses relative to optimal policy for domestic representative household is

$$W = - \left( \frac{1 - \alpha}{2} \right) \left( \frac{\epsilon}{\lambda} \right) \sum_{t=0}^{\infty} \beta^t \left[ \pi_{h,t}^2 + \frac{\lambda(1 + \varphi)}{\epsilon} \chi_t^2 \right]$$

- Taking conditional expectations and letting $\beta \to 1$,

$$V = - \left( \frac{1 - \alpha}{2} \right) \left( \frac{\epsilon}{\lambda} \right) \left[ \text{var}(\pi_{h,t}) + \frac{\lambda(1 + \varphi)}{\epsilon} \text{var}(\chi_t) \right]$$
Comparing policies

- G-M compare optimal policy to a Taylor rule based on domestic price inflation (DITR), a Taylor rule based on CPI inflation (CITR), and an exchange rate peg. They calibrate domestic and world productivity shocks using Canadian and U.S. data respectively.

- In face of a domestic productivity shock, DITR leads to a rise in the CPI (there is a real depreciation due to the productivity shock) while under CITR, the domestic price level falls (which requires a negative output gap).

- Under a peg, responses are similar to under CITR but peg makes domestic and CPI price levels stationary (as they are under optimal policy).

- Optimal policy involves greater terms of trade and nominal exchange rate volatility than DITR or CITR (but domestic inflation and output gap are keep equal to zero under the optimal policy).

- Peg yields large welfare losses as it makes TOT too smooth. DITR does better than CITR (but that is presumably because under the case considered, it is optimal to stabilize the domestic price level).
The assumptions of the law of one price and UIP inconsistent with data.

Introducing new distortions:

- Imperfect pass through arises because importers prices are sticky (Monacelli 2003).
- For UIP, ALLV note that risk premia are strongly negatively correlated with expected exchange rate changes.

\[ \hat{R}_t = \hat{R}_t^* + E_t \Delta S_{t+1} + \Phi_t \]

They assume risk premium is

\[ \Phi (a_t, S_t, \tilde{\phi}_t) = -\tilde{\phi}_s (E_t \Delta S_{t+1} - \Delta S_t) - \tilde{\phi}_a \hat{a}_t \]
Role of deviations from UIP: ALLV (2008)

Fig. 1. Impulse response functions (posterior median and 95% uncertainty intervals) to a one standard deviation monetary policy shock, with standard (dashed lines) and modified UIP condition (solid lines). Note: Inflation and interest rates are reported as annualized quarterly rates while quantities as log-deviations from steady state.
The monetary union model of Benigno (2004)

- Two regions and a single monetary authority. Two fiscal authorities.
- Preferences of household \( j \) in region \( i \):

\[
U_t^i = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ U(C_s^j) + L \left( \frac{M_s^j}{P_s^i}, \xi_s^i \right) - V(y_s^j, z_s^i) \right] \right\}
\]

and \( i = H \) is \( j \in [0, n] \), \( i = F \) if \( j \in [n, 1] \).
- Each household is a consumer of all goods and a producer of a differentiated product.
- Consumption bundle of household \( j \) is

\[
C^j \equiv \frac{\left( C_H^j \right)^n \left( C_F^j \right)^{1-n}}{n^n (1-n)^{(1-n)}}, \; n \in [0, 1]
\]

where \( \sigma > 1 \) and

\[
C_H^j \equiv \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \left( \int_0^n c_s^j(h) \frac{\sigma-1}{\sigma} dh \right) \right]^{\frac{\sigma}{\sigma-1}}
\]
The model: prices

- Price of consumption bundle for region $i$:

$$P^i = \left( P^i_H \right)^n \left( P^i_F \right)^{1-n}$$

where

$$P^i_H = \left[ \left( \frac{1}{n} \right) \left( \int_0^n p^i_t(h)^{1-\sigma} dh \right) \right]^{\frac{1}{1-\sigma}}$$

$$P^i_F = \left[ \left( \frac{1}{1-n} \right) \left( \int_n^1 p^i_t(f)^{1-\sigma} df \right) \right]^{\frac{1}{1-\sigma}}$$

- $p^i(h)$ is the price of good $h$ sold in region $i$.
- No transportation costs and $p^H(h) = p^F(h)$ and $p^H(f) = p^F(f)$ which implies $P^H = P^F$.
- Terms of trade $T = P^F / P^H$. 
The model: demand for individual goods

- For goods $h$ and $f$,

$$c^j(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} T^{1-n} C^j; \quad c^j(f) = \left[ \frac{p(f)}{P_F} \right]^{-\sigma} T^{-n} C^j$$

- Fiscal authorities only purchase goods from its own region and has demand elasticity $\sigma$.

- Hence, total demand for goods $h$ and $f$ are

$$y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} \left[ T^{1-n} C^W + G^H \right]; \quad y^d(f) = \left[ \frac{p(f)}{P_F} \right]^{-\sigma} \left[ T^{-n} C^j + G^F \right]$$

where $C^W = \int_0^1 C^j \, dj$. 
The model: asset markets

- Complete within regions; incomplete across regions.
- Within each region, identical initial wealth, perfect risk sharing – representative household
- Internationally traded nominal bond with interest rate $R_t$. Euler equation is

$$U_C(C_t^i) = (1 + R_t) \beta E_t \left[ \frac{P_t}{P_{t+1}} U_C(C_{t+1}^i) \right], \; i = H, F$$

- Aggregate resource constraint for each region:

$$C_t^i + \frac{B_t^i}{P_t(1 + R_t)} = \frac{B_{t-1}^i}{P_t} + C_t^W.$$  

- Assumption that $B_{t-1}^i = 0$ and unitary elasticity of substitution implies $C^W = C^H = C^F$. 


The flexible-price equilibrium

- The following conditions characterize the equilibrium under flexible prices in terms of log deviations around the steady state:

\[ \tilde{C}_t^W = \left( \frac{\eta}{\rho + \eta} \right) \left( \tilde{Y}_t^W - g_t^w \right) \]

\[ \tilde{Y}_t^W = \left( \frac{\eta}{\rho + \eta} \right) \tilde{Y}_t^W + \left( \frac{\rho}{\rho + \eta} \right) g_t^w \]

\[ \tilde{R}_t = \left( \frac{\rho \eta}{\rho + \eta} \right) \mathbb{E}_t \left[ \left( \tilde{Y}_{t+1}^W - \tilde{Y}_t^W \right) - \left( g_{t+1}^w - g_t^w \right) \right] \]

\[ \tilde{T}_t = \left( \frac{\eta}{1 + \eta} \right) \left( g_t^R - \tilde{Y}_t^R \right) \]

where \( \tilde{Y}^W \) and \( g^W \) are world productivity and government purchase shocks and \( Y^R \) and \( g^R \) are relative shocks (e.g., \( \tilde{Y}^R = \tilde{Y}^F - \tilde{Y}^H \) so a relative productivity shock to \( F \) reduces \( P^F \) relative to \( P^H \)).
The sticky-price equilibrium

- The following conditions characterize the equilibrium under flexible prices in terms of log deviations around the steady state:

\[
\hat{C}_t^W = E_t \hat{C}_{t+1} - \left( \frac{1}{\rho} \right) \left( \hat{R}_t - E_t \pi_{t+1}^W \right)
\]

\[
\hat{Y}^H = (1 - n) \hat{T}_t + \hat{C}_t^W + g_t^H; \quad \hat{Y}^F = -n \hat{T}_t + \hat{C}_t^W + g_t^F
\]

- In terms of gaps,

\[
c_t^W = \hat{C}_t^W - \bar{C}_t^W = \tilde{y}_t^W = E_t c_{t+1}^W - \left( \frac{1}{\rho} \right) \left( \hat{R}_t - E_t \pi_{t+1}^W - \tilde{R}_t \right)
\]
The sticky-price equilibrium

- Inflation rates:

\[ \pi_t^H = \beta E_t \pi_{t+1}^H + k_C^H y_t^W + (1 - n) k_T^H (\hat{T}_t - \tilde{T}_t) \]

\[ = \beta E_t \pi_{t+1}^H + k_C^H y_t^H + (1 - n) (k_T^H - k_C^H) (\hat{T}_t - \tilde{T}_t) \]

\[ \pi_t^F = \beta E_t \pi_{t+1}^F + k_C^F y_t^W - nk_T^F (\hat{T}_t - \tilde{T}_t) \]

\[ = \beta E_t \pi_{t+1}^F + k_C^F y_t^F - n (k_T^F - k_C^F) (\hat{T}_t - \tilde{T}_t) \]

\[ \hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H \]

- Looks like the wage-price block when wages and prices are sticky.
Welfare

- Assume a fiscal subsidy to offset the distortions due to monopolistic competition.
- Benigno then shows that the second-order approximation to the welfare of the representative household is

\[ W_t = -\Omega E_0 \sum_{t=0}^{\infty} \beta^t L_t \]

\[ L_t = \Lambda \left( y_t^W \right)^2 + n(1-n)\Gamma \left( \hat{T}_t - \tilde{T}_t \right)^2 + \gamma \left( \pi_t^H \right)^2 + (1-\gamma) \left( \pi_t^F \right)^2 + t.i.p. + o \left( || \zeta ||^3 \right) \]

- Union-wide output gap, terms of trade gap, and inflation rates in each region matter.
  - Movements in \( \hat{T}_t \) reduce welfare if they deviate from \( \tilde{T}_t \). Relative prices in the two regions should move to reflect changes in \( \tilde{T} \).
- Inflation enters as weighted average of inflation squared in two regions.
Welfare

- Basic intuition similar to that with sticky wages and prices, or any two sector model with differing degrees of nominal rigidity.
- With multiple nominal rigidities, the single instrument of monetary policy cannot eliminate all distortions.
- Stabilizing a measure of inflation does not ensure that relative prices can adjust appropriately.
- Will be optimal to focus most on
Which inflation rate?

- The loss function involves

\[ \gamma \left( \pi^H_t \right)^2 + (1 - \gamma) \left( \pi^F_t \right)^2 \]

where

\[ \gamma = \frac{nd^H^H}{nd^H^H + (1 - n)d^F} \]

\[ d^H = \frac{\alpha^H}{(1 - \alpha^H)(1 - \beta\alpha^H)} \]

\[ d^F = \frac{\alpha^F}{(1 - \alpha^F)(1 - \beta\alpha^F)} \]
Which inflation rate?

- If prices are equally sticky in each region, $\alpha^H = \alpha^F$ and $\gamma = n$. Weight related to size.
  - In this case, if $\pi^R \equiv \pi^F - \pi^H$,
    \[
    \gamma \left( \pi^H_t \right)^2 + (1 - \gamma) \left( \pi^F_t \right)^2 = \left( \pi^W_t \right)^2 + n(1 - n) \left( \pi^R_t \right)^2.
    \]

- If $\alpha^H = \alpha^F$, optimal to set $\pi^W = n\pi^H + (1 - n)\pi^F = 0$ since relative prices are out of the monetary authority’s control.

- $\pi^W = \pi^{HICP}$: harmonized index of consumer price inflation.

- If prices are flexible in one regions (say $F$), then $d^F = 0$ and $\gamma = 1 - \text{welfare only depends on } H \text{ inflation.}$
Currency union (Galí and Monacelli JIE 2008)

- Currency union consists of a continuum of small open economics.
- Each individual country, indexed by \( i \in [0, 1] \), is of measure zero.
- Shared preferences, technologies and market structure.
- Preferences of household
Households

- Representative household of country \( i \) maximizes

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, G_t); \quad U(C_t, N_t, G_t) = (1 - \chi) \log C_t + \chi \log G_t - \frac{(N_t)}{1 + \lambda}
\]

where \( G_t^i \) is a measure of public consumption and

\[
C_t^i \equiv \frac{(C_t^i)^{1-\alpha} (C_F^i)^{\alpha}}{(1 - \alpha)^{(1-\alpha)} \alpha^\alpha}, \quad \alpha \in [0, 1]
\]

where

\[
C_{i,t}^i \equiv \left( \int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}
\]

and \( j \in [0, 1] \) is in index of the type of good.

- \( \alpha \) is the weight of imported goods in private consumption. \( \alpha < 1 \) implies home bias; \( \alpha \) can be viewed as an index of openness.
Households

- In addition, $C_{F_i,t}$ is an index of imported goods consumed by country $i$, and

$$C_{F_i,t} = \exp \int_0^1 c_{f,t}^i df, \quad c_{f,t}^i = \log C_{f,t}^i$$

where $C_{f,t}^i$ is the index of country $i$’s consumption of goods imported from country $f$ and

$$C_{f,t}^i = \left( \int_0^1 C_{f,t}^i (j) \frac{\varepsilon - 1}{\varepsilon} dj \right) ^{\frac{\varepsilon}{\varepsilon - 1}}.$$

- Budget constraint is

$$\int_0^1 P_t^i (j) C_{i,t}^i (j) dj + \int_0^1 \int_0^1 P_t^f (j) C_{f,t}^i (j) djf + E_t \left( Q_{t,t+1} D_{t+1}^i \right) \leq D_t^i +$$

where $P_t^i (j)$ and $P_t^f (j)$ are prices (in the single currency). $D_{t+1}^i$ is the nominal payoff in $t + 1$ of portfolio held at end of $t$.

- Complete set of contingent claims markets and $Q_{t,t+1}$ is stochastic discount factor for one-period ahead nominal payoffs – common across countries.
Household choices

- Linearized versions of labor supply and Euler conditions

\[ w^i_t - p^i_{c,t} = c^i_t + \varphi n^i_t - \log (1 - \chi) \]

\[ c^i_t = E_t c^i_{t+1} - (r^*_t - E_t \pi^i_{c,t+1} - \rho) \]

where \( \rho \equiv -\log \beta \). These hold for all \( i \).

- \( p^i_{c,t} \) is log consumer price index for country \( i \).
  - Consumer price index is

\[ p^i_{c,t} = (1 - \alpha)p^i_t + \alpha p^*_t \]

where for each individual country, \( p^*_t \) is log index of imported goods prices.
Definitions

- Bilateral terms of trade:
  \[ S_{f,t}^i \equiv \frac{P_f^t}{P_t^i} \]
  which is the price of country f’s domestically produced goods in terms of country i’s.

- Effective terms of trade for country i:
  \[ S_t^i \equiv \frac{P_t^*}{P_t^i} = \exp \int_0^1 \left( P_f^t - P_t^i \right) df = \exp \int_0^1 s_{f,t}^i df \]
  where \( s_{f,t}^i = \log S_{f,t}^i \). In logs, \( s_t^i = \int_0^1 s_{f,t}^i df \).

- Then
  \[ P_{c,t}^i = P_t^i (S_t^i)^\alpha \Rightarrow p_{c,t}^i = p_t^i + \alpha s_t^i \quad (39) \]
  and
  \[ \pi_{c,t}^i = \pi_t^i + \alpha \Delta s_t^i \]
International risk sharing and gov’t purchases

- Assume symmetric initial conditions with zero asset holdings,

\[ c_t^i = c_t^f + (1 - \alpha) s_{f,t}^i \]

and then integrating over \( f \)

\[ c_t^i = c_t^* + (1 - \alpha) s_t^i; \quad c_t^* \equiv \int_0^1 c_t^f \, df \] (40)

- Government buys individual goods in same proportions as households:

\[ G_t^i \equiv \left( \int_0^1 G_t^i(j) \left( \frac{\epsilon - 1}{\epsilon} \right) \, dj \right)^{\frac{\epsilon}{\epsilon - 1}} \]

- Government only buys domestic goods.
Firms

- Continuum of firms in each country with technology
  \[ Y_t^i(j) = A_t^i N_t^i(j) \]

- Real marginal cost is the same for all firms in \( i \)
  \[ mc_t^i = \log(1 - \tau_t^i) + w_t^i - p_t^i - a_t^i \]
  where \( \tau_t^i \) is an employment subsidy.

- Aggregate output and employment (to first order) given by
  \[ y_t^i = n_t^i + a_t^i \]

- Calvo with \( 1 - \theta \) firms resetting prices each period and optimal price setting rule is
  \[ \bar{p}_t^i = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t (mc_{t+k}^i + p_{t+k}^i) \]  \( (41) \)
Market clearing

- For good \( j \) produced in country \( i \):

\[
Y_t^i(j) = C_{i,t}^i(j) + \int_0^1 C_{i,t}^f(j) df + G_t^i(j)
\]

\[= \left( \frac{P_t^i(j)}{P_t^i} \right)^{-\varepsilon} \left[ C_t^i \left( S_t^i \right)^{\alpha} + G_t^i \right] \tag{42}
\]

- Integrating (42) over \( j \), and taking log linear approximation yields

\[
\hat{y}_t^i = (1 - \gamma) \left( \hat{c}_t^i + \alpha \hat{s}_t^i \right) + \gamma \hat{g}_t^i \tag{43}
\]

where \( \hat{x} \) is a deviation from steady state and \( \gamma \equiv G/Y \).

- But \( c_t^i = c_t^* + (1 - \alpha)s_t^i \) and \( s_t^i = p_t^* - p_t^i \), so

\[
\hat{y}_t^i = (1 - \gamma) \hat{c}_t^* + (1 - \gamma) \left( p_t^i - p_t^* \right) + \gamma \hat{g}_t^i \tag{44}
\]

- Integrating over \( i \) yields

\[
\hat{y}_t^* = (1 - \gamma) \hat{c}_t^* + \gamma \hat{g}_t \tag{45}
\]
Inflation

- NKPC:
  \[ \pi_t^i = \beta E_t \pi_{t+1}^i + \lambda \hat{mc}_t^i \] (46)

- Marginal cost is
  \[ mc_t^i = (w_t^i - p_{c,t}^i) + (p_{c,t}^i - p_t) - a_t^i + \log(1 - \tau_t^i) \]
  \[ = c_t^i + \phi n_t^i + \alpha s_t^i - a_t^i + \log(1 - \tau_t^i) - \log(1 - \chi) \] (47)

- Use (47), \( \hat{y}_t^i = (1 - \gamma) (\hat{c}_t^i + \alpha \hat{s}_t^i) + \gamma \hat{g}_t^i \) and \( y_t^i = n_t^i + a_t^i \) to obtain
  \[ \hat{mc}_t^i = \left( \frac{1}{1 - \gamma} + \phi \right) \hat{y}_t^i - \left( \frac{\gamma}{1 - \gamma} \right) \hat{g}_t^i - (1 + \phi) a_t^i \] (48)

  Given output, a rise in government spending generates an appreciation which lowers the real product wage and so lowers marginal cost.

- NKPC for entire union is
  \[ \pi_t^* = \beta E_t \pi_{t+1}^* + \lambda \hat{mc}_t^* \]
  \[ = \beta E_t \pi_{t+1}^* + \lambda \left( \frac{1}{1 - \gamma} + \phi \right) \hat{y}_t^* - \lambda \left( \frac{\gamma}{1 - \gamma} \right) \hat{g}_t^* - \lambda (1 + \phi) a_t^* \]
With sticky prices

- Let $\tilde{y}$ be the deviation of $\hat{y}$ from the flexible-price, efficient equilibrium levels (in which $y_t^i = a_t^i$ and $g_t^i = \gamma + a_t^i$) where $a_t^i$ is country $i$ productivity shock and let $\tilde{f}_t = \tilde{g}_t - \tilde{y}_t$ be the fiscal gap (equal to zero in the efficient equilibrium).

- Then from

$$\hat{m}_t^i = (1 + \varphi) \tilde{y}_t^i - \left( \frac{\gamma}{1 - \gamma} \right) \tilde{f}_t^i$$

- Then

$$\pi_t^i = \beta E_t \pi_{t+1}^i + \lambda (1 + \varphi) \tilde{y}_t^i - \left( \frac{\lambda \gamma}{1 - \gamma} \right) \tilde{f}_t^i$$

- For the union wide,

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \lambda (1 + \varphi) \tilde{y}_t^* - \left( \frac{\lambda \gamma}{1 - \gamma} \right) \tilde{f}_t^*$$
With sticky prices

- The fiscal gap acts like a cost shock and forces policy trade-offs between union-wide price stability and maintaining a zero output gap.
- If the fiscal gap is zero, no trade-off is faced by the union central.
- If the individual country decisions about fiscal policy causes $\tilde{f}_t^i$ to deviate from zero, then it is just like a cost shock.
Optimal monetary and fiscal policy

- Second order approximation to the average utility loss of union households due to fluctuations around the efficient steady state is

\[ W \simeq \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int_0^1 \left[ \frac{\varepsilon}{\lambda} (\pi^i_t)^2 + (1 + \varphi)(\tilde{y}^i_t)^2 + \frac{\gamma}{1 - \gamma} (\tilde{f}^i_t)^2 \right] di + t.i.p. \]

- Policy problem:

\[
\min \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int_0^1 \left[ \frac{\varepsilon}{\lambda} (\pi^i_t)^2 + (1 + \varphi)(\tilde{y}^i_t)^2 + \frac{\gamma}{1 - \gamma} (\tilde{f}^i_t)^2 \right] di
\]

subject to

\[
\pi^i_t = \beta E_t \pi^i_{t+1} + \lambda (1 + \varphi) \tilde{y}^i_t - \left( \frac{\lambda \gamma}{1 - \gamma} \right) \tilde{f}^i_t
\]

\[
\Delta \tilde{y}^i_t - \Delta \tilde{y}^i_* - \left( \frac{\gamma}{1 - \gamma} \right) (\Delta \tilde{f}^i_t - \Delta \tilde{f}^i_*) + \left[ (\pi^i_t - \pi^*_t) + (\Delta a^i_t - \Delta a^*_t) \right] = 0
\]

and aggregation definitions (i.e., \( \pi^*_t = \int \pi^i_t \, di \), \( \tilde{y}^*_t = \int \tilde{y}^i_t \, di \) and \( \tilde{f}^*_t = \int \tilde{f}^i_t \, di \)).
First order conditions

- Let $\psi_{\pi,t}^i$ be Lagrangian multipliers on the NKPC for country $i$.
- Let $\psi_{y,t}^i$ be Lagrangian multipliers on (??) for country $i$.
- Let $\psi_{\pi,t}^*$, $\psi_{y,t}^*$, and $\psi_{f,t}^*$ be the multipliers on the definitions of the aggregates.
First order conditions

For $\pi^i_t$:
$$\left(\frac{\varepsilon}{\lambda}\right) \pi^i_t + \Delta \psi^i_{\pi,t} + \psi^i_{y,t} - \psi^*_{\pi,t} = 0$$

For $y^i_t$:
$$(1 + \varphi) \tilde{y}^*_t - \lambda (1 + \varphi) \psi^i_{\pi,t} + \psi^i_{y,t} - \beta \psi^i_{y,t+1} - \psi^*_y,t = 0$$

For $\tilde{f}^i_t$:
$$\tilde{f}^i_t + \lambda \psi^i_{\pi,t} - \psi^i_{y,t} + \beta \psi^i_{y,t+1} - \left(\frac{1 - \gamma}{\gamma}\right) \psi^*_f,t = 0$$

For $\pi^*_t$:
$$- \int \psi^i_{y,t} di + \psi^*_\pi,t = 0$$

For $y^*_t$:
$$-(1 - \beta L^{-1}) \int \psi^i_{y,t} di + \psi^*_y,t = 0$$

For $\tilde{f}^*_t$:
$$\left(\frac{\gamma}{1 - \gamma}\right) (1 - \beta L^{-1}) \int \psi^i_{y,t} di + \psi^*_f,t = 0$$
First order conditions

- FOCs (timeless precommitment) imply

\[
\left( \frac{\varepsilon}{\lambda} \right) \pi_t^* + \int_0^1 \Delta \psi_{\pi, t}^i \, di \Rightarrow \varepsilon \pi_t^* + \Delta \tilde{y}_t^* = 0
\]

- And

\[
\tilde{f}_t^* + \lambda \int \psi_{\pi, t}^i \, di \Rightarrow \tilde{f}_t^* - \tilde{y}_t^* = \tilde{g}_t^* = 0.
\]

- So \(\varepsilon \pi_t^* + \Delta \tilde{y}_t^*\) and \(\tilde{f}_t^* = \tilde{y}_t^*\) and aggregate NKPC imply that if \(\tilde{y}_0^* = 0\),

\[
\pi_t^* = \tilde{f}_t^* = \tilde{y}_t^* = 0.
\]

- So union wide policy is zero inflation, zero output gap, and zero fiscal gap.
For union member

- The second, third, fifth, and sixth FOC imply

\[(1 + \varphi)\bar{y}_t^i + \bar{f}_t^i = \lambda \varphi \psi_{\pi,t}^i\]

- So, since \(\psi_{\pi,t}^i > 0\) if prices are sticky, setting \(\bar{y}_t^i = \bar{f}_t^i = 0\) for each country is not feasible as an equilibrium under sticky prices.
Implications

- Optimal monetary and fiscal policy from the union perspective would always maintain zero inflation, output gap, and fiscal gap. However, this does not mean inflation, output gap, and the fiscal gap are zero for all \( i \).

- Consider an asymmetric productivity shock:
  - With *flexible prices*, union wide output gap and fiscal gaps could be kept at zero.
  - Individual countries maintain output and government spending at first-best levels while terms of trade adjust in response to relative productivity.
  - With *sticky prices*, relative productivity shocks can’t generate required movement in terms of trade (because prices are sticky and the exchange rate is effectively fixed for members of the currency union).
  - To increase demand in countries with high productivity shocks (i.e., to boost demand to match supply), fiscal policy must expand.
  - Three distortions (output gap, fiscal gap, and inflation), but individual countries only have one instrument – fiscal policy.
Fiscal and monetary interactions

1. Fiscal or monetary dominance
2. Fiscal policy in the new Keynesian model
3. Optimal monetary and fiscal policies.
Several alternative assumptions possible about the relationship between monetary and fiscal policies.

- Fiscal policy assumed to adjust to ensure the government’s intertemporal budget is always in balance, while monetary policy is free to set the nominal money stock or the nominal rate of interest – described as a Ricardian regime (Sargent 1982), monetary dominance, or one with fiscal policy passive and monetary policy active (Leeper 1991).

- The fiscal authority sets its expenditure and taxes without regard to intertemporal budget balance. Seigniorage must adjust to ensure intertemporal budget constraint is satisfied. Case of fiscal dominance (or active fiscal policy) and passive monetary policy.
Fiscal and monetary interactions

The fiscal theory of the price level

The government’s intertemporal budget constraint may not be satisfied for arbitrary price levels. Following Woodford (1995), these regimes are described as non-Ricardian. The intertemporal budget constraint is satisfied only at the equilibrium price level, and the government’s nominal debt plays a critical role in determining the price level.
Intertemporal budget balance and seigniorage

- The intertemporal budget constraint implies that any government with a current outstanding debt must run, in present value terms, future surpluses.
- One way to generate a surplus is to increase revenues from seigniorage.
- Let $s^f_t \equiv t_t - g_t$ be the primary fiscal surplus excluding seigniorage revenue.
- The government’s budget constraint can be written as

$$b_{t-1} = R^{-1} \sum_{i=0}^{\infty} R^{-i} s^f_{t+i} + R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}.$$  (49)

- The current real liabilities of the government must be financed by, in present value terms, either a fiscal primary surplus $R^{-1} \sum_{i=0}^{\infty} R^{-i} s^f_{t+i}$ or by seigniorage.
Intertemporal budget balance and seigniorage

Unpleasant arithmetic

- Sargent and Wallace (1981) – “unpleasant monetarist arithmetic” in a regime of fiscal dominance:
  - If the present value of the fiscal primary surplus is reduced, the present value of seigniorage must rise to maintain intertemporal budget balance.
  - Reducing inflation now can mean higher inflation in the future.
Suppose that the initial nominal stock of money is set exogenously by the monetary authority. Does this mean that the price level is determined solely by monetary policy, with no effect of fiscal policy?

Fiscal policy can still affect the initial equilibrium price level, even when the initial nominal quantity of money is given and the government’s intertemporal budget constraint must be satisfied at all price levels.

It does so if it affects the equilibrium real rate of interest.
Assume perfect foresight equilibrium.

The government’s budget constraint must be satisfied and the real demand for money must equal the real supply of money.

Assume money demand is

\[ \frac{M_t}{P_t} = f(R_{m,t}) , \]  

(50)

Given \( R_m \), (50) implies a proportional relationship between the \( M \) and \( P \). If the initial money stock is \( M_0 \), then the initial price level is \( P_0 = M_0 / f(R_m) \).
The government’s budget constraint given by

\[ g_t + rb_{t-1} = t_t + (b_t - b_{t-1}) + m_t - \left( \frac{1}{1 + \pi_t} \right) m_{t-1}. \] (51)

Consider a stationary equilibrium. The budget constraint becomes

\[ g + \left( \frac{1}{\beta} - 1 \right) b = t + \left( \frac{\pi_t}{1 + \pi_t} \right) m = t + \left( \frac{\beta R_m - 1}{\beta R_m} \right) f(R_m), \] (52)

Suppose the fiscal authority sets \( g, t, \) and \( b \). Then (52) determines the nominal interest rate \( R_m \).

Given the interest rate, \( P_0 \) is given by equation (50) as \( P_0 = M_0 / f(R_m) \), where \( M_0 \) is the initial money stock.
In subsequent periods, the price level is equal to $P_t = P_0 (\beta R_m)^t$ where $\beta R_m = (1 + \pi_t)$ is the gross inflation rate. The nominal stock of money in each future period is endogenously determined by $M_t = P_t f(R_m)$.

Nominal interest rate must be such as to generate enough seigniorage to satisfy the government’s budget constraint.

In this example, even though the monetary authority has set $M_0$ exogenously, the initial price level is determined by the need for fiscal solvency.
Recently, a number of researchers have examined models in which fiscal factors replace the money supply as the key determinant of the price level.

Two ways fiscal policy might matter for the price level:

1. If fiscal variables affect real money demand, the price level will also depend on fiscal factors (see previous example).

2. If there are multiple price levels consistent with a given nominal quantity of money, fiscal policy may determine which of these is the equilibrium price level.
The fiscal theory of the price level

Multiple equilibria

- Assume a constant nominal money supply $M_0$.
- Assume real interest rate fixed (or exogenous to money and prices).
- The equilibrium between the real money supply and real money demand is
  \[
  \frac{M_0}{P_t} = g \left( \frac{P_{t+1}}{P_t} \right), \quad g' < 0.
  \]

- Rewrite this as
  \[
  P_{t+1} = P_t g^{-1} \left( \frac{M_0}{P_t} \right) \equiv \phi(P_t) \quad (53)
  \]

- One (unique, stationary) solution is $P_{t+i} = P^*$ for all $i \geq 0$, where $P^* = M_0 g (1)$.
- In this equilibrium, the quantity theory holds, $P$ is proportional to $M$. 

Carl E. Walsh (UC Santa Cruz)  
Gerzensee Study Center  
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This may not be the only equilibrium.

- Any price path starting at $P_0 = P' > P^*$ is consistent with (53) and involves a speculative hyperinflation.
- Equilibria originating to the left of $P^*$ eventually violate a transversality condition since $M/P$ is exploding as $P \to 0$.
- By itself, equation (53) is not sufficient to unique determine the equilibrium value of $P_0$, even though the nominal quantity of money is fixed.
The fiscal theory of the price level

Multiple equilibria
The fiscal theory of the price level

The basic idea

- If multiple equilibrium price levels are possible, does fiscal policy pin down a unique equilibrium?
- Restrict analysis to perfect-foresight equilibria for simplicity.
- Under standard assumptions, the household intertemporal budget constraint takes the form

\[
d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} (y_{t+i} - \tau_{t+i}) = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ c_{t+i} + \left( \frac{i_{t+i}}{1+i_{t+i}} \right) m_{t+i}^d \right].
\]

(54)

where \( \lambda_{t,t+i} = \prod_{j=1}^{i} \left( \frac{1}{1+r_{t+j}} \right) \) with \( \lambda_{t,t} = 1 \).
The fiscal theory of the price level

The basic idea

- The budget constraint for the government sector, in real terms,

\[ g_t + d_t = \tau_t + \left( \frac{i_t}{1 + i_t} \right) m_t + \left( \frac{1}{1 + r_t} \right) d_{t+1}. \]

Recursively substituting for future values of \( d_{t+i} \), this budget constraint implies

\[ d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ g_{t+i} - \tau_{t+i} - \bar{s}_{t+i} \right] = \lim_{T \to \infty} \lambda_{t,t+T} d_T, \quad (55) \]

where \( \bar{s}_t = i_t m_t / (1 + i_t) \) is the government’s real seigniorage revenue.
The fiscal theory of the price level

The basic idea

Definition

Policy paths for \((g_{t+i}, \tau_{t+i}, s_{t+i}, d_{t+i})_{i \geq 0}\) such that

\[
d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} [g_{t+i} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T \to \infty} \lambda_{t,t+T} d_T = 0
\]

for all price paths \(p_{t+i}, i \geq 0\) are called *Ricardian* policies.

Definition

Policy paths for \((g_{t+i}, \tau_{t+i}, \bar{s}_{t+i}, d_{t+i})_{i \geq 0}\) for which \(\lim_{T \to \infty} \lambda_{t,t+T} d_T\) may not equal zero for all price paths are called *non-Ricardian*.
Now consider a perfect-foresight equilibrium; \( y_t = c_t + g_t \) and \( m_t^d = m_t \). Substituting in (54) and rearranging yields

\[
d_t + \sum_{i=0}^{\infty} \lambda_{t,i} \left[ g_{t+i} - \tau_{t+i} - \left( \frac{i_{t+i}}{1+i_{t+i}} \right) m_{t+i} \right] = 0. \tag{56}
\]

Thus, an implication of the representative household’s optimization problem is that equation (56) must hold in equilibrium.
Under a non-Ricardian policy, budget balance imposes an additional condition that must be satisfied in equilibrium.

This requirement can be written as

\[ \frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ \tau_{t+i} + \bar{s}_{t+i} - g_{t+i} \right]. \]  (57)

At time \( t \), the government’s outstanding nominal liabilities \( D_t \) are predetermined by past policies.

Given the present discounted value of the government’s future surpluses, the only endogenous variable is the current price level \( P_t \). The price level must adjust to ensure equation (57) is satisfied.
Suppose the real demand for money is given by

\[ \frac{M_t}{P_t} = f(1 + i_t). \] (58)

Equations (57) and (58) must both be satisfied in equilibrium.

Which two variables are determined jointly by these two equations depends on the assumptions that are made about fiscal and monetary policy.
The fiscal theory of the price level
Non-Ricardian policies

- Suppose the fiscal authority determines $g_{t+i}$ and $\tau_{t+i}$ for all $i \geq 0$, and the monetary authority pegs the nominal rate of interest $i_{t+i} = \bar{i}$ for all $i \geq 0$.

- Seigniorage is equal to $\bar{i}f(1 + \bar{i})/(1 + \bar{i})$ and so is fixed by monetary policy. With this specification of monetary and fiscal policy, the right side of

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ \tau_{t+i} + \bar{s}_{t+i} - g_{t+i} \right]$$

is given.
The fiscal theory of the price level

Non-Ricardian policies

- Since $D_t$ is predetermined at date $t$, this equation can be solved for the equilibrium price level $P_t^*$ given by

$$P_t^* = \frac{D_t}{\sum_{i=0}^{\infty} \lambda_{t,t+i} [\tau_{t+i} + \bar{s}_{t+i} - g_{t+i}]}.$$ (59)

- The current nominal money supply is then determined by

$$M_t = P_t^* f (1 + \bar{i}).$$

- One property of this equilibrium is that changes in fiscal policy ($g$ or $\tau$) directly alter the equilibrium price level, even though seigniorage is unaffected.
The fiscal theory of the price level
Non-Ricardian policies

- In standard infinite horizon, representative agent models, a tax cut (current and future government expenditures unchanged) has no effect on equilibrium (i.e., Ricardian equivalence holds) – the government cannot engineer a permanent tax cut unless government expenditures are also cut (in present value terms).

- If budget balance holds only when evaluated at the equilibrium price level, the government can plan a permanent tax cut. If it does, the price level must rise to ensure the new, lower value of discounted surpluses is again equal to the real value of government debt.
The fiscal theory of the price level

Empirical evidence

- Under the fiscal theory of the price level, intertemporal budget balance holds at the equilibrium value of the price level.
- Under traditional theories of the price level, it holds for all values of the price level.
- If we only observe equilibrium outcomes, it will be impossible empirically to distinguish between the two theories.
  - As Sims (1994) puts it, “Determinacy of the price level under any policy depends on the public’s beliefs about what the policy authority would do under conditions that are never observed in equilibrium.”
The fiscal theory of the price level

Empirical evidence

- Under the fiscal theory of the price level, intertemporal budget balance holds at the equilibrium value of the price level.
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The fiscal policy in a neoclassical model

- Suppose government imposes lump-sum taxes to finance exogenous stream of expenditures.
- A rise in government expenditures has a negative wealth effect on the private sector because taxes must be raised.
- This increases labor supply (so output increases) and reduces private consumption.
- With distorting taxes, steady-state equilibrium output is reduced by taxes.
The fiscal policy in a neoclassical model: example

- If \( Y_t = A_t H_t^\alpha, H_t = f^{-1}(Y_t) \Rightarrow H_t = (A_t^{-1} Y_t)^{\frac{1}{\alpha}}. \)
- If \( U = C_t^{1-\sigma}/(1-\sigma) - H_t^{1+\phi}/(1+\phi). \) labor supply condition is \( MRS = MPL \) or

\[
(A_t^{-1} Y_t)^{\frac{\phi}{\alpha}} C_t^{-\sigma} = \alpha A_t^{\frac{1}{\alpha}} Y_t^{1-\frac{1}{\alpha}} \Rightarrow C_t^{\sigma} = \alpha A_t^{\frac{1+\phi}{\alpha}} Y_t^{1-\frac{1+\phi}{\alpha}}
\]

- Goods market clearing condition is

\[
Y_t = C_t + G_t.
\]

- These imply

\[
(Y_t - G_t)^{-\sigma} = \alpha A_t^{\frac{1+\phi}{\alpha}} Y_t^{1-\frac{1+\phi}{\alpha}} \Rightarrow \frac{dY_t}{dG_t} = \frac{1}{1 + \left( \frac{C}{Y} \right) \left( \frac{1+\alpha\phi}{\sigma\alpha} \right)} < 1.
\]
Fiscal policy in a new Keynesian model

- Basic building blocks of the new Keynesian model:
  - Euler condition for optimal consumption:
    \[ c_t = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) \]
  - Inflation adjustment:
    \[ \pi_t = \beta E_t \pi_{t+1} + \bar{\kappa} m c_t \]
  - Marginal costs:
    \[ mc_t = \sigma c_t + \eta n_t - z_t. \]
Fiscal policy in a new Keynesian model

- With government spending, \( Y_t = C_t + G_t \), or
  \[
  c_t = \left( \frac{Y}{C} \right) y_t - \left( \frac{G}{C} \right) g_t.
  \]

- So model becomes
  \[
  \left( \frac{Y}{C} \right) y_t - \left( \frac{G}{C} \right) g_t = \left( \frac{Y}{C} \right) E_t y_{t+1} - \left( \frac{G}{C} \right) E_t g_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1})
  \]
  or
  \[
  x_t \equiv y_t - y_t^{\text{flex}} = E_t (y_{t+1} - y_{t+1}^{\text{flex}}) - \left( \frac{1}{\sigma} \right) \left( \frac{C}{Y} \right) (i_t - E_t \pi_{t+1})
  \]
  \[
  - \left( \frac{G}{Y} \right) (E_t g_{t+1} - g_t) + E_t y_{t+1}^{\text{flex}} - y_t^{\text{flex}},
  \]
Fiscal policy in a new Keynesian model

- We can write this as

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\bar{\sigma}} \right) (i_t - E_t \pi_{t+1} - r^n_t), \]

where

\[ r^n_t = -\bar{\sigma} \left( \frac{G}{C} \right) (E_t g_{t+1} - g_t) + \bar{\sigma} \left( E_t y^f_{t+1} - y^f_t \right). \]

- So fiscal shocks affect the IS curve.
Fiscal policy in a new Keynesian model

- Fiscal shocks can also affect inflation via marginal cost.
- If $Y_t = e^{z_t} N_t$, marginal cost is $mc_t = \sigma c_t + \eta n_t - z_t$, or
  \[
  mc_t = \bar{\sigma} \left[ y_t - \left( \frac{G}{Y} \right) g_t \right] + \eta (y_t - z_t) - z_t,
  \]
  or
  \[
  mc_t = (\bar{\sigma} + \eta) y_t - \bar{\sigma} \left( \frac{G}{Y} \right) g_t - (1 + \eta) z_t = (\bar{\sigma} + \eta) \left( y_t - y_t^{\text{flex}} \right),
  \]
  where
  \[
  y_t^{\text{flex}} = \frac{\bar{\sigma} \left( \frac{G}{Y} \right) g_t + (1 + \eta) z_t}{\bar{\sigma} + \eta}.
  \]
Fiscal policy in a new Keynesian model

- IS equations becomes

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\bar{\sigma}} \right) \left( i_t - E_t \pi_{t+1} - r_t^n \right), \]

which is the same as before, but \( r^n \) depends on \( E_t g_{t+1} - g_t \).

- Inflation equation becomes

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \]

where \( \kappa = \bar{\kappa} (\sigma + \eta) \), which is also the same as before.
Fiscal policy in a new Keynesian model

- But what about the loss function?
  - If loss function is $\pi_t^2 + \lambda_x x_t^2$, then $g$ only becomes a source of real rate shocks.
    - In this case, optimal monetary policy will neutralize the effects of $g$ on the output gap and inflation.
    - Flexible-price output is still affected via neoclassical channels.
  - If loss is $\pi_t^2 + \lambda_x (y_t - y_t^*)^2$ with $y_t^* \neq y_t^{\text{flex}}$, then things are different.
- So key issue is how fiscal policy affects the efficient level of output $y^*$.
Fiscal policy and monetary policy

Optimal policy when flex-price and efficient output differ

- Policy problem is to

\[
\min E_t \sum_{i=0}^{\infty} \beta^i \left\{ \pi^2_{t+i} + \lambda_x (x_{t+i} - x^*_{t+i})^2 + \theta_{t+i} (\pi_t - \beta_t \pi_{t+1} - \kappa x_t) \right\},
\]

where \( x \equiv y - y^{\text{flex}} \) and \( x^* = y^* - y^{\text{flex}} \).

- First order conditions (timeless perspective):

\[
\pi_t + \theta_t - \theta_{t-1} = 0;
\]

\[
\lambda_x (x_t - x^*_t) - \kappa \theta_t = 0.
\]

- Combining,

\[
\pi_t = - \left( \frac{\lambda_x}{\kappa} \right) \Delta (x_t - x^*_t).
\]
Fiscal policy and monetary policy
Optimal policy when flex-price and efficient output differ

- Suppose $y_t^* = (1 + \eta)z_t / (\tilde{\sigma} + \eta)$.
- Then
  
  $$y_t^{\text{flex}} - y_t^* = \frac{\tilde{\sigma} \left( \frac{G}{Y} \right) g_t + (1 + \eta)z_t}{\tilde{\sigma} + \eta} - \frac{(1 + \eta)z_t}{\tilde{\sigma} + \eta} = \left( \frac{\tilde{\sigma}}{\tilde{\sigma} + \eta} \right) \left( \frac{G}{Y} \right) g_t =$$

- So FOC becomes

  $$\pi_t = - \left( \frac{\lambda_x}{\kappa} \right) \Delta (x_t + \gamma g_t).$$

- For optimal discretion,

  $$\pi_t = - \left( \frac{\lambda_x}{\kappa} \right) (x_t + \gamma g_t).$$
Fiscal policy at the ZLB

- Suppose monetary policy cannot neutralize fiscal policy, for example because interest rates are at the ZLB.
- Argument is that at ZLB, fiscal policy is more powerful.
- At ZLB, old fashion Keynesian multiplier returns.
  - No crowding out since real interest rate does not rise.
  - Output expansion raises marginal cost and inflation, lowering the real interest rate, so a form of crowding in occurs.
Output at the ZLB

- Assume \( i = 0 \) and with probability \( \mu \) economy is still at ZLB in following period. If economy exits ZLB, output and inflation are zero.

- Equilibrium given by solutions to

\[
y^Z = \mu y^Z - \left( \frac{1}{\sigma} \right) \left( -\mu \pi^Z - r^n \right)
\]

\[
\Rightarrow y^Z = \left( \frac{1}{1 - \mu} \right) \left( \frac{1}{\sigma} \right) \left( \mu \pi^Z + r^n \right)
\]

\[
\pi^Z = \beta \pi^Z + \kappa y^Z
\]

where \( \mu y^Z \) and \( \mu \pi^Z \) are expected future output and inflation.
Eggertsson (2010) – adds wage and sales taxes (plus other taxes)

Marginal cost:

\[ mc_t = (w_t - p_t) - mpl_t \]

\[
\begin{align*}
th_{\text{after tax}} - p_t^{\text{cpi}} &\approx (w_t - \tau^w_t) - (p_t + \tau^s_t) \\
&= \eta n_t + \sigma \left[ \left( \frac{Y}{C} \right) y_t - \left( \frac{G}{C} \right) g_t \right]
\end{align*}
\]

so

\[
\begin{align*}
mc_t &= \eta (y_t - z_t) + \sigma \left[ \left( \frac{Y}{C} \right) y_t - \left( \frac{G}{C} \right) g_t \right] + \tau^s_t + \tau^w_t - z_t \\
&= \left[ \eta + \sigma \left( \frac{Y}{C} \right) \right] y_t - \sigma \left( \frac{G}{C} \right) g_t + \tau^s_t + \tau^w_t - (1 + \eta) z_t
\end{align*}
\]
Adding taxes:

- The NKPC becomes

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \left\{ \eta + \sigma \left( \frac{Y}{C} \right) y_t - \sigma \left( \frac{G}{Y} \right) g_t + \tau_s^t + \tau_w^t - (1 + \eta) z_t \right\}
\]

- Note that a rise in the wage tax or sales tax increases marginal costs.
- So wage tax cuts are deflationary
Adding taxes:

- Aggregate demand:

\[
y_t = E_t y_t - \left( \frac{1}{\sigma} \right) \left( i_t - E_t \pi_{t+1}^{cpi} - r_t^n \right) - \left( \frac{G}{Y} \right) (E_t g_{t+1} - g_t) \\
= E_t y_t - \left( \frac{1}{\sigma} \right) \left( i_t - E_t \pi_{t+1} - r_t^n \right) - \left( \frac{G}{Y} \right) (E_t g_{t+1} - g_t) \\
+ \left( \frac{1}{\sigma} \right) (E_t \tau_{t+1}^s - \tau_t^s)
\]
Adding taxes: fiscal policy at the ZLB

Now consider the case of the ZLB. Equilibrium is given by

\[ \pi_t^Z = \beta E_t \pi_{t+1} + \kappa \left[ \eta + \sigma \left( \frac{Y}{C} \right) \right] y_t^Z - \kappa \sigma \left( \frac{G}{Y} \right) g_t + \kappa (\tau^s + \tau^w) - \kappa (1 + \eta) z_t \]

\[ y_t^Z = E_t y_{t+1} + \left( \frac{1}{\sigma} \right) \left( \mu \pi_{t+1}^Z + r^*_t \right) - \left( \frac{G}{Y} \right) (E_t g_{t+1} - g_t) + \left( \frac{1}{\sigma} \right) (E_t \tau^s_{t+1} - \tau^s) \]

Expansionary policies:
- temporary sales tax cut (boosts demand and inflation)
- temporary rise in government spending (boosts demand but lowers inflation) (possibly)
- Expansionary policies: increase in wage tax (raises inflation and demand)
Optimal taxation with distortionary taxes

Tax smoothing

- Review of Barro’s result on optimal intertemporal taxation
- Equalize marginal distortionary costs per dollar of revenue across
  - tax instruments
  - time
If there are two tax rates $\tau_1$ and $\tau_2$, and distortions per dollar raises are $D(\tau_1, \tau_2)$,

$$\frac{\partial D(\tau_1 t, \tau_2 t)}{\partial \tau_1 t} = \frac{\partial D(\tau_1 t, \tau_2 t)}{\partial \tau_2 t}$$

and

$$\frac{\partial D(\tau_1 t, \tau_2 t)}{\partial \tau_1 t} = E_t \left[ \frac{\partial D(\tau_1 t+1, \tau_2 t+1)}{\partial \tau_1 t+1} \right].$$

If $D$ is quadratic, this second condition implies

$$\tau_{jt} = E_t \tau_{jt+1} \Rightarrow \tau_{jt+1} = \tau_{jt} + \xi_{jt+1}$$
Distortionary taxes

Tax smoothing

- Tax rates follow random walks.
- Level is set to finance expected present discounted value of government expenditures.
- This is like a permanent income model of taxes
  - Taxes are like consumption – smooth them and use debt to finance transitory fluctuations in spending.
Suppose flexible-price equilibrium output depends negatively on tax on final output:

\[ y_t^{\text{flex}} = -\psi \tau_t + \varepsilon_t. \]

Inflation adjustment equation is

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \left( y_t - y_t^{\text{flex}} \right). \]

Suppose \( y_t^* \) is the efficient or welfare maximizing output. Loss function is

\[ E_t \sum_{i=0}^{\infty} \beta^i \left\{ \pi_{t+i}^2 + \lambda_x (y_{t+i} - y_{t+i}^*)^2 \right\}. \]
Government budget constraint

- Assume level of nominal debt (as fraction of GDP) constant at $\bar{d}$.
- Let $b$ be public debt as fraction of GDP.
- Then

\[
\frac{D_{t-1}}{P_t} + g_t = \tau_t + z_t + \left( \frac{1}{1 + R_t} \right) \frac{D_t}{P_t},
\]

where $z$ represents (potential) lump-sum transfers.

- This can be approximated by

\[
b_{t-1} + g_t = \tau_t + z_t + \bar{d} (\pi_t - \beta E_t \pi_{t+1}) + \beta b_t
\]

since $1/P_t \approx (1 - \pi_t)/P_{t-1}$ and $(1 + R_t)^{-1} \approx \beta (1 - E_t \pi_{t+1})$ since $\beta$ is equal to the steady-state inverse of the gross real rate of interest.
Ramsey optimal policy problem

- The joint fiscal and monetary policy becomes

\[
\min_{\pi, y, \tau} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \pi_{t+i}^2 + \lambda_x (y_{t+i} - y_{t+i}^*)^2 \right\}
\]

subject to

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t + \psi \tau_t - \varepsilon_t).
\]

and

\[
b_{t-1} + g_t = \tau_t + z_t + \bar{d} (\pi_t - \beta E_t \pi_{t+1}) + \beta b_t
\]
First order conditions

- The first order conditions with respect to $y_t$, $\pi_t b_t$, and $\tau_t$ (timeless perspective) are

  \[
  \lambda_x (y_t - y_t^*) - \kappa \theta_t = 0; \\
  \pi_t + \theta_t - \theta_{t-1} - \bar{d} (\varphi_t - \varphi_{t-1}) = 0; \\
  \varphi_t - E_t \varphi_{t+1} = 0; \\
  -\kappa \psi \theta_t - \varphi_t = 0,
  \]

  where $\theta$ and $\varphi$ are the Lagrangians on the inflation adjustment equation and the budget constraint.
First order conditions

- These FOCs imply that $\theta_t = (\lambda_x / \kappa)(y_t - y^*_t)$ and $\varphi_t = -\kappa \psi \theta_t$.
- Hence, we can write the FOCs as

$$y_t - y^*_t = \left(\frac{\kappa}{\lambda_x}\right) \theta_t = -\left(\frac{1}{\psi \lambda_x}\right) \varphi_t;$$

$$\pi_t = \left(\frac{1}{\psi \lambda_x}\right) \left(\frac{\lambda_x}{\kappa} + \kappa \psi \bar{d}\right) \Delta \varphi_t;$$

$$\varphi_t = E_t \varphi_{t+1}.$$ 

- Output gap and inflation are functions of the shadow cost of government budget resources and this cost is a random walk.
Implications

Unrestricted lump-sum taxes

Suppose z is unrestricted

- First best – if there are no restrictions on z, optimal choice implies $\varphi_t = 0$, so $\pi_t = 0$ and $y_t = y_t^{\text{flex}} = y^*_t$.
- With $y = y^*$,

$$y_t = y^*_t = -\psi \tau_t + \varepsilon_t \Rightarrow \tau_t = -\left(\frac{1}{\psi}\right) (y^*_t - \varepsilon_t) \equiv \tau^*_t,$$

where $\tau^*_t$ is the tax (or subsidy) needed to ensure $y^{\text{flex}} = y^*$.
- Fiscal policy used to stabilize output at $y^*_t$; inflation kept equal to zero.
Implications

Restricted lump-sum taxes

- Suppose lump-sum transfers are not available: \( z_t \equiv 0 \).
- From FOC’s, \( \varphi_t = E_t \varphi_{t+1}, E_t \Delta \varphi_{t+1} = 0 \), so from

\[
\pi_t = \left( \frac{1}{\psi \lambda_x} \right) \left( \frac{\lambda_x}{\kappa} + \kappa \psi \bar{d} \right) \Delta \varphi_t
\]

it follows that \( E_t \pi_{t+1} = 0 \).
- Hence, the government’s budget constraint becomes

\[
b_{t-1} + g_t = \tau_t + \bar{d} \pi_t + \beta b_t
\]

- Solving forward,

\[
(1 - \beta) b_{t-1} = (1 - \beta) \bar{d} \pi_t - f_t + (1 - \beta) \sum_{i=0}^{\infty} \beta^i E_t (\tau_{t+i} - \tau_{t+i}^*)
\]

where

\[
f_t = (1 - \beta) \sum_{i=0}^{\infty} \beta^i E_t (g_{t+i} - \tau_{t+i}^*)
\]
Implications
Restricted lump-sum taxes

- The variable $f_t$ is the annuity value of government spending plus the subsidy necessary to implement the first best.

Implications: restricted $z$

- FOC for $y_t$ implies

$$y_t - y_t^* = -\psi \tau_t + \varepsilon_t - y_t^* = -\psi (\tau_t - \tau_t^*) = - \left( \frac{1}{\psi \lambda_x} \right) \varphi_t.$$

- This implies

$$E_t (\tau_{t+i} - \tau_{t+i}^*) = (1/\psi^2 \lambda_x) E_t \varphi_{t+i} = (1/\psi^2 \lambda_x) \varphi_t = \tau_t - \tau_t^*.$$
Implications
Restricted lump-sum taxes

Hence,

\[(1 - \beta) \sum_{i=0}^{\infty} \beta^i E_t (\tau_{t+i} - \tau^*_{t+i}) = (1 - \beta) \sum_{i=0}^{\infty} \beta^i (\tau_t - \tau^*_t) = \tau_t - \tau^*_t.\]

Therefore,

\[\phi_t = \psi^2 \lambda_x (\tau_t - \tau^*_t) = (\psi^2 \lambda_x) [(1 - \beta) b_{t-1} + f_t - (1 - \beta) \bar{d} \pi_t] \]
Taking expectations as of $t - 1$:

$$\varphi_{t-1} = E_{t-1} \varphi_t = (\psi^2 \lambda_x) \left[ (1 - \beta) b_{t-1} + E_{t-1} f_t \right]$$

So

$$\varphi_t - \varphi_{t-1} = \Delta \varphi_t = (\psi^2 \lambda_x) \left[ (f_t - E_{t-1} f_t) - (1 - \beta) \bar{d} \pi_t \right]$$

From FOCs,

$$\pi_t = \left( \frac{1}{\psi \lambda_x} \right) \left( \frac{\lambda_x}{\kappa} + \kappa \psi \bar{d} \right) \Delta \varphi_t = A \Delta \varphi_t.$$
Implications
Restricted lump-sum taxes

Hence,

\[
\Delta \varphi_t = (\psi^2 \lambda_x) \left[ (f_t - E_{t-1} f_t) - (1 - \beta) \bar{d}A \Delta \varphi_t \right]
\]

\[
= \left( \frac{\psi^2 \lambda_x}{1 + \psi^2 \lambda_x (1 - \beta) \bar{d}A} \right) (f_t - E_{t-1} f_t)
\]

\[
= \eta (f_t - E_{t-1} f_t)
\]

where

\[
\eta = \left( \frac{\psi^2 \lambda_x}{1 + \psi^2 \lambda_x (1 - \beta) \bar{d}A} \right)
\]
Implications
Restricted lump-sum taxes

Inflation, output, and the tax rate satisfy

\[ \pi_t = A \eta (f_t - E_{t-1} f_t) \]

\[ \Delta (y_t - y_t^*) = - \left( \frac{1}{\psi \lambda_x} \right) \Delta \varphi_t = - \left( \frac{1}{\psi \lambda_x} \right) \eta (f_t - E_{t-1} f_t) \]

\[ y_t - y_{t-1} = y_t^* - y_{t-1}^* - \left( \frac{1}{\psi \lambda_x} \right) \eta (f_t - E_{t-1} f_t) \]

\[ \tau_t - \tau_{t-1} = \tau_t^* - \tau_{t-1}^* + \left( \frac{1}{\psi^2 \lambda_x} \right) \eta (f_t - E_{t-1} f_t) \]
Implications

Summary

- Inflation is a white noise process, depending on unexpected shifts in the fiscal variable $f$.
- Expected future inflation always equal to zero.
- The output gap and the tax rate follow martingales plus a component related to fluctuations in the first best level of output.
- Both monetary and fiscal policy need to be used, but shocks are not completely stabilized.
Conclusions on monetary and fiscal policy

- Integration of monetary and fiscal policy in optimizing models with nominal rigidities is new area of work.
- Intertemporal budget constraint links monetary and fiscal decisions.
- Fiscal theory reminds us that monetary policy cannot ignore fiscal policy.
- New Keynesian framework can be expanding to study joint determination of optimal policy.
Summing up

Evolving views: then

- policy as systematic

  ...[equilibrium methods] will focus attention on the need to think of policy as the choice of stable rules of the game, well understood by economic agents. Only in such a setting will economic theory help predict the actions agents will choose to take. (Lucas and Sargent 1978)

- but unpredictable

  ..  the government countercyclical policy must itself be unforeseeable by private agents...while at the same time be systematically related to the state of the economy. Effectiveness, then, rests on the inability of private agents to recognize systematic patterns in monetary and fiscal policy. (Lucas and Sargent 1978).
Summing up
Evolving views: now

- policy as systematic *and* predictable

...the central bank’s stabilization goals can be most effectively achieved only to the extent that the central bank not only acts appropriately, but is also understood by the private sector to predictably act in a certain way. The ability to successfully steer private-sector expectations is favored by a decision procedure that is based on a rule, since in this case the systematic character of the central bank’s actions can be most easily made apparent to the public. (Woodford 2003, p. 465)