Thursday Lecture 1
The Maturity Rat Race

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Is maturity transformation necessary?

- Households have long-term saving needs
- Firms have long-term borrowing needs
- Why do banks use short-term liabilities to fund long-term assets?
- Bryant (1980); Diamond and Dybvig (1983)
- Calomiris and Kahn (1991); Diamond and Rajan (2001)
- Brunnermeier and Oehmke argue that “there may be excessive mismatch in the financial system”
A borrower raises financing for a long-term project
  ▶ from multiple creditors
  ▶ at (potentially) different maturities

Short-term financing is favored because
  ▶ shorter maturities provide “seniority” by diluting longer maturity claims
  ▶ depending on the type of information received at rollover dates

Mechanism can apply generally but is particularly relevant for financial intermediaries

Successively unravels all long-term financing and leads to a *Maturity Rat Race*
The model

- Long-term project
  - investment at $t = 0$: $1$
  - payoff at $t = T$: $\theta \sim F(\cdot)$ on $[0, \bar{\theta}]$.

- Firm is risk neutral, there is no discounting and

$$NPV = E[\theta] - 1 = \int \theta dF - 1 > 0.$$  

- Information is learned over time:
  - $s_t$ is observed at date $t = 1, ..., T$
  - $S_t$ is a sufficient statistic for the history $\{s_1, ..., s_t\}$
  - $S_t$ orders $F(\cdot)$ according to FOSD

- Premature liquidation is costly: $\lambda E[\theta|S_t]$, $\lambda < 1$
Credit markets

- The firm has no equity capital and raises financing by issuing debt to a large number of small, risk-neutral lenders.
- A debt contract specifies a face value and a date at which the debt is due. A debt contract of maturity $T$ is referred to as a long-term contract. The face value of $t$-period debt issued at date 0 is denoted by $D_{0,t}$. The face value of $\tau$-period debt issued at date $t$ is denoted by $D_{t,t+\tau}(S_t)$.
- If the firm defaults at $t$, all debt is accelerated and there is equal priority among holders of different maturities of debt. There is no distinction between principal and accrued interest.
- The firm’s maturity structure is opaque. This makes sense for financial institutions, but not for industrial companies.
The borrower (firm) simultaneously offers debt contracts to creditors. Bilateral contracts are not conditioned on the contracts of other creditors.

**Definition**

An equilibrium maturity structure must satisfy two conditions:

(i) zero profits for lenders;
(ii) no incentive for firm to deviate from equilibrium maturity structure by forming a coalition with a lender, holding all else constant.
For simplicity, consider the special case $T = 2$. Suppose a fraction $\alpha$ enter into short-term debt contracts, i.e., that mature at $t = 1$.

To roll over short term debt, the firm must issue new debt with face value $D_{1,2}(S_1)$ that has the same value as the maturing debt $D_{0,1}$. Thus,

$$\int_{0}^{\tilde{D}_2(S_1)} \frac{D_{1,2}(S_1)}{\tilde{D}_2(S_1)} \theta F(d\theta|S_1) + D_{1,2}(S_1) \int_{\tilde{D}_2(S_1)}^{\infty} F(d\theta|S_1) = D_{0,1}, \quad (1)$$

for every value of $S_1$, where

$$\tilde{D}_2(S_1) = \alpha D_{1,2}(S_1) + (1 - \alpha) D_{0,2}.$$
Breakeven conditions II

- It is impossible to roll over the short-term debt when the total conditional return is less than the face value of the short-term debt:

\[ \alpha D_{0,1} > E[\theta|S_1] = \int_{0}^{\infty} \theta F(d\theta|S_1). \]  

(2)

- Let \( \tilde{S}_1(\alpha) \) denote the signal at which (2) is just satisfied:

\[ \alpha D_{0,1} = E[\theta|\tilde{S}_1(\alpha)]. \]  

(3)

When the signal \( S_1 < \tilde{S}(\alpha) \), the firm cannot roll over its obligations and dispersed lenders join in an uncoordinated ‘run.’

- The face value of short-term debt at \( t = 0, D_{0,1} \), is determined by the creditors’ zero-profit condition, assuming early liquidation if \( S_1 < \tilde{S}(\alpha) \),

\[ 1 = \int_{S_L}^{\tilde{S}(\alpha)} \lambda E[\theta|S_1] dG(S_1) + \left[ 1 - G(\tilde{S}(\alpha)) \right] D_{0,1}. \]  

(4)
Breakeven conditions III

- Now consider the zero-profit condition for the long-term creditors. There are three possible outcomes:
  - early liquidation, in which case long-term creditors receive $\lambda E[\theta|S_1]$;
  - default at time $t = 2$, in which case they receive $D_{0,2}\theta/\bar{D}_2(S_1)$;
  - and no default, in which case they receive $D_{0,2}$.

- Thus,

$$
1 = \int_{S_L}^\delta \alpha \lambda E[\theta|S_t] \, dG(S_t) + \\
\int_0^\delta \alpha \frac{D_{0,2}}{\bar{D}_2(S_1)} \theta F(d\theta|S_1) + D_{0,2} \int_0^\delta \frac{F(d\theta|S_1)}{\bar{D}(S_1)} \, dG(S_1),
$$
Expected profit for the firm is

\[ \Pi = \int_{\tilde{S}(\alpha)} \int_{\tilde{D}_2(S_1)} \left[ \theta - \tilde{D}_2(S_1) \right] F(d\theta|S_1) \, dG(S_1), \quad (5) \]

To calculate the optimal maturity structure, consider the effect of a small change in \( \alpha \), bearing in mind that

\[ \tilde{D}_2(S_1) = \alpha D_{1,2}(S_1) + (1 - \alpha) D_{0,2}. \]

Differentiating with respect to \( \alpha \) yields

\[ \frac{\partial \Pi}{\partial \alpha} = \int_{\tilde{S}(\alpha)} \int_{\tilde{D}_2(S_1)} \left[ D_{0,2} - D_{0,2}(S_1) \right] F(d\theta|S_1) \, dG(S_1). \quad (6) \]
Optimum

- An optimum occurs where either
  \[
  \frac{\partial \Pi}{\partial \alpha} = 0,
  \]
  or
  \[
  \frac{\partial \Pi}{\partial \alpha} > 0 \text{ and } \alpha = 1,
  \]
  or
  \[
  \frac{\partial \Pi}{\partial \alpha} < 0 \text{ and } \alpha = 0.
  \]

- Since short-term finance exposes the firm to the risk of early liquidation, the efficient maturity structure is to have 100% long-term debt. But this may not be an equilibrium. Intuition?
Example: News about probability of default

Suppose \( \theta \) takes only two values

- \( \theta^H \) with probability \( p \)
- \( \theta^L \) with probability \( 1 - p \)

\( \theta^L < 1 \) and \( \theta^H \) is high enough to avoid default

\( p \) is random and is revealed at \( t = 1 \)

If all finance is long term, i.e., \( \alpha = 0 \),

\[
(1 - p_0) \theta^L + p_0 D_{0,2} = 1
\]

\[
D_{0,2} = \frac{1 - (1 - p_0) \theta^L}{p_0}
\]
Rollover

- Breakeven condition for \( t = 1 \) rollover creditor:

\[
(1 - p_1) \frac{D_{1,2}}{D_{0,2}} \theta^L + p_1 D_{1,2} = 1,
\]

or

\[
D_{1,2} = \left[ (1 - p_1) \frac{\theta^L}{D_{0,2}} + p_1 \right]^{-1} = \frac{D_{0,2}}{(1 - p_1) \theta^L + p_1 D_{0,2}} = \frac{1 - (1 - p_0) \theta^L}{p_0 (1 - p_1) \theta^L + p_1 \left( 1 - (1 - p_0) \theta^L \right)}.
\]
Deviation payoff

From (6) we know there is an incentive to deviate if

$$\frac{\partial \Pi}{\partial \alpha} = \int \tilde{S}(\alpha) \int_{D_2(S_1)} [D_{0,2} - D_{0,2}(S_1)] F(d\theta|S_1) \, dG(S_1)$$

$$= p_0 D_{0.2} - E[p_1 D_{1,2}]$$

$$= \frac{p_0}{p_0} \frac{1 - (1 - p_0) \theta^L}{p_0} - E \left[ p_1 \frac{1 - (1 - p_0) \theta^L}{p_0 \theta^L + p_1 (1 - \theta^L)} \right]$$

$$= 1 - (1 - p_0) \theta^L \left\{ 1 - E \left[ \frac{p_1}{p_0 \theta^L + p_1 (1 - \theta^L)} \right] \right\} > 0.$$ 

Thus,

$$\frac{\partial \Pi}{\partial \alpha} > 0 \iff 1 > E \left[ \frac{p_1}{p_0 \theta^L + p_1 (1 - \theta^L)} \right].$$
Promised face values and probability of repayment
The integrand is a strictly concave function when $0 < \theta^L < 1$, so application of Jensen’s inequality shows that

$$E \left[ \frac{p_1}{p_0 \theta^L + p_1 (1 - \theta^L)} \right] < \frac{E [p_1]}{p_0 \theta^L + E [p_1] (1 - \theta^L)} = 1,$$

since $E [p_1] = p_0$.

Note the argument doesn’t work in the extreme cases where $\theta^L = 0, 1$. 
Multiplying face value and probability of repayment

\[ A > B \] implies rollover is cheaper in expectation.
Counter-example: News about recovery value

- Again $\theta$ only takes two values
  - $\theta^H$ with probability $p = \frac{1}{2}$
  - $\theta^L$ with probability $1 - p$

- the low cash flow $\theta^L$ is random and the highest value is less than one, so default occurs for sure when $\theta = \theta^L$.

- If all financing is long term ($\alpha = 0$),
  \[
  \frac{1}{2} D_{0,2} + \frac{1}{2} E \left[ \theta^L \right] = 1, \quad D_{0,2} = 2 - E \left[ \theta^L \right].
  \]

- The breakeven condition at the rollover date $t = 1$ is
  \[
  \frac{1}{2} D_{1,2} + \frac{1}{2} D_{0,2} \theta^L = 1, \quad D_{1,2} = 2 \left( \frac{2 - E \left[ \theta^L \right]}{2 - E \left[ \theta^L \right] + \theta^L} \right)
  \]
Deviation condition

- The necessary condition for a deviation is
  \[
  \frac{\partial \Pi}{\partial \alpha} = \frac{1}{2} D_{0,2} - \frac{1}{2} E \left[ D_{1,2} \left( \theta^L \right) \right] > 0
  \]

- But \( D_{1,2} \left( \theta^L \right) \) is a decreasing, convex function of \( \theta^L \), which implies that
  \[
  E \left[ D_{1,2} \left( \theta^L \right) \right] > E \left[ D_{1,2} \left( \theta^L \right) \right] = D_{0,2}.
  \]

So there is no incentive to deviate.

- The product of two quantities matters: Promised face value under ST and LT debt and probability that face value is repaid.
Multiplying promised face value and repayment probability

Note:

$A' < B'$ implies rolling over more expensive in expectation
One-step Deviation

Assumption 1 \( D_{1,2} (S_1) \int_{D_2(S_1)}^{\infty} F (d\theta|S_1) \) is weakly increasing in \( S_1 \) on the interval \( S_1 \geq \tilde{S}_1 (\alpha) \).

The assumption implies that it is cheaper to compensate a short-term lender in states \( S_1 \) where the firm is more likely to be the residual claimant.

**Theorem**

*Suppose that Assumption 1 holds. Then in any conjectured equilibrium maturity structure with some amount of long-term financing, \( \alpha \in [0,1) \), the financial institution has an incentive to increase the amount of short-term financing by switching one additional creditor from maturity 2 to the shorter maturity 1, since*

\[
\frac{\partial \Pi}{\partial \alpha} > 0.
\]

*The unique equilibrium maturity structure involves all short-term financing.*
Assumption 2 \( D_{t-1,t} \left( S_{t-1} \right) \int_{\tilde{S}_t}^{\infty} dG \left( S_t \mid S_{t-1} \right) \) is weakly increasing in \( S_{t-1} \) on the interval \( S_{t-1} \geq \tilde{S}_{t-1} \).

**Theorem**

*Suppose that Assumption 2 holds. When many rollover dates are possible, successive application of the one-step deviation principle results in a complete unraveling of the maturity structure to the minimum rollover interval.*
Figure 1: **Illustration of the Maturity Rat Race.** Start in a conjectured equilibrium in which all financing has maturity $T$ (dashed line). In that case there is a profitable deviation to move some creditors to and initial maturity of $T - 1$ and then roll over from $T - 1$ to $T$. However, once all creditors’ initial maturity is $T - 1$, there is an incentive to move some creditors to an initial maturity of $T - 2$. The process repeats until all financing has the shortest possible maturity and is rolled over from period to period.
Conclusions

- An example of *seniority through maturity*
- Information about Probability of Default
- Information about Loss Given Default
- Only one Rat in the Race
  - Investors receive the opportunity cost of funds
  - In the Rat Race case, it is the firm that deviates out of equilibrium ...
  - ... but in equilibrium the firm appears to have cut its own throat
- Brunnermeier and Oehmke argue (convincingly) that banks’ funding is opaque and easily changed ...
  - ... but there is nothing to stop them from setting up SIVs with long-term funding